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IMPACT PARAMETER INTERPRETATION OF PROTON-PROTON SCATTERING
FROM A CRITICAL REVIEW OF ALL ISR DATA

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ABSTRACT

This paper contains a critical review of all the data produced at the ISR on proton-proton elastic scattering and total cross-sections. This coherent and complete set of data is used to compute the impact parameter distribution of the proton-proton inelastic overlap integral. This impact parameter analysis has smaller errors than any other previously made, and confirms the good agreement with the geometrical scaling model while strongly disagreeing with models based on factorizing eikonals. For the first time we find indications of a second contribution to the peripheral rising of the proton-proton cross-section in a region around 2.2 fm.

1. INTRODUCTION

In 1971, a few months after the successful start-up of the CERN Intersecting Storage Rings (ISR) [1], the first data on elastic scattering were obtained. In the following eight years, various groups measured differential cross-sections, total cross-sections, and real parts of the forward nuclear amplitude at the ISR. All these groups have published their results and no new information is expected to come from the ISR in the next years. Having been personally involved in many of these experiments, we felt the need to conclude this extensive program of measurements by producing a coherent picture of the phenomena under study. For this reason the present paper contains a critical review of the data, which takes into account the quality of the various experiments by eliminating the (unavoidable) contradictions through a careful consideration of the sources of systematic errors.

The numerical results of such an analysis, with all the relevant tables of cross-sections, are to be published by one of us (K.R.S.) in the numerical tables of the series of Landolt-Börnstein [2]. In Section 2 of the present paper we discuss the available experiments and the choices made in their compilation.

The output of this work represents the most coherent set of data on proton-proton scattering available in the largest and (up till now) highest energy range. This allows us to complete a program that was first started in 1973 by one of us (U.A.), immediately after the discovery of the rising proton-proton cross-sections [3,4]. By computing the impact parameter distribution of the proton-proton overlap integral with the few data then available, it was possible to conclude that the rise of the total cross-section in the ISR energy range is a peripheral phenomenon [5]. This result contradicted the only previous analysis [6]. The first analysis was soon followed by others [7-10] that confirmed the result and led to the application of the very fruitful idea of geometrical scaling to proton-proton scattering [11]. With the complete and coherent set of data now at our disposal, we have performed a new overlap-integral analysis, which is presented in Section 3 together with the discussion of the propagation of the statistical and systematic errors.

Section 4 summarizes the results and compares them with the ones obtained previously.

2. COMPILATION OF THE DATA

2.1 Total cross-sections

After the first experiments, which discovered that total proton-proton cross-sections rise by about 10% in the ISR energy range [3,4], three other measurements were published [12-14]. All the published data agree within the errors (see fig. 13 of ref. 14). The data of the CERN-Pisa-Rome-Stony Brook Collaboration [13] were obtained by the authors of refs. 3 and 4 and, being much more accurate, supersede their previous results. For this reason table 1 reproduces only the final data of refs. 12, 13, and 14. Note that in experiments 13 and 14 *three* different methods were applied to obtain σ_{tot} : i) extrapolation of the elastic rate to zero angle and measurement of the luminosity by means of the Van der Meer method [15] (this is the so-called CERN-Rome method); ii) measurement of the total interaction rate and of the luminosity (Pisa-Stony Brook method); iii) luminosity-independent method, which combines the elastic rate with the total interaction rate. In ref. 12 only the second method was used, and this explains the larger errors. In the experiments of refs. 13 and 14 the largest contribution to the scale error (i.e. to the error which is equal at all energies) comes from the error in the step size of the vertical displacement of the beams used in the Van der Meer method. The errors in the experiments are thus strictly correlated, and the averaging procedure hardly implies a reduction of the scale error.

2.2 Real part of the forward nuclear amplitude

Two experiments have been performed by the CERN-Rome Collaboration to measure the ratio ρ between the real and the imaginary part of the forward nuclear amplitude [16,17] by detecting the interference effect between Coulomb and nuclear scattering. The experiments found a destructive interference that increases with energy and, through the use of a dispersion relation, have allowed the authors to reach the conclusion that the proton-proton total cross-section rises continuously at least up to a laboratory momentum of about 40 TeV/c [17]. The results are summarized in table 2.

