Ref.TH.2788-CERN

A FRESH LOOK AT NEUTRINO OSCILLATIONS

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A B S T R A C T

We analyze the present experimental situation on neutrino oscillations in the standard weak interaction theory with three neutrinos. Sizeable oscillations $\nu_e \rightleftharpoons \nu_\tau$ are compatible with, and perhaps indicated by, present data.

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1. INTRODUCTION

There is no good reason or principle requiring neutrinos to be massless. For neutrinos with non-zero masses, there is no good reason or principle to avoid a mismatch between mass eigenstates and weak eigenstates. In this situation, neutrino oscillations are to be expected at some level¹⁾⁻³⁾. Our utter theoretical ignorance on the subject of fermion masses and weak mixings fully justifies whatever progress can be made in the experimental study of neutrino masses or oscillations.

All previous experiments on neutrino oscillations, perhaps with the exception of the observed scarcity of solar neutrinos, have yielded negative results within errors. But the data have, more often than not, been analyzed in terms of two neutrinos. The existence of at least a third neutrino, ν_τ , makes the possibilities of neutrino oscillations considerably richer and their analysis more complex^{4),5)}. We study the experimental situation with an open mind on the neutrino mixing problem, in the framework of the standard weak interaction theory⁶⁾ with three neutrino types. We find that the available experimental constraints are too few to determine the parameters (masses and mixing angles), too few even to imply a useful set of constraints in the most general analysis of parameter-space. But even in simplified situations we will show that the oscillation problem is wide open in two respects. First, indications may exist that neutrinos do indeed oscillate. Second, not only $\nu_\mu(\bar{\nu}_\mu)$ induced but also, and particularly, $\nu_e(\bar{\nu}_e)$ induced neutrino oscillation experiments should be improved. Because accelerator neutrino beams are mainly ν_μ beams, the best limits on neutrino oscillations concern the absence of $\nu_\mu \rightarrow \nu_e$ transitions. Limits on $\nu_\mu \rightarrow \nu_\tau$ oscillations are also quite stringent.

Let ν_e , ν_μ and ν_τ be the objects weakly coupled to e , μ and τ , respectively. Let $\nu_{1,2,3}$ be neutrino mass eigenstates. We find that all negative evidence and positive indications of neutrino oscillations are compatible, for instance, with a mixing pattern

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} .65 & .65 & -.38 \\ -.69 & .72 & .05 \\ .31 & .23 & .92 \end{pmatrix} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

with mass differences in the ranges

$$10^{-5} \text{ eV} \gtrsim \sqrt{|m_1^2 - m_2^2|} \gtrsim 1 \text{ eV}$$

and

$$\sqrt{|m_3^2 - m_1^2|} \sim 10 \text{ eV}.$$

Here, for the sake of illustration, we neglected CP violating effects, to be allowed for in the main text.

In Section 2 we introduce the notation relevant to the analysis of oscillations in the standard weak interaction theory. Section 3 is a reminder of the present experimental situation. In Section 4 we marry Sections II and III to obtain restrictions on the parameters of the theory. Section 5 is an analysis of oscillations in beam dump experiments. In Section 6 we come back to the problem of solar neutrinos. Section 7 touches upon the problem of new neutrinos beyond the three known ones. Conclusions are offered in Section 8.

2. NEUTRINO OSCILLATIONS IN THE STANDARD MODEL: GENERALITIES AND NOTATION

Let weak and electromagnetic interactions be described by the standard $SU(2) \times U(1)$ gauge theory⁶⁾ with leptons in weak doublets $(\nu_e, e)_L$; $(\nu_\mu, \mu)_L$; $(\nu_\tau, \tau)_L$; ...? More often than not we will refer to a three-neutrino situation involving the known leptons. Let neutrinos have masses m_i , $i = 1, \dots, n$ and let the corresponding mass eigenstates be labelled ν_i . We shall assume neutrino masses to be Dirac rather than Majorana masses. The non-Abelian gauge invariance of the theory demands that the weak interaction "eigenstates" $\nu_e, \nu_\mu, \nu_\tau, \dots$ (defined as the objects that couple to e, μ and $\tau, \dots$ respectively) and the mass eigenstates ν_i be related by a unitary transformation U :

$$\gamma_w = U_{wi} \nu_i \quad ; \quad w = e, \mu, \tau, \dots ; i = 1, \dots, n ; U U^\dagger = 1 \quad (2.1)$$

In general, for n lepton doublets, and given the freedom to redefine $2n - 1$ relative lepton phases, the matrix U can be parametrized via the $n(n - 1)/2$ "weak angles" of an n dimensional rotation plus $(n - 1)(n - 2)/2$ "CP violating phases".

