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ON A NATURAL HIERARCHY IN DYNAMICALLY BROKEN GAUGE MODELS

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A B S T R A C T

A model based on dynamical symmetry breaking provides a naturally large "mass hierarchy". Few fermions are needed at intermediate energies, and asymptotic freedom of usual interactions is therefore not imperiled.

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In this note we wish to address the problem of dynamically generating those extraordinary different mass scales which are vital to unifying theories of weak, electromagnetic and strong interactions. In the most conventional procedure, masses of order 10^{15} and 1 GeV, respectively, are generated via vacuum expectation values of scalar fields. It has been observed¹⁾ that actually obtaining such a large mass ratio requires in such models a very careful tuning of the renormalized parameters of the scalar potential.

There is clearly nothing wrong with that, at least if those parameters are considered as fundamental coupling constants, in the same way as the gauge coupling itself. The acceptance of a relatively large set of parameters with precise values, however, appears in some way incompatible with the idea of a grand unified theory. The vast arsenal of discrete symmetries does not seem, until now, to have provided a "natural" pattern for this breaking.

In this context, Susskind¹⁾, and Weinberg²⁾ revived the suggestion³⁾ of a dynamical (without scalars) breakdown of symmetry. The scheme proposed by Susskind hinges on a repetition at energies of order 1 TeV of the experimentally known chiral symmetry breaking by strong colour interactions around 1 GeV. The original toy model of Ref. 1) was later implemented⁴⁾ and is now characterized by a proliferation of particles - both confined fermions and gauge bosons - between, say 1 and 100 TeV, whose pattern is governed by a sequential breaking of gauge symmetries.

We would like in this note to propose another mechanism, in which a very strong hierarchy is automatically obtained. Although we will limit ourselves to the description of a toy model, it seems important to note that the number of particles at intermediate energies can be reduced with respect to the technicoloured model of Susskind, a feature which seems to be of some importance for the preservation of asymptotic freedom of the usual strong interactions and in fact, of the concept of grand unification itself⁵⁾.

Let us suppose that, at some mass scale M we have a gauge theory characterized by the following group structure: (no colour is introduced in this oversimplified model)

$$G = SU(N) \times SU(2) \times U(1)_L \times U(1)_R \quad (1)$$

with the idea of grand unification in mind we suppose that all four coupling constants $g_i(p^2)$ are equal for $p^2 = -M^2$, or possibly related by simple numerical coefficients. We choose three sets of fermions which transform as

$$\begin{aligned} S &= (N, 1, X, X) \\ L &= (1, 2, X, 0) \\ R &= (1, 1, 0, X) \end{aligned} \quad (2)$$

The symbols (X) and (0) above respectively mean full g coupling and no coupling to the corresponding $U(1)$ group.

For definiteness, let us suppose for instance that $g^2(-M^2)/4\pi = 1/10$ and $N = 10$, $M = 10^{16}$ GeV (these values should not be taken seriously, they are mainly given to illustrate the mechanism in this simple model). The usual renormalization group arguments teach us that the coupling constant of the $SU(N)$ group will rise rapidly. At a mass scale around 10^{15} GeV it reaches the value 1, and, according to the technicolour scheme, we expect chiral symmetry breaking to occur in this sector. S fermions get "massive" and confined, $SU(N)$ vector bosons are confined: all of these particles disappear from low energy interactions.

At the same time, a mixing is induced between $U(1)_L$ and $U(1)_R$ gauge bosons through the graph of Fig. 1. If we accept^{*)} the "technicoloured dogma" that the fermion masses are soft (which is needed for renormalizability) it is clear that this $U(1)_L - U(1)_R$ mixing is finite. After diagonalization of the corresponding mass matrix, this gives a mass ρ to the $U(1)_A$ boson coupled to the difference of generators and leaves massless the boson coupled to the sum ($U(1)_V$). As advocated in Ref. 1) the mass will be of order:

$$\rho \approx \sqrt{g^2 N} \cdot 10^{15} \text{ GeV}$$

It is essential at this point to remark that the fermions L, R , in Eq. (2) stay temporarily massless. This is due to the particular structure of the group Eq. (1) and should be preserved when embedding in a grand unification group. (No mixing between the heavy left and right bosons which have to be added to (1) to unify the theory.)

^{*)} If we feel more reluctant about this, it is sufficient to add a $SU(N)$ axial group with same coupling. This will effectively make the "masses" decrease rapidly, as discussed below in the text.