2.3 Differential cross-sections

In the compilation of the differential cross-sections we have used the data of all the experiments [14,17,18-22] listed in table 3.

At each of the five standard ISR energies, we have normalized all the available data to the optical point and relative to each other by taking into account both the statistics and the systematic errors. The results of this procedure are summarized in table 4. We distinguish between "normalized", "unnormalized", "interpolated" and "extrapolated" data.

"Normalized" data are given in the original paper in $\text{mb}/(\text{GeV}/c)^2$ and are accompanied by a scale error. In the course of our procedure we readjust their normalization according to the general behaviour of the differential cross-section, and scale them, if necessary, by interpreting the scale error as a 1 standard deviation value. (The scaling factors are given in column 5 of table 4.)

The "unnormalized" data are given, in the original paper, in number of events per bin; by joining them with the data at smaller and larger momentum transfers we obtain the normalization constants of column 6 in table 4. In this case the scale error of each set of data (column 7 of table 4) is estimated on the basis of the scale errors of the contiguous sets. When two experiments overlap, we take the data from the experiment that has the smaller errors.

In some t -ranges there are no data and we interpolate them between the data of the two contiguous sets. For the present analysis we forget about the Coulomb interference region and include only data points above $|t| = 0.01 (\text{GeV}/c)^2$. The data in this gap are obtained by interpolation between the optical point and the first set of data using the slope b of the forward nuclear cross-section, usually written for small momentum transfers in the form

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt} \right)_0 e^{-b|t|} . \quad (1)$$

By following the procedure of ref. 13, we use for b a linearly interpolated value in $\ln s$ between the slope data measured at the largest FNAL energies [23] and the ISR measurements at large energies [18,24]. The scale error of these "interpolated" data is obtained by assuming an uncertainty $\Delta b = \pm 0.3 (\text{GeV}/c)^{-2}$. In the computations

to be described in the next section, this is the only error attributed to these fake data, since the points are taken to lie on the interpolated curves without statistical errors. At momentum transfer larger than 3.0 (GeV/c)^2 we "extrapolate" the cross-section with an energy-independent slope [22] of $1.85 \text{ (GeV/c)}^{-2}$. This is legitimate because this t -region gives a very small contribution to all the results of our analysis.

We now discuss each energy in turn, first giving the value of the optical point obtained from the values of σ_{tot} and ρ appearing in tables 1 and 2, according to the equation

$$\left(\frac{d\sigma}{dt}\right)_0 = \frac{1 + \rho^2}{16\pi(\hbar c)^2} \sigma_t^2. \quad (2)$$

At 23.5 GeV the forward cross-section is $(77.5 \pm 0.7) \text{ mb/(GeV/c)}^2$. The unnormalized data of 72B2 start at $|t| = 0.04 \text{ (GeV/c)}^2$, and to normalize them we use the slope b of the forward nuclear cross-section [13] $b = (11.8 \pm 0.3) \text{ (GeV/c)}^{-2}$. The normalization of the data of paper 72B2 turns out to be $32.7 \text{ } \mu\text{b/event}$, and to join them smoothly with the unnormalized event of paper 74B1 these last data have to be normalized with the constant 102 nb/event . The data of paper 78N1 start at $|t| = 0.82 \text{ (GeV/c)}^2$, where the data of 74B1 have larger statistical errors. A smooth join is obtained by multiplying the data of paper 78N1 by 1.075, corresponding to 1.5 standard deviations of the quoted normalization uncertainty. The data of refs. 20 and 21 are in agreement with the behaviour obtained by applying the described procedure.