The probability amplitude $V_{ww'}$, for a neutrino of weak type w and energy E to "oscillate" into a neutrino of weak type w' at a distance L is³⁾:

$$V_{ww'} = (U D U^\dagger)_{ww'} \quad (2.2)$$

$$D_{ij} = \delta_{ij} \exp \left\{ -i m_i^2 L / 2E \right\} ; (E \gg m_i^2) \quad (2.3)$$

In words, the operation $UDU^\dagger$ expresses the newly born weak eigenstate neutrino in the mass basis $[U]$, propagates it $[D]$ and re-expresses it in the weak basis.

It is the difference of squares of neutrino masses that is relevant to the oscillation problem and it is convenient to introduce the notation:

$$\Delta_{ij} \equiv (m_i^2 - m_j^2) L / 2E \quad (2.4)$$

In terms of the Δ_{ij} , the oscillation amplitude V (up to an irrelevant overall phase) can be written in n equivalent ways. One of them (labelled j) is:

$$V_{ww'} = \delta_{ww'} - \sum_{i \neq j}^n (1 - e^{-i\Delta_{ij}}) U_{wi} U_{w'i}^* \quad (2.5)$$

The probability for a neutrino of type w to turn into type w' is the square of the corresponding V matrix element

$$P(\nu_w \rightarrow \nu_{w'}) = |V_{ww'}|^2 \quad (2.6)$$

The n neutrino oscillation problem is described by the $(n-1)^2$ parameters defining the weak mixing matrix U plus the $n-1$ parameters $m_i^2 - m_j^2$, $i \neq j$, a total of $n(n-1)$ parameters.

3. BRIEF RECOLLECTION OF THE EXPERIMENTAL SITUATION

As can be explicitly read off Eqs 2.4) - 2.6), for there to be neutrino oscillations ($V_{ww'} \neq \delta_{ww'}$) it is necessary that some neutrino pair mass difference be different from zero:

$$\Delta_{ij} \equiv \frac{(m_i^2 - m_j^2) L}{2E} \neq 0 \quad (3.1)$$

Thus, a "figure of merit" of an oscillation experiment, giving a rough idea of the values of

$$\sqrt{|m_i^2 - m_j^2|}$$

to which the experiment (for non-vanishing mixing angles) may be sensitive, is:

$$M = \sqrt{\frac{2E}{L}} \quad (3.2)$$

Here E is a typical (or lowest) energy and L a typical (or maximal) distance in the range covered by the experiment.

A list of some characteristic experiments and their typical figures of merit is given in the Table below:

Experiment	Oscillations	L (meters)	E (MeV)	M (eV)
Solar neutrinos ⁷⁾	$\nu_e \rightarrow \nu_e$	$1.5 \cdot 10^{11}$	5	$3.6 \cdot 10^{-6}$
Reactor neutrinos ⁸⁾	$\bar{\nu}_e \rightarrow \bar{\nu}_e$	6	4	0.5
Low energy accelerators ⁹⁾	$\nu_\mu \rightarrow \nu_e$ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ $\nu_e \rightarrow \nu_e$ $\bar{\nu}_e \rightarrow \bar{\nu}_e$	50	$2 \cdot 10^3$	4
High energy accelerators ^{10),11)}	$\nu_\mu \rightarrow \nu_e, \nu_\tau$ $\bar{\nu}_\mu \rightarrow \bar{\nu}_e, \bar{\nu}_\tau$ $\nu_e \rightarrow \nu_e$ $\bar{\nu}_e \rightarrow \bar{\nu}_e$	450	$5 \cdot 10^4$	7

The actual limits on mass differences given by an experiment depend on the one hand, on details of energy distributions, the size of the decay region and the various experimental cuts, and on the other hand, on the theoretical framework used to interpret the results (the mixing angle in a two-neutrino situation, the various mixing parameters in a general situation). To give an idea of the precision of different experiments, we quote the result of the most restrictive experiment of each type. When an interpretation of the raw data is necessary we use the standard weak model. Brackets will indicate averaging over distances and energies.

i) Solar neutrino experiment⁷⁾:

$$\langle P(\nu_e \rightarrow \nu_e) \rangle = \frac{\text{observed events}}{\text{calculated expectation}} = \frac{1.75 \pm .4 \text{ SNU}}{\sim 4.7 \text{ SNU}} \quad (3.3)$$

