Ref.TH.2789-CERN

ON THE DEEP INELASTIC STRUCTURE FUNCTIONS
OF π 's AND K's

P. Castorina and G. Nardulli
Istituto di Fisica dell'Università di Bari
and
Sezione INFN di Bari

and

G. Preparata
CERN - Geneva
and
Istituto di Fisica dell'Università di Bari

A B S T R A C T

We show that in the framework of the massive quark model a very accurate description of meson deep inelastic structure functions is obtained in terms of only one parameter. No other theoretical approach can claim to represent the data as accurately.

Ref.TH.2789-CERN

17 December 1979

Scaling and its violations in deep inelastic lepton-hadron scattering are rightly considered today as one of the key issues of sub-nuclear physics. The peculiar scaling behaviour, which was first observed more than ten years ago, has given a strong impulse to the belief that the world of hadrons is correctly described by a field theory of coloured quarks and gluons, quantum chromodynamics (QCD). Prompted by the scaling phenomenon, asymptotic freedom (AF) - with its theoretical apparatus based on the notions of the renormalization group - is the basis for ambitious attempts to unify all known interactions, through the so-called grand unified theories. Thus, at the present time, it is particularly important to be able to assess, with a high degree of confidence, whether asymptotic freedom is really observed in the experimental data.

In order to provide for a strong "theoretical contrast" to the problems raised by perturbative QCD, and its attending AF, we thought it appropriate to carry out a thorough and up-to-date analysis ^{1),2)} of the experimental information available on deep inelastic processes within the theoretical framework of the massive quark model (MQM) ³⁾.

From the results we have obtained so far it has become quite clear that the new generation experiments on deep inelastic scattering will have to provide us with data so precise that we will be able to discriminate between the logarithmic scaling violations predicted by AF, and the MQM subasymptotic corrections whose effects die out like $1/\sqrt{Q^2}$.

However, at this stage of our knowledge, one thing has clearly emerged from our analysis ; the MQM qualifies itself as the most predictive, economical and realistic description of the available experimental information. In this letter we wish to offer further confirmation of the power of the MQM by applying it to the calculation of deep inelastic structure functions for π 's and K's.

Our theoretical analysis closely follows the one carried out in Ref. 1) ; we consider the MQM amplitude of Fig. 1, and by applying to it the usual Mueller-Regge analysis in the Bjorken limit

we obtain, for $W_1^{(M)}(\nu, Q^2) \rightarrow F_1^M(x)$ the following results

$$F_1^M(x) \xrightarrow{x \rightarrow 0} \gamma \beta_{PMM} \frac{1}{x} + \frac{\delta}{\sqrt{x}} \quad (1)$$

and

$$F_1^M(x) \xrightarrow{x \rightarrow 1} \eta (1-x)^{1-2\alpha_q(0)}, \quad (2)$$

where β_{PMM} is the residue of the Pomeron MM vertex and $\alpha_q(0) = 0$ ³⁾ is the intercept of the super-exotic quark-like Regge singularity. These are the general results from a Mueller-Regge analysis. But if we want to have a full description of the scaling functions F_1^M , and F_2^M ^{*)} we need, as in Ref. 1), a detailed dynamical model

For small values of the scaling variable x such a model ^{**))} is depicted in Fig. 2, where, as dictated by the MQM, the $q\bar{q}$ Green's function G in the limit $s \rightarrow \infty$ is given by a sum of Regge poles. The calculation of the diagram in Fig. 2 has been carried out using the geometrodynamical rules ⁴⁾, and at the end of a lengthy but straightforward procedure we obtain :

$$F_1^M(x) \xrightarrow{\text{small } x} \frac{A \xi^{(M)}}{3x} (1-x)^3 F(2,1,4;1-x) + \frac{B \xi'^{(M)}}{5\sqrt{x}} (1-x)^{5/2} F(1,2,7/2;1-x). \quad (3)$$

The first term in (3) corresponds to double Pomeron exchange, while the second originates from Pomeron-meson double Regge pole exchange ; $F(a,b,c;z)$ are hypergeometric functions and A and B are constants to be determined. Finally $\xi^{(M)}$ and $\xi'^{(M)}$ are $SU(3)$ Clebsch-Gordan coefficients whose expressions are given in the Table.

For large values of x , the dominant contribution comes from the diagram of Fig. 3. The term for $x \rightarrow 1$ is easily determined as

^{*)} Which are easily seen to obey the Callan-Gross relation $2xF_1^M(x) = F_2^M(x)$.

^{**))} Suggested by the quark geometrodynamical theory of Ref. 4).

$$F_1^M(x) \xrightarrow{x \rightarrow 1} C \zeta^{(M)}(1-x), \quad (4)$$

where C is a constant and $\zeta^{(M)}$ is the Clebsch-Gordan coefficient whose expression for the different distributions is given in the Table. Notice that Eq. (7) is in agreement with the Drell-Yan-West connection⁵⁾ between the $x \rightarrow 1$ behaviour of the meson structure function and the meson form factor behaviour

$$F^M(Q^2) \xrightarrow{Q^2 \rightarrow \infty} \left(\frac{m_0^2}{Q^2} \right). \quad (5)$$

The complete expression for the meson structure function $F_1^{(M)}(x)$ is obtained by adding Eqs. (3) and (4). Thus, we get for the scaling part (we are here neglecting scaling violations)

$$F_1^M(x) = \frac{A \zeta^{(M)}}{3x} (1-x)^3 F(2,1,4;1-x) + \frac{B \zeta'^{(M)}}{5\sqrt{x}} (1-x)^{5/2} F(1,2,7/2;1-x) + C \zeta^{(M)}(1-x). \quad (6)$$