At 30.7 GeV the optical point computed from eq. (1) is $82.5 \pm 0.7 \text{ mb/(GeV/c)}^2$. The minimum momentum transfer of the data of paper 72B2 is small $[0.016 \text{ (GeV/c)}^2]$, and the normalization of the first set of data is easily found: $12.7 \text{ } \mu\text{b/event}$. The second set of data coming from the same source $[0.011 \leq t \leq 0.46 \text{ (GeV/c)}^2]$ are similarly normalized with the constant $7.76 \text{ } \mu\text{b/event}$. The unnormalized data of paper 74B1 from the same group cover a contiguous t -range $[0.46\text{--}1.14 \text{ (GeV/c)}^2]$, and a smooth join requires 20.4 nb/event . The data of paper 78N1 are kept with their normalization starting from $|t| = 1.17 \text{ (GeV/c)}^2$. In the common range of the last two experiments $[0.88\text{--}1.16 \text{ (GeV/c)}^2]$ we take the average of the two sets

with the normalization fixed as described above. We remark that the points of ref. 21 agree with the data normalized as described above.

At 44.7 GeV the optical point is at $(89.6 \pm 0.7) \text{ mb}/(\text{GeV}/c)^2$. The data of papers 77A2 and 78B1 are absolutely normalized and cover contiguous ranges $[0.01-0.019 \text{ and } 0.022-0.052 (\text{GeV}/c)^2]$. These already join very well, but we apply normalization corrections of +0.6% and -1.0%, corresponding to the differences between the cross-section given in the papers and the average cross-section of table 1. These shifts correspond to 0.3 and 0.8 standard deviations of the systematic errors quoted in the two papers. From $0.053 \text{ to } 0.29 (\text{GeV}/c)^2$ we use the unnormalized data of paper 72B2 with the constant $8.2 \text{ } \mu\text{b}/\text{event}$. The next set of points starts at $|t| = 0.45 (\text{GeV}/c)^2$, but the first three points are left out because their trend does not fit the general behaviour of the differential cross-section. From $0.60 (\text{GeV}/c)^2 \text{ to } 1.16 (\text{GeV}/c)^2$ the points are normalized with the constant $1.30 \text{ nb}/\text{event}$. From $1.17 (\text{GeV}/c)^2$ onward, the data of paper 78N1 are used with their own normalization. Note that the slope of the differential cross-section around $0.05 (\text{GeV}/c)^2$ turns out to be $13.2 (\text{GeV}/c)^{-2}$, 2 standard deviations larger than the value obtained in ref. 13, by smoothly interpolating between FNAL and ISR data: $b = (12.8 \pm 0.2) (\text{GeV}/c)^{-2}$.

At 52.8 GeV the optical point is at $(93.6 \pm 0.8) \text{ mb}/(\text{GeV}/c)^2$. The normalized data of the papers 77A2 and 78B1 are slightly changed, as at 44.7 GeV (factors 1.01 and 0.99, respectively). The unnormalized data of papers 72B2 and 74B1 are treated as at the lower energy: the normalization factors turns out to be $4.62 \text{ } \mu\text{b}/\text{event}$ and $0.72 \text{ nb}/\text{event}$, while the first two points of 74B1 are left out. For momentum transfer larger than $1.27 (\text{GeV}/c)^2$ the points of paper 78N1 are used with a normalization factor of 1.03, corresponding to a shift by 0.6 standard deviations.

At 62.5 GeV the optical point is at $(96.8 \pm 1.1) \text{ mb}/(\text{GeV}/c)^2$, and the data of papers 77A2 and 78B1 are treated as for the two lower energies (the normalization factors here are 1.01 and 0.98) but, since there is an overlapping region, data from the first (second) paper are taken below (above) $0.36 (\text{GeV}/c)^2$. Data have been interpolated between $0.1 \text{ and } 0.26 (\text{GeV}/c)^2$, and from here onwards the points

of papers 75K1 [$|t| \leq 0.8 \text{ (GeV/c)}^2$] and 78N1 [$|t| \geq 0.8 \text{ (GeV/c)}^2$] can be used without changing the normalization.

The differential cross-sections at the five energies are plotted in figs. 1 and 2. As mentioned in the Introduction, the interested reader can find the numerical tables in ref. 2.

2.4 Integrated elastic cross-sections

By numerical integration of the differential cross-section we find the elastic cross-sections appearing in the last column of table 5. The other columns contain the values of the elastic cross-section as measured in previous experiments. The data of ref. 14 are systematically higher than the results of the present analysis by 0.16 mb, i.e. by one and a half times the scale error. (Note that the main source of scale error in both experiments comes from the luminosity measurement, which means that the scale errors are strongly correlated.)