Taken at face value this is a strong indication of oscillations. Because of the obvious uncertainties in the expectation we will not use this experiment as an input to our analysis, but comment on it a posteriori.

ii) Reactor experiment⁸⁾:

$$\langle P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rangle = \frac{\text{observed events}}{\text{calculated expectation}} = 0.88 \pm 0.13 \quad (3.4)$$

iii) Gargamelle "low energy" experiment⁹⁾:

$$\frac{\langle P(\nu_\mu \rightarrow \nu_e) \rangle}{\langle P(\nu_\mu \rightarrow \nu_\mu) \rangle} = (-0.03 \pm 0.1) 10^{-2} \quad (3.5a)$$

$$\frac{\langle P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) \rangle}{\langle P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \rangle} = (0.02 \pm 0.07) 10^{-2} \quad (3.5b)$$

These results are the strongest indication of the absence of ν_μ induced transitions. The experiment also yields the limits:

$$\frac{\langle P(\nu_e \rightarrow \nu_e) \rangle}{\langle P(\nu_\mu \rightarrow \nu_\mu) \rangle} = 0.95 \pm 0.30 \quad (3.6a)$$

$$\frac{\langle P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rangle}{\langle P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) \rangle} = 0.89 \pm 0.30 \quad (3.6b)$$

iv) Fermilab 15-foot bubble chamber experiment¹⁰⁾:

$$\frac{\langle P(\nu_\mu \rightarrow \nu_\tau) \rangle}{\langle P(\nu_\mu \rightarrow \nu_\mu) \rangle} < 0.025 \quad (3.7)$$

where a 20% branching ratio of $\tau^- \rightarrow e^- + \text{anything}$ has been assumed.

v) The recent CERN beam dump experiments¹¹⁾. The interpretation of their results in terms of neutrino oscillations is speculative (and possibly incorrect), so that we must give some details.

The experiment with the cleanest interpretation (and smallest statistics) is the BEBC bubble chamber experiment, where electrons can be directly observed. The experiments quote results for "prompt" events, from which the conventional neutrinos from π , K and μ decay are allegedly subtracted by extrapolation to an "infinite

density" target. The conventional interpretation of prompt neutrinos is that they originate almost exclusively from charmed particle decay. Barring unforeseen surprises, this implies the following relations between the fluxes of different neutrinos: $\phi(\nu_e) = \phi(\nu_\mu) \gg \phi(\nu_\tau)$ and $\phi(\bar{\nu}_e) = \phi(\bar{\nu}_\mu) \gg \phi(\bar{\nu}_\tau)$. The quoted ratio R of prompt-neutrino induced electrons and muons

$$R = \frac{\#(e^+e^-)}{\#(\mu^+\mu^-)} = 0.59 \pm 0.22 \quad (3.8)$$

is about two standard deviations below unity and may be an indication of ν_e oscillations. Since $\nu_\mu \rightarrow \nu_e, \nu_\tau$ oscillations seem to be absent, ($P(\nu_\mu \rightarrow \nu_\mu) \sim 1$) a way to allow for $R \neq 1$ is to have $P(\nu_e \rightarrow \nu_e) < 1$.

Upper limits on neutrino masses are also relevant to the analysis of oscillations. Laboratory experiments are usually analyzed under the assumption that charged leptons couple to ν mass eigenstates: $\nu_e = \nu_1$, $\nu_\mu = \nu_2$, $\nu_\tau = \nu_3$. The most stringent quoted limits are: $m[\nu_e] < 35$ eV¹²⁾, $m[\nu_\mu] < 0.57$ MeV¹³⁾ and $m[\nu_\tau] < 250$ MeV¹⁴⁾. We have not reanalyzed these experiments in the framework of a mixed-neutrino world, since we will remain within the "cosmological" bounds for the sum of neutrino masses and these bounds are approximately as stringent as the most restrictive laboratory limit on a single neutrino mass. The commonly accepted "cosmological" bound on the sum of masses of "light" ($m \lesssim 3$ MeV) neutral leptons¹⁵⁾ is

$$\Sigma \equiv \sum_i m_i < 40 \text{ eV} \quad (3.9)$$

There is, however, at least one claim of a more stringent cosmological bound¹⁶⁾ ($\Sigma < 1.2$ eV) that we will have to choose to violate*).