Let us write down the coupling of the massless fermions:

$$\begin{aligned} \mathcal{L}_{int} = & g (\bar{L}_1, \bar{L}_2) \frac{\vec{\tau}}{2} \gamma^\mu \frac{(1-\gamma_5)}{2} \begin{pmatrix} L_1 \\ L_2 \end{pmatrix} \vec{W}^\mu \\ & + \frac{g}{\sqrt{2}} \bar{L} \gamma^\mu \frac{1-\gamma_5}{2} L (V^\mu - A^\mu) + \frac{g}{\sqrt{2}} \bar{R} \gamma^\mu \frac{1+\gamma_5}{2} R (V^\mu + A^\mu) \end{aligned} \quad (3)$$

Since A^μ now possesses a large mass, this is essentially the usual $SU(2) \times U(1)$ weak interaction Lagrangian. We now assume that a self-consistent left-right transition takes place and write down the Schwinger-Dyson equation for this (Fig. 2). If a non-trivial solution exists, it will in general give contributions to both $(L_1 - R)$ and $(L_2 - R)$ and yield a matrix of the form:

$$M = \begin{array}{ccc} & \begin{matrix} L_1 & L_2 & R \end{matrix} \\ \begin{matrix} L_1 \rightarrow \\ L_2 \rightarrow \\ R \rightarrow \end{matrix} & \begin{pmatrix} . & . & \alpha \\ . & . & \beta \\ \alpha & \beta & . \end{pmatrix} \end{array} \quad (4)$$

This is, as well known, only a technical problem: since the electric charge is not yet defined, neither α nor β will break it. An isospin rotation brings this matrix back to the more usual form of a single $(L_1 - R)$ transition, and defines simultaneously the charge axis. The self-consistent equation for the $(L_1 - R)$ transition (or lepton mass) now reads, $(\not{p} - \Sigma)$ being the inverse propagator of the (L_1, R) Dirac particle:

$$\Sigma(-q^2) = \frac{3}{16\pi} \left(\frac{g}{\sqrt{2}} \right)^2 \int \frac{\Sigma(-p^2)}{p^2 + \Sigma^2(-p^2)} \frac{p^2 d^4 p}{(p+q)^2 [(p+q)^2 + p^2]} \quad (5)$$

where a Wick rotation has been performed. The coupling constant g^2 which appears in front of Eq. (5) is of course momentum dependent. This dependence will vary according to the precise actual model envisaged (in particular, the number of low mass fermions required). For this reason, we will simply consider here the case of constant g .

Equation (5) has been studied previously⁶⁾ and shown to possess a non-trivial solution $\Sigma(-p^2)$ which decreases like $1/p^4$ for $+p^2 > p^2$, and is regular at the origin. The behaviour of Σ in the range $\Sigma^2(0) < p^2 < p^2$ is more difficult to establish, and requires a numerical solution, which proves very delicate in the case of small g .

An empirical estimation of $\Sigma_2(0)$ can be obtained by assuming that Σ keeps an appreciable value up to $p^2 = \rho^2$. This leads to the formula:

$$\Sigma(0)/\rho = \exp \left[- \left(\frac{3g^2}{16\pi^2} \right)^{-1} \right] \quad (6)$$

which we have shown in Ref. 6) to be in qualitative agreement with the results of numerical computation. This is indeed a big mass hierarchy ! ($g^2/4\pi = 0(1/10)$ gives the right order of magnitude).

The chiral symmetry breaking associated with (5) leads to the appearance of Goldstone bosons. [This is not the case when the axial boson is absent, as discussed in Ref. 6)]. These can no longer be absorbed by the $U(1)_A$ boson, which is already massive. Instead they will provide mass to the $SU(2) \times U(1)$ bosons in the now familiar way¹⁾, as is easily checked. For example, the mass of the neutral Z boson^{*} which results from the mixing of W^0 of $SU(2)$ with the $U(1)$ is found to be:

$$M_Z^2(q^2 \rightarrow 0) = \frac{g^2}{(2\pi)^4} \int \frac{d^4k}{(k^2 + \Sigma^2)^2} \Sigma(-k^2) \quad (7)$$

Depending on the behaviour of Σ at intermediate momenta $\Sigma^2(0) < p^2 < \rho^2$ (this behaviour depends itself of the detailed model considered, and in particular of the number of low mass fermions present) this integral may take values between $g^2\Sigma^2(0)$ and $g^2\Sigma^2(0) \ln \rho/\Sigma_2$.

It is thus quite clear that the fermions (L,R) cannot be the low mass particles we know, since their mass should lie between 0.1 and 1 TeV. More fermions need then be introduced, as well as the boson appropriate to "feed down" the mass generated in Eq. (5). These bosons may for instance constitute a "horizontal" generation linking group. As announced, we do not want to enter into these considerations within the framework of this note.

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^{*}) We hope to comment soon on the status of Susskind's $SU(2)$ problem in this model.

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FIGURES CAPTIONS

Fig. 1 : "Masses" of S fermions mix $U(1)_L$ and $U(1)_R$.

Fig. 2 : Self-consistent equations for $(L_1 - R)$ and $(L_2 - R)$ transitions.

FIG. 1

FIG. 2