By subtracting σ_{el} from σ_{tot} we get the values of the inelastic cross-section σ_{inel} reported in table 6. The quoted errors take into account the correlation between the errors on σ_{el} and σ_{tot} due to the fact that our determination of σ_{el} contains the optical point, computed from the knowledge of σ_{tot} .

The ratios of σ_{el} to σ_{tot} are given in table 7. In this case, too, the strong correlations between the errors are taken into account.

The ratio σ_{el}/σ_{tot} is plotted in fig. 3 versus $\ln s$ together with two linear fits. The probability of the fit that has free slope is small but still acceptable ($\chi^2 = 10.0$ for NDF = 3) and the slope is compatible with zero:

$$\frac{d}{d \ln s} \frac{\sigma_{el}}{\sigma_{tot}} = (0.0007 \pm 0.0040) ,$$

where s is measured in GeV^2 . The quoted error is multiplied by $\sqrt{\chi^2/\text{NDF}}$. By assuming zero slope we find the ratio

$$\sigma_{el}/\sigma_{tot} = 0.1747 \pm 0.0012 , \quad (3)$$

where the error is scaled as above.

3. OVERLAP INTEGRALS IN IMPACT PARAMETER SPACE

By neglecting nucleon spins, the differential cross-section can be written in the form

$$d\sigma/dt = |f(t)|^2 = |R(t) + i A(t)|^2, \quad (4)$$

where $R(t)$ and $A(t)$ are the real and the imaginary parts of the scattering amplitude. At high energies the scattering angles are so small that the momentum transfer $|t|$ is the square of a vector $\vec{q}$ that lies in a plane perpendicular to the direction of the incoming protons: $|t| = q^2$.

When the incoming momentum is larger than a few GeV/c, the scattering is mainly diffractive, i.e. it is due to the absorption of the incoming proton waves due to the many open inelastic channels. In the ISR energy range $R(q) \ll A(q)$ for most values of $q = \sqrt{|t|}$, and an approximate knowledge of $R(q)$ is sufficient to invert eq. (3) and determine $A(q)$:

$$A(q) = [(d\sigma/dt) - R^2(q)]^{1/2}. \quad (5)$$

It follows from the unitarity of the S-matrix that the absorptive part $A(q)$ of the elastic scattering amplitude receives contributions both from the open inelastic channels and from the elastic channel itself. These contributions have been named elastic and inelastic overlap integrals [25]. By applying a Fourier transformation, the unitarity condition can be written in impact parameter space (variable $\vec{a}$) instead of in momentum space (variable $\vec{q}$). In a corpuscular description of the collision, $\vec{a}$ is the vector joining the directions of flight of the colliding protons.

In the notations of the review paper of ref. 26, the unitarity condition is

$$2 \operatorname{Re} \Gamma(a) = |\Gamma(a)|^2 + G_{\text{in}}(a), \quad (6)$$

where $\Gamma(a)$ is the elastic "profile function" defined as

$$\Gamma(a) = \frac{1}{i\hbar c\sqrt{\pi}} \int_0^\infty dq \, q \, f(q) J_0(qa/\hbar c), \quad (7)$$

and $G_{\text{in}}(a)$ is the (real) inelastic overlap integral in impact parameter space; J_0 is the Bessel function of zero order, and $|\Gamma(a)|^2$ in eq. (6) represents the contribution of the shadow of the elastic channel. We use eqs. (5), (6), and (7)

to compute $\Gamma(a)$ and $G_{in}(a)$ at the five standard ISR energies. Before presenting the results, we discuss the model used to obtain $R(q)$ and the treatment of the experimental errors.