4. ANALYSIS OF EXPERIMENTS IN A THREE-NEUTRINO WORLD

The three-neutrino oscillation problem is described, in the standard model, by six parameters. Data exist on the probabilities $P(\bar{\nu}_e \rightarrow \bar{\nu}_e)$, $P(\nu_e \rightarrow \nu_e)$, $P(\nu_\mu \rightarrow \nu_e)$, $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$, $P(\nu_\mu \rightarrow \nu_\tau)$ and $P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau)$. Since most of the data are in the form of upper limits, we can at best determine a six-dimensional domain of allowed values of the parameters. The domain is sufficiently large and unbecoming to be neither very useful nor easy to visualize. Rather than attempting a general analysis at this point, we have chosen to start by restricting the

*) The three neutrino mixing problem for a heavy and unstable third neutrino ($m \gtrsim 10$ MeV) has recently been analyzed¹⁷⁾.

parameters to a region where the analysis of oscillations is particularly transparent. This will be enough to illustrate our main points.

Earth-based neutrino experiments (see the Table) are sensitive to

$$\sqrt{|m_i^2 - m_j^2|} \sim \text{a few eV}$$

only the classic solar neutrino experiment is sensitive to much smaller mass differences³⁾. To simplify our three-neutrino analysis we first make the assumption *) that for earth-based experiments

$$|\Delta_{12}| = |(m_1^2 - m_2^2)L/2E| \ll 1 \quad (4.1)$$

The oscillation amplitude Eq. (2.5) immediately simplifies to

$$V = \mathbb{1} - (1 - e^{-i\Delta}) \begin{pmatrix} |U_{e3}|^2 & U_{e3}U_{\mu 3}^* & U_{e3}U_{\tau 3}^* \\ U_{e3}^*U_{\mu 3} & |U_{\mu 3}|^2 & U_{\mu 3}U_{\tau 3}^* \\ U_{e3}^*U_{\tau 3} & U_{\mu 3}^*U_{\tau 3} & |U_{\tau 3}|^2 \end{pmatrix} \quad (4.2)$$

where $\Delta = \Delta_{31} \simeq \Delta_{32}$ and we have made the notation very explicit. In this limit of a small squared mass difference between two neutrinos, CP violating effects are not observable [i.e., $P(\nu_e \rightarrow \nu_\tau) = P(\nu_\tau \rightarrow \nu_e)$], but all manners of oscillations are still possible. In particular, the tightest no-oscillation limit: the fact that

$$P(\nu_\mu \rightarrow \nu_e) = |(1 - e^{-i\Delta})U_{e3}^*U_{\mu 3}|^2 \quad (4.3)$$

is small, continues to place stringent limits on the mixing matrix U . This constraint and all the other constraints on non-diagonal transitions [$P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ and $P(\nu_\mu \rightarrow \nu_\tau)$] can be simultaneously satisfied with just the requirement that $U_{\mu 3}$ be small. (The precise meaning of "small" will be discussed in the Appendix.)

*) For charged leptons $m_\mu^2 - m_e^2 \ll (m_\tau^2 - m_\mu^2) \sim (m_\tau^2 - m_e^2)$. Quarks follow a similar pattern. Unfortunately, we know of no good reason to infer that neutrino masses should be similarly related.

We choose the easy path first and set $U_{\mu 3} = 0$. In such a limit and with the definition $U_{e3} = \sin \alpha$, we obtain

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) = 1 \quad (4.4)$$

$$P(\nu_\mu \rightarrow \nu_e) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e) = P(\nu_\mu \rightarrow \nu_\tau) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\tau) = 0 \quad (4.5)$$

$$\begin{aligned} P(\nu_e \rightarrow \nu_\tau) &= P(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) = \sin^2 2\alpha \cdot \sin^2 \Delta/2 = \\ &= \sin^2 2\alpha \sin^2 \left[\frac{(m_3^2 - m_1^2)L}{4E} \right] \end{aligned} \quad (4.6)$$

$$P(\nu_e \rightarrow \nu_e) = P(\nu_\tau \rightarrow \nu_\tau) = 1 - P(\nu_e \rightarrow \nu_\tau) \quad (4.7)$$

Thus, though there is only one small non-diagonal entry, rather than four of them, in the mixing matrix U , the three-neutrino oscillation problem reduces to a two-neutrino oscillation problem ($\nu_e \rightleftharpoons \nu_\tau$). It is now relatively straightforward to analyze the relevant earth-based experiments in terms of two parameters:

$$\sin 2\alpha \quad \text{and} \quad \delta m \equiv \sqrt{|m_3^2 - m_1^2|}$$

Because of the appearance of the factor L/E in Eqs (4.6) and (4.7), and because neutrino beams are not monochromatic and generally experiments cover a range of distances, knowledge of the neutrino flux as a function of energy and distance $\phi(E, L)$ is required. The most stringent limits on δm and $\sin 2\alpha$ come from the BEBC beam dump experiment¹¹⁾ and the reactor experiment⁸⁾.