From the Clebsch-Gordan coefficient of the Table, we easily derive the quark sea- and valence-distributions of π 's and K 's. It turns out that both q_s and q_v are $SU(3)$ symmetric and are given by :

$$q_s(x) = \frac{2A}{3} (1-x)^3 F(2,1,4;1-x) \\ q_v(x) = \frac{2B\sqrt{x}}{5} (1-x)^{5/2} F(1,2,7/2;1-x) + 2Cx(1-x). \quad (7)$$

In the MQM the value of A is fixed in the following way. For $x \rightarrow 0$, employing the notation of Ref. 1), we derive

$$\frac{F_1^{ep}}{F_1^{en}} \xrightarrow{x \rightarrow 0} \frac{3}{4} \frac{\tilde{\theta}_0}{A}, \quad (8)$$

where $\tilde{\theta}_0$ was determined ¹⁾ to have the value $\tilde{\theta}_0 = 0.13$. On the other hand, as we can see from Eq. (1),

$$\frac{F_1^{ep}}{F_1^{en+}} \xrightarrow{x \rightarrow 0} \frac{\beta_{pNN}}{\beta_{p\pi\pi}} = \lim_{s \rightarrow \infty} \frac{\sigma_{TOT}(NN)}{\sigma_{TOT}(\pi N)}. \quad (9)$$

Using the experimental cross-section ratio $\simeq 1.7$, we obtain :

$$A = 0.06. \quad (10)$$

Furthermore, the Adler sum rule, being a current algebra relation, must be valid in the MQM and gives the following relation between the C and the B parameters of Eq. (7) :

$$C + 1.06 B = 1. \quad (11)$$

Thus from (7), (10) and (11) we see that we can express the π and K structure functions in terms of only one parameter ^{*)}.

As no direct determination yet exists of the meson structure functions, we have compared our predictions for the quark distribution with the available experimental results ^{6),7)} on massive lepton-pair production in high energy $\pi^\pm$ -p interactions. In Fig. 4 we have utilized our distributions (7) and fixed parameter C at the value 0.36, to predict the experimental quantity

$$F_\pi = K [q_v(x) + G q_s(x)] \quad (12)$$

defined by the authors of Ref. 7), where K is a rescaling factor of the naïve parton model formula ⁸⁾ whose value has been determined to be 2.4 ± 0.6 when use is made of the CDHS parametrization ⁹⁾ of the nucleon structure functions : G is an experimental quantity (depending on acceptances) whose values are 1.4 and 3.7 for π^- and π^+ data, respectively.

*) When this parameter is fixed (see below), we obtain the interesting result that the fraction of the meson momentum carried away by quarks and antiquarks is equal to 0.48.

In Fig. 5, we have plotted our prediction for $q_V(x)$ and compared it with the results given in Ref. 6). The small discrepancies that one sees are probably due to experimental uncertainties on the over-all normalization and on the neglect in the analysis of the $q_S(x)$ contribution.

We thus conclude that the MQM provides an excellent description of meson structure functions. We know of no other approach which is able to achieve results so accurate and predictive ; on the contrary some well-known descriptions ^{10),11)} of these structure functions are seen to disagree strongly with the experimental data ^{*)}.

^{*)} In particular the expression

$$F_1^\pi(x) \underset{x \rightarrow 1}{\simeq} A \left[(1-x)^2 + \frac{2}{9} \frac{C}{Q^2} \right]$$

proposed in Ref. 10), where C is a constant of order 1 GeV^2 , can under no circumstances be claimed to represent the data.

REFERENCES

- 1) P. Castorina, G. Nardulli and G. Preparata - CERN Preprint TH. 2670 (1979), to be published in Nuclear Phys. B.
- 2) P. Castorina, G. Nardulli and G. Preparata - CERN Preprint TH. 2773 (1979).
- 3) G. Preparata - Phys.Rev. D7 (1973) 2973 ; and in Lepton-Hadron Structure, A. Zichichi Ed., Academic Press (1975) p. 54.
- 4) G. Preparata and N. Craigie - Nuclear Phys. B102 (1976) 478 ; and for a recent review, see : G. Preparata - "Quark Geometrodynamics : A New Approach to Hadrons and Their Interactions", Proceedings 1977 Erice Summer School, A. Zichichi Ed., to appear.
- 5) S.D. Drell and T.M. Yan - Phys.Rev. D1 (1970) 2402 ;
G.B. West - Phys.Rev.Letters 24 (1970) 1206.
- 6) C.B. Newman et al. - Phys.Rev.Letters 42 (1979) 951.
- 7) J. Bodier et al. - CERN Preprint EP/79-67 (1979), presented at the EPS Conference on High Energy Physics, Geneva (1979).
- 8) S.D. Drell and T.M. Yan - Phys.Rev.Letters 25 (1970) 316 ; 25 (1970) 902 ; Ann.Phys.(N.Y.) 66 (1971) 578.
- 9) J.G.H. de Groot et al. - Phys.Letters 82B (1979) 456.
- 10) E.L. Berger and S.J. Brodsky - Phys.Rev.Letters 42 (1979) 940.
- 11) G.R. Farrar and D.R. Jackson - Phys.Rev.Letters 35 (1975) 1416.

FIGURE CAPTIONS

- Fig. 1 The MQM amplitude for virtual Compton scattering off a meson.
- Fig. 2 The diagram dominating $W_{\mu\nu}$ for small x . G is the quark-antiquark Green's function.
- Fig. 3 The MQM diagram dominating in the large x region.
- Fig. 4 F_{π} defined in Eq. (12) with $C=0.36$. Data come from Ref. 7) for π^- and π^+ .
- Fig. 5 $q_v(x)$ defined in Eq. (7). Data come from C.I.P. collaboration, Ref. 6).

Figure 1

Figure 2

Figure 3

Figure 4

Figure 5

12