It can be checked *a posteriori* that $R(q)$ has a very small effect on $\Gamma(a)$ (less than 1% for all values of a). It is thus sufficient to compute $R(q)$ from an expression of $f(t)$ that fits the data and has almost the correct analytical properties:

$$f(t) = i\alpha [A_1 \exp(-b_1\alpha t/2) + A_2 \exp(-b_2\alpha t/2)] - i A_3 \exp(-b_3 t/2), \quad (8)$$

where α is complex and is given by

$$\alpha = \alpha(s) = [\sigma_{tot}(s)/\sigma_{tot}(23 \text{ GeV})] [1 - i\rho(s)] \quad (9)$$

and ρ is the ratio R/A measured at $t = 0$ (table 2). This parametrization gives the correct value of $R(0)/A(0)$, fills the diffraction minimum by the observed amount [22], and makes the real part of the amplitude equal to zero for $|t| = 0.2 \text{ (GeV/c)}^2$, as required by an analysis based on dispersion relations [9]. Figure 4 shows the t -dependence of the ratio R/A computed from eq. (8), where we used the parameters of ref. 22:

$$\begin{aligned} A_2^2 &= 11.29 \text{ mb}/(\text{GeV/c})^2, & A_3^2 &= 0.0025 \text{ mb}/(\text{GeV/c})^2 \\ b_2 &= 7.67 \text{ (GeV/c)}^{-2}, & b_3 &= 1.858 \text{ (GeV/c)}^{-2}, \end{aligned} \quad (10)$$

and A_1 and b_1 were chosen to meet the optical points:

$$A_1^2 = 30.14 \text{ mb}/(\text{GeV/c})^2, \quad b_1 = 14.3 \text{ (GeV/c)}^{-2}.$$

The divergence of the ratio R/A near $|t| = 1.4 \text{ (GeV/c)}^2$ clearly indicates the main hypothesis made in writing eq. (8): the diffraction minimum is due to a single zero in the absorptive part $A(q)$, and the minimum is filled by the real part $R(q)$ [27].

We now discuss the treatment of the experimental errors in the numerical calculation of the integral entering eq. (7), which main contribution is of the form

$$I(a) = \sum_{i=1}^N I_i(a) = \sum_{i=1}^N \int_{q_i}^{Q_i} dq q A(q) J_0(qa/\hbar c). \quad (11)$$

The sum extends on the N sets of data available at each energy and listed in table 4, each covering a range $q_i - Q_i$; $A(q)$ is computed from eq. (5) and is

essentially given by $\sqrt{d\sigma/dt}$. Since we assume that $A(q)$ has a single zero around $|t| = 1.4 \text{ (GeV/c)}^2$, its contribution to the integral is positive below this t -value and negative above it. Each set of data is represented by a histogram of binning Δq . For each bin Δq the integral is numerically computed by adapting the integration step to the variation of J_0 within the considered bin.

Four types of errors have to be taken into account:

- i) The statistical errors on the points within each set of "normalized" and "unnormalized" data: the statistical errors in each bin Δq are combined quadratically by taking into account the contribution of each bin to the total integral.
- ii) The scale errors of each set of data (last column of table 4): similarly, these errors are properly weighted and combined quadratically.
- iii) The errors on the "interpolated" and "extrapolated" data: these are represented by points that have no statistical error, but have a generously estimated scale error that is treated as under point (ii).
- iv) The error on the optical point, to which $d\sigma/dt$ has been normalized: the error on the total cross-section affects the over-all normalization of the profile function $\Gamma(a)$ by an amount which is smaller than the other errors. For this reason we take the pessimistic choice of combining it quadratically with the others. The over-all scale error of $\pm 0.25 \text{ mb}$ (table 1) influences all energies in the same way and is kept separate.

Figure 5 shows the results of our analysis as far as the inelastic overlap functions $G_{in}(a)$ are concerned. The vertical bars are the total errors computed as described above, and have the meaning of ± 1 standard deviation. It must be noted that the errors at different values of the impact parameter are strongly correlated. In fig. 6 we have plotted G_{in} versus the c.m. energy for various values of the impact parameter a . (Numerical tables are available from the authors.)

4. DISCUSSION OF RESULTS

At fixed energy, the inelastic overlap function $G_{in}(s,a)$ is a nearly Gaussian function of the impact parameter a . As may be seen in fig. 5f for one energy, $G_{in}(a)$ is slightly below a Gaussian for small impact parameters and has a long tail above for large impact parameters. This property has been noted before [5,8,10].