We proceed to discuss the BEBC beam dump experiment. In this case L is fixed. Estimates of the flux $\phi(E)$ have been kindly provided to us by G. Conforto and F. Ferroni. We have assumed $\phi(\nu_e) = \phi(\nu_\mu)$ and $\phi(\bar{\nu}_e) = \phi(\bar{\nu}_\mu)$ as is appropriate to a prompt neutrino flux of charmed particle decay origin. We have also assumed $\phi(\nu_e) = \phi(\bar{\nu}_e)$ for lack of a better ansatz. The ratio R defined in Eq. (3.8) is

$$R = \frac{\langle P(\nu_e \rightarrow \nu_e) \rangle + \langle P(\nu_e \rightarrow \nu_\tau) f(\nu_\tau \rightarrow e) \rangle + \{ \nu_e \leftrightarrow \bar{\nu} \}}{1 + \langle P(\nu_e \rightarrow \nu_\tau) f(\nu_\tau \rightarrow \mu) \rangle + \{ \nu_e \leftrightarrow \bar{\nu} \}} \quad (4.8)$$

where $f(\nu_\tau \rightarrow e) \simeq f(\nu_\tau \rightarrow \mu)$ is the product of the branching ratio of τ decay into electrons (or muons)¹⁴⁾ $BR \sim 17\%$, times the cross-section ratio $\sigma(\nu_\tau \rightarrow \tau)/\sigma(\nu_\mu \rightarrow \mu)$. Whenever correlations due to the spin of the fermions are important (as in the study of energy distributions), we take them into account.

Curve (a) in Fig. 1 is a lower limit on δm and $\sin 2\alpha$ imposed, at the one standard deviation level (1σ , 68% confidence) by the beam dump BEBC result $R < 0.81$, see Eqs (3.8) and (4.8). This evidence in favour of oscillations is shaky: it disappears at the 2σ level, for which R is compatible with unity. Curve (b) in Fig. 1 is an upper limit on oscillations imposed at the 1σ level by the reactor experiment $\langle P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \rangle \geq 0.75$, see Eq. (3.4). In the region of δm allowed by our optimistic interpretation of the beam dump results, the analysis of the reactor experiment is very trivial. Indeed, $(\delta m)^2 L/E \gg 1$ for the distances and energies covered by the experiment so that the oscillatory factor in Eqs (4.6) and (4.7) averages to $1/2$. Thus curve (b) is nothing but

$$1 - \frac{1}{2} \sin^2 2\alpha > 0.75$$

The hypothesis $\Delta_{21} = 0$ may seem too stringent for reactor experiments. We note, however, that to first order in Δ_{21}

$$P(\nu_e \rightarrow \nu_e) = 1 - 2(1 - \cos \Delta_{31}) |U_{e3}|^2 (1 - |U_{e3}|^2) + \\ + 2 \Delta_{21} \sin \Delta_{31} \cdot |U_{e3}|^2 |U_{e2}|^2 + O(\Delta_{21}^2)$$

Since $\Delta_{31} \gg 1$ the first order term averages to zero, so that only $O(\Delta_{21}^2)$ corrections appear.

To complete Fig. 1 a cosmologist would add, somewhere at the few tens of eV level, an upper limit on δm , independent of the mixing angle α , see Eq. (3.9) and the comments around it. To summarize: $\nu_e \rightleftharpoons \nu_\tau$ oscillations with mixing angles up to $\sim 20^\circ$ are not excluded. Such oscillations with δm from a few to a few tens of eV may even be indicated by the beam dump experiments.

5. ENERGY DISTRIBUTIONS IN BEAM DUMP EXPERIMENTS

We proceed to discuss the "visible energy" distributions predicted by the $\nu_e \rightleftharpoons \nu_\tau$ oscillation ansatz, for beam dump experiments of the type recently performed at CERN¹¹⁾. The visible energy E_v is the energy of all reaction products except eventual outgoing neutrinos. The distance L between source and detector is much bigger than either, and fixed. Thus, oscillations of sufficiently large amplitude would show as ups and downs in the number of events as a function of visible energy, to the extent that they are not washed off by $E_v \neq E$ effects.

Figures 2 and 3 are relevant to a bubble chamber experiment capable of detecting electrons. They are drawn for $L = 820\text{m}$ and two choices of oscillation parameters in the allowed domain of Fig. 1, points A and B, respectively, for which

$(\sin^2 2\alpha, \delta m) = (0.5, 7\text{eV})$ and $(0.4, 12\text{eV})$. The doubly dash-dotted line in Fig. 2 is $E\phi(E)$, with ϕ the assumed flux of ν_e , $\bar{\nu}_e$, ν_μ or $\bar{\nu}_\mu$. This line would be the E_ν distribution of events with muons in the final state if no oscillations took place. The continuous line in Figs 2 and 3 is the calculated E_ν distribution of events with μ (and/or $\bar{\mu}$) in the final states, in the presence of oscillations. It is above the flux line because of the contribution of events where electron neutrinos oscillate into tau neutrinos, and make tau-leptons that decay into muons. The dashed line in Figs 2 and 3 is the E_ν distribution of events with electrons (and/or positrons) in the final state. The bulk of the curve comes from the production of electrons by surviving e type neutrinos $P(\nu_e \rightarrow \nu_e)\sigma(\nu_e \rightarrow e)$. Also contributing are ν_e 's that oscillate into ν_τ 's, and make τ 's that decay into e 's. This contribution is the dot-dashed line in Fig. 2. The electron signal oscillates as a function of E_ν in a manner whose observation would be proof of the existence of oscillations. The shape of the curves, drawn for $\phi(\nu_e) = \phi(\bar{\nu}_e)$, is not very sensitive to this assumption.

Figures 4 and 5 are relevant to a calorimeter-type detector unable to tell electrons from hadrons. They are drawn for $L = 920\text{m}$ and for the same values of $(\sin^2 2\alpha, \delta m)$ as Figs 2 and 3, respectively. The continuous lines are the visible energy distributions of events with one muon and coincide with the corresponding curves for the bubble chamber detectors. The dot-dashed lines represent events with no muons. They are the sum of neutral current events induced by the three types of neutrinos, the charged current $\nu_e \rightarrow e$ events, and the events where a ν_e converts into a ν_τ that makes a τ that decays into anything but a muon (ooph!). Following experimental tradition, we have drawn as dashed lines the events with no muons minus the contribution to them of ν_μ induced neutral currents. The curves, though obviously less bumpy than the bubble chamber curves, still show oscillations that one may hope to observe. The shape of energy distributions for muonless events is sensitive to the assumption $\phi(\nu_e) = \phi(\bar{\nu}_e)$. An excess of anti-neutrinos tends to enhance the low energy region.

The previous analysis has been performed under the simplifying assumption $U_{\mu 3} = 0$. We show in the Appendix that the results are practically independent of this hypothesis and that the actual bound is $|U_{\mu 3}| < 0.07$.

6. SOLAR NEUTRINOS

So far we have set

$$|\Delta_{12}| = (|m_1^2 - m_2^2|L)/2E \ll 1$$

Due to the moderate energies of and enormous distances travelled by solar neutrinos, it is not unreasonable to expect $|\Delta_{12}| \gg 1$, in this case. Thus:

$$P_{\odot}(\nu_e \rightarrow \nu_e) = \left| e^{-i\Delta_{12}} |U_{e1}|^2 + |U_{e2}|^2 + e^{-i\Delta_{32}} |U_{e3}|^2 \right|^2 \quad (6.1)$$

$$\simeq |U_{e1}|^4 + |U_{e2}|^4 + |U_{e3}|^4$$

since all the oscillating factors average to zero. Within the bounds on $|U_{e3}|$ allowed by our previous analysis, $P_{\odot}(\nu_e \rightarrow \nu_e)$ is in the range

$$0.39 \leq P_{\odot}(\nu_e \rightarrow \nu_e) \leq .86$$

The minimum value is obtained for $|U_{e3}|$ maximum, and maximal ν_1, ν_2 mixing ($|U_{e1}| = |U_{e2}|$). This lower bound is close to the absolute minimum (1/3) that one can obtain with three Dirac neutrinos.

If one really believes in the existence^{3),7)} of a "solar neutrino puzzle" and wants to solve it constructively as an indication of neutrino oscillations, a possible mixing matrix, consistent with all other constraints is

$$U = \begin{pmatrix} 0.65 & 0.65 & -0.38 \\ [-0.71 e^{i\delta} \mp |<.02|] & [.71 e^{i\delta} \mp |<.02|] & \mp |<.06| \\ [0.27 \mp e^{i\delta} |<.04|] & [.27 \pm e^{i\delta} |<.04|] & .92 \end{pmatrix} \quad (6.2)$$

where all the upper (or lower) signs should be simultaneously taken. The CP violating phase δ is arbitrary, but CP violating effects would anyway be quite small.