Figure 6 shows the energy dependence of $G_{in}(s,a)$ at four values of a . The values at $a = 0$ are compatible with an energy independence in the ISR energy range, but the χ^2 of a straight line fit in $\ln s$ is rather poor ($\chi^2/NDF = 15/3$). The largest contribution to χ^2 comes from the point at 44.7 GeV, which is low because the forward slope of the used data is 2 standard deviations away from the smooth energy interpolation, as mentioned in Section 3. The dashed error of fig. 6a shows the displacement of $G_{in}(0)$ as expected from this effect. In fig. 7a we have added the $a = 0$ results at lower energies, as obtained in the analysis of refs. 28 and 29. The central opacity of the pp collision above $\sqrt{s} = 7$ GeV is first decreasing with energy and then reaches an energy-independent value.

This conclusion is in agreement with refs. 5 and 7-10 but is at variance with refs. 6 and 30. The recent analysis [30] confirms an older result [6] by finding again a rise of $G_{in}(s,0)$ in the ISR energy range, which is incompatible with our results. We attribute this difference to two choices taken in ref. 30: i) at $\sqrt{s} = 62$ GeV, the forward slope was taken to be $12.4 (\text{GeV}/c)^{-2}$ instead of the measured value [13] 13.3 ± 0.3 ; ii) at $\sqrt{s} = 23$ GeV and small t -values, a single experiment [20] was selected, whereas we use all available data.

The mean interaction radius of the (inelastic) pp collision may be derived from $G_{in}(s,a)$. We adopt the definition of ref. 29:

$$\sqrt{\langle a^2 \rangle} = \left[\frac{\int_0^{a_{\max}} a^2 G_{in}(a) a da}{\int_0^{a_{\max}} G_{in}(a) a da} \right]^{1/2}, \quad (12)$$

where the result depends strongly on the choice of $a_{\max}$. This choice is important since the uncertainties in the tail of G_{in} , originating from very small details

in the small t -region of the elastic cross-section, are so large that their weighting with a^3 leads to large uncertainties in $\langle a^2 \rangle$. The authors of ref. 29 took $a_{\max} = 1.6$ fm, and their results are shown in fig. 7b. In order to compare our results and to demonstrate the energy dependence of $\langle a^2 \rangle$ over a wide energy range, in fig. 7b we show our results with the following choice of $a_{\max}$:

$$G_{\text{in}}(s, a_{\max}) = 0.085 \cdot G_{\text{in}}(s, 0) . \quad (13)$$

In the presence of an increasing interaction region, this choice is more adequate than a fixed value of $a_{\max}$, and the factor 0.085 is arbitrary but matches the choice of ref. 29 at $\sqrt{s} \approx 20$ GeV. Using eq. (13), we find $\sqrt{\langle a^2 \rangle} = (0.888 \pm 0.005)$ and (0.926 ± 0.005) fm at $\sqrt{s} = 23.5$ and 62.5 GeV, corresponding to a rise of $(4.3 \pm 1.0)\%$ in the ISR range. Note, however, that these values are obtained from an integration which covers only 90% of σ_{inel} . Going out to 99% of σ_{inel} ($a_{\max} \approx 2.8$ fm) leads to mean radii of 1.03 and 1.08 fm, which are 15% larger.

In fig. 8 we show the energy variation $\Delta G_{\text{in}}(a)$ of the inelastic overlap function as a function of the impact parameter. This quantity is defined as

$$\Delta G_{\text{in}}(a) = \frac{d G_{\text{in}}(\ln s, a)}{d \ln s} (\ln s_5 - \ln s_1) , \quad (14)$$