7. FURTHER NEUTRINOS

So far, we have restricted ourselves to a three-neutrino world. There might be more neutrino types, and it may turn out that the electron neutrino oscillates with one of those, ν_{σ} , rather than with ν_{τ} . If, continuing the sequential pattern, a new doublet $(\nu_{\sigma}, \sigma)_L$ exists, very little changes in our analysis, even if σ is too heavy to be produced at present energy. The bounds given in Fig. 1 are practically unchanged, and so are the shapes of the curves given in Figs 2 and 3. This is because the τ contribution (which may be absent in the $\nu_e \leftrightarrow \nu_{\sigma}$ case) appears there multiplied by the small branching ratio $B(\tau \rightarrow e) = B(\tau \rightarrow \mu)$.

The visible energy distribution of muonless events in calorimetric experiments would also change somehow (see Figs 3 and 4 where the τ contribution is indicated by a double dot-dashed line). Of course, a real distinction between $\nu_e \rightleftharpoons \nu_\tau$ and $\nu_e \rightleftharpoons \nu_\sigma$ has to rely on the possibility of detecting the presence of τ (or σ) in the final state.

8. CONCLUSIONS

There are stringent limitations on $\nu_\mu (\bar{\nu}_\mu)$ induced oscillations; the limits on the probabilities $P(\nu_\mu \rightarrow \nu_e, \nu_\tau)$ are tight.

There are disputable indications of $\nu_e (\bar{\nu}_e)$ induced oscillations, from the solar neutrino experiment and the CERN beam dump experiment. These experiments share two features:

- i) They indicate that $P(\nu_e \rightarrow \nu_e) < 1$, a ν_e fluctuat nec mergitur;
- ii) Knowledge of the neutrino flux is subject to uncertainties; the solar neutrino flux in particular is not yet under the experimentalist's control.

In this situation two conclusions are possible:

- a) Dismiss the "positive evidence" because of its doubtfulness.
- b) Accept that oscillations of ν_e 's into ν_τ 's (or, perhaps, ν_σ ?) may exist, and are not particularly contrived in the framework of the standard weak interaction theory, as we believe to have shown.

Should one choose the second of the above conclusions, a corollary to it seems to us inescapable. Better experiments on $\nu_e (\bar{\nu}_e)$ induced oscillations are necessary. These could be improvements of solar, reactor or beam dump experiments or specific experiments with a ν_e enriched (K decay) source. The indications are that the relevant mixing angles are comfortably large, $\alpha \sim 22^\circ$ and that the relevant mass difference

$$\delta m = \sqrt{m_1^2 - m_3^2}$$

may be in the range of a few to a few tens of eV. Thus, the "figure of merit" of future experiments $\sqrt{2E/L}$ should be of the same order of magnitude, somewhere in the vicinity of $20(\text{MeV/m})^{1/2}$.

ACKNOWLEDGEMENTS

We would like to thank many a member of the experimental groups at CERN for discussions.

APPENDIX

In this Appendix we discuss the bounds on mixing parameters, allowing $U_{\mu 3} \neq 0$. In the approximation $\Delta_{12} = 0$ (Eq. (4.1)) the 68% confidence level experimental bound on $\nu_{\mu} \rightarrow \nu_e$ oscillations⁹⁾ is

$$|U_{e3}|^2 |U_{\mu 3}|^2 \leq 5 \cdot 10^{-4} \text{ for } \delta m \sim 7 \text{ eV} \quad (\text{A.1})$$

and decreases monotonically to the level of 3.5×10^{-4} for large δm . If $|U_{\mu 3}|$ is not equal to zero, the ratio R of Eq. (3.8) becomes