where $s_5 = (62.5 \text{ GeV})^2$, $s_1 = (23.5 \text{ GeV})^2$, and $d G_{\text{in}}/d \ln s$ is the slope of a χ^2 -fitted straight line through the five G_{in} results at fixed impact parameter. The errors on ΔG_{in} are the fit errors, increased by $\sqrt{\chi^2/\text{NDF}}$ if χ^2 is larger than the number of degrees of freedom; $\Delta G_{\text{in}}(a)$ has a peripheral shape with a maximum at 1 fm and a weakly pronounced second structure at 2.2 fm. At zero impact parameter we find $\Delta G_{\text{in}} = 0.0002 \pm 0.0086$, and the maximum of ΔG_{in} is reached at 1.00 fm with 0.0390 ± 0.0031 . This is in qualitative agreement with the results of previous analyses [5,7-10] and also with very recent work by Henzi and Valin [31]. As discussed before, the value of $\Delta G_{\text{in}}(0)$ is ~ 2 standard deviations away from the result of ref. 30, where $\Delta G_{\text{in}}(0) = 0.042 \pm 0.022$. The second structure, not previously found in any analysis, has an estimated confidence level of 2 standard deviations. Looking at fig. 5f, it can be noticed that the position of this structure coincides with the onset of the tail above the Gaussian that approximates $G_{\text{in}}(a)$ at small impact parameters.

The energy variation of the inelastic overlap function may be compared with the expectations of two simple models: geometrical scaling (GS) and factorizing eikonals (FE). The elastic scattering eikonal $\Omega(s,a)$ is defined by

$$\Gamma(s,a) = 1 - e^{-\Omega(s,a)} , \quad (15)$$

and one may factorize it in the form [32]

$$\Omega(s,a) = w(s) \cdot g[a/r(s)] ,$$

where w and g are only functions of the energy. The two much-discussed extremes of this ansatz are: GS [11] with $w(s) = \text{const}$, $r(s) \propto \sqrt{\sigma_{\text{tot}}(s)}$, and FE [33,34] with $r(s) = \text{const}$, $\Omega(s,a) = w(s) \cdot g(a)$.

In order to test GS, fig. 9a shows the quantity

$$\Delta G_{\text{in}}\left(\frac{a}{r}\right) = \frac{d G_{\text{in}}(\ln s, a/r)}{d \ln s} (\ln s_5 - \ln s_1) . \quad (16)$$

Here s_5 and s_1 are the same as in eq. (14), and $d G_{\text{in}}/d \ln s$ is the slope of a χ^2 -fitted straight line through the five G_{in} results at fixed values of a/r , where

$$r = r(s) = \left[\sigma_{\text{tot}}(s) / \sigma_{\text{tot}}(23 \text{ GeV}) \right]^{1/2} . \quad (17)$$

GS predicts $\Delta G_{\text{in}}(a/r) \equiv 0$, and our results are in good agreement with this expectation.

In order to test FE, one has to examine $d \ln \Omega / d \ln s$, because

$$\Omega(s,a) = \Omega(s_0,a) \cdot f(\ln s)$$

with an a -independent function $f(\ln s)$. From this it follows that

$$\begin{aligned} \ln \Omega(s,a) &= \ln \Omega(s_0,a) + \ln f(\ln s) , \\ \frac{d \ln \Omega(s,a)}{d \ln s} &= \frac{d \ln f}{d \ln s} \quad (\text{independent of } a) . \end{aligned} \quad (18)$$

Figure 9b shows the quantity

$$\Delta \ln \Omega(a) = \frac{d \ln \Omega(\ln s, a)}{d \ln s} (\ln s_5 - \ln s_1) , \quad (19)$$

where the slope of a χ^2 -fitted straight line appears through five results for $\ln \Omega$ at fixed impact parameters. The errors are increased by $\sqrt{\chi^2/\text{NDF}}$ if $\chi^2 > \text{NDF}$. As may be seen in the figure, the hypothesis of FE is *not* fulfilled since $\Delta \ln \Omega(a)$ depends strongly on a .

Our results rule out FE models, while they are in good agreement with the GS hypothesis. An independent argument, leading to the same conclusion, is the measured energy dependence of the differential cross-section in the second maximum around $t = -2(\text{GeV}/c)^2$. It is expected to be strong in FE [34], and it is found to be weak [22] in rather good agreement with GS.

5. CONCLUSIONS

From seven ISR experiments on elastic pp scattering, one can extract a coherent set of data that leads to an impact parameter representation having smaller errors than in any other analysis. This impact parameter analysis supports previous conclusions on a constant central opacity and an increasing mean interaction radius over the ISR energy range, and is in good agreement with the geometrical scaling model. It is in strong disagreement with models based on a factorizing eikonal. The main contribution to the increasing mean interaction radius comes from a rise of the overlap function around 1 fm, but we also find for the first time an indication of a second contribution from a region around 2.2 fm.