$$\begin{aligned} R &= \frac{\langle P(\nu_e \rightarrow \nu_e) \rangle + \langle P(\nu_{\mu} \rightarrow \nu_e) \rangle + \langle (P(\nu_e \rightarrow \nu_{\tau}) + P(\nu_{\mu} \rightarrow \nu_{\tau})) f(\nu_{\tau} \rightarrow e) \rangle + \{ \nu \leftrightarrow \bar{\nu} \}}{\langle P(\nu_{\mu} \rightarrow \nu_{\mu}) \rangle + \langle P(\nu_e \rightarrow \nu_{\mu}) \rangle + \langle (P(\nu_e \rightarrow \nu_{\tau}) + P(\nu_{\mu} \rightarrow \nu_{\tau})) f(\nu_{\tau} \rightarrow \mu) \rangle + \{ \nu \leftrightarrow \bar{\nu} \}} = \\ &= \frac{1 - 2 \langle 1 - \cos \Delta \rangle |U_{e3}|^2 |U_{\tau 3}|^2 + 2 \langle f(1 - \cos \Delta) \rangle |U_{\tau 3}|^2 (|U_{e3}|^2 + |U_{\mu 3}|^2)}{1 - 2 \langle 1 - \cos \Delta \rangle |U_{\mu 3}|^2 |U_{\tau 3}|^2 + 2 \langle f(1 - \cos \Delta) \rangle |U_{\tau 3}|^2 (|U_{e3}|^2 + |U_{\mu 3}|^2)} \end{aligned} \quad (\text{A.2})$$

where $\Delta = \Delta_{32} \approx \Delta_{31}$ and $|U_{\tau 3}|^2 = 1 - |U_{e3}|^2 - |U_{\mu 3}|^2$.

In Fig. 6 we present, for a typical value of δm ($\delta m = 12$ eV), the bounds determined at 68% confidence level by the beam dump preliminary result¹¹⁾ $R < 0.81$ (full line), by the Gargamelle PS experiment⁹⁾ (dashed line) and by the reactor experiment⁸⁾ (dash-dotted lines). The allowed regions are dashed in the figure and they demand $|U_{\mu 3}| \lesssim 0.07$. We recall that the lines in this figure move with δm and that the allowed regions shrink to a point for $\delta m \approx 7$ eV. On the other hand, if one would make an analysis at the two standard deviation level (95% confidence level), the beam dump would provide no restriction at all.

The allowed range of values of $|U_{e3}|^2$ is practically independent of $|U_{\mu 3}|^2$. The allowed domain in Fig. 1 is, in fact, the largest projection of the six-dimensional domain on the $(\sin 2\alpha, \delta m)$ plane. From Eqs (A.1), (A.2) and $R < 0.81$ one can obtain an absolute upper bound

$$|U_{\mu 3}| \leq 0.07 \quad (\text{A.3})$$

corresponding to $\delta m = 11$ eV and $|U_{e3}| \approx 0.29$. Therefore, one has

$$P(\nu_\mu \rightarrow \nu_\tau) = 4|U_{\mu 3}|^2|U_{\tau 3}|^2 \sin^2(\Delta/2) < 0.025$$

so that the experimental bounds on $\nu_\mu \rightarrow \nu_\tau$ oscillations¹⁰⁾ are automatically satisfied.

Thus far we have assumed $\Delta_{12} = 0$. It can be shown, by a first order expansion in Δ_{12} of the exact expression for $P(\nu_\mu \rightarrow \nu_e)$, that the upper bound on $|U_{\mu 3}|$ can only be moved up to ~ 0.12 provided that

$$\sqrt{|m_2^2 - m_1^2|} \lesssim 1 \text{ eV}$$

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FIGURE CAPTIONS

- Fig. 1 : Bounds in the $\delta m, \sin 2\alpha$ plane coming from beam dump (curve a) and reactor (curve b) experiments.
- Fig. 2 : Visible energy spectra for muons (full line), electrons (dashed line) for a typical charm decay neutrino flux (double dot-dashed line). The electron yield coming from τ decay is also drawn (dot-dashed line). The choice of parameters δm and $\sin 2\alpha$ corresponds to point A of Fig. 1.
- Fig. 3 : Same as Fig. 2 for δm and $\sin 2\alpha$ values corresponding to point B of Fig. 1.
- Fig. 4 : Visible energy spectra for muons (full line), total muonless events (dot-dashed line), muonless events without ν_μ neutral current contribution (dashed line) and muonless events coming from τ decay (double dot-dashed line). Parameters δm and $\sin 2\alpha$ correspond to the point A of Fig. 1
- Fig. 5 : The same as Fig. 4 with parameters corresponding to point B of Fig. 1.
- Fig. 6 : Limitations on $|U_{\mu 3}|$ versus $|U_{e 3}|$ imposed at 68% confidence level by the different experiments (see the Appendix for details). The allowed regions are the dashed ones.

FIG. 1

FIG. 2

FIG. 2

FIG. 4.

FIG. 5

FIG.6