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Table 1

Total proton-proton cross-sections

σ_{tot} (mb) $\sqrt{s}$ (GeV)	From ref. 12	From ref. 13	From ref. 14	Average
23.5	38.70 ± 0.70	38.89 ± 0.21	39.1 ± 0.3	38.94 ± 0.17
30.7	40.00 ± 0.60	40.17 ± 0.21	40.1 ± 0.3	40.14 ± 0.17
44.7	42.50 ± 0.80	41.66 ± 0.19	42.0 ± 0.3	41.79 ± 0.16
52.8	42.90 ± 0.70	42.46 ± 0.26	42.9 ± 0.3	42.67 ± 0.19
62.5	44.10 ± 0.90	43.04 ± 0.29	43.7 ± 0.4	43.32 ± 0.23
Scale error		± 0.28	± 0.32	± 0.25

Table 2

Ratio ρ between the real and the imaginary part of
the forward nuclear amplitude

$\sqrt{s}$ (GeV)	ρ	Ref.
23.5	0.02 ± 0.05	16
30.7	0.042 ± 0.011	17
44.7	0.062 ± 0.011	17
52.8	0.078 ± 0.010	17
62.5	0.095 ± 0.011	17
Scale error	± 0.015	

Table 3

Sources of differential proton-proton cross-sections

Papers (as indicated in ref. 2)	Ref.	Energies $\sqrt{s}$ (GeV)	Remarks
72B2	18	23.5, 30.7, 44.7, 52.8	The data are unnormalized and are based on a subset of the published data (private information from P. Strolin)
74B1	19	23.5, 30.7, 44.7, 52.8	The data are not normalized
75K1	20	23.5, 62.5	The t-range is 0.21-0.8 (GeV/c) ² . Above 0.8 (GeV/c) ² the data are replaced by those of 78N1
76A2	21	23.5, 30.7, 32.4, 35.2, 38.3	Normalized to $\pm 10\%$
77A2	17	30.7, 44.7, 52.8, 62.5	These data are in the Coulomb interference region. At $\sqrt{s} = 62.5$ they extend further
78B1	14	44.7, 52.8, 62.5	The data correspond to the published figure (private communication from A. Böhm)
78N1	22	23.5, 30.7, 44.7, 52.8, 62.5	The data are in the large t-region: $ t \gtrsim 0.8$ (GeV/c) ²

Table 4

Compilation of the data
on the proton-proton differential cross-section

$\sqrt{s}$ (GeV)	t-range (GeV/c)	Paper	Type of data	Scaling factors	Normaliz. const. (nb/event)	Scale error (%)
23.5	0.0 -0.04	72B2 74B1 78N1	interpolated	1.075	32700 102	1.0
	0.04 -0.24		unnormalized			1.2
	0.24 -0.80		unnormalized			3.0
	0.80 -3.0		normalized			5.0
	3.0 -6.0		extrapolated			10.0
30.7	0.0 -0.015	72B2 72B2 74B1 78N1	interpolated	0.985 1.000	12700 7760 20.4	0.4
	0.015-0.112		unnormalized			0.5
	0.112-0.458		unnormalized			2.0
	0.458-0.88		unnormalized			3.5
	0.88 -1.16		averaged			5.0
	1.16 -3.0		normalized			5.0
44.7	0.0 -0.009	77A2 78B1 72B2 74B1 78N1	interpolated	1.006 0.990 1.000	8200 1.3	0.25
	0.009-0.02		normalized			1.0
	0.02 -0.053		normalized			1.0
	0.053-0.29		unnormalized			2.0
	0.29 -0.60		interpolated			3.5
	0.60 -1.16		unnormalized			5.0
	1.16 -3.0		normalized			5.0
	3.0 -6.0		extrapolated			10.0
52.8	0.0 -0.009	77A2 78B1 72B2 74B1 78N1	interpolated	1.010 0.990 1.030	4.620 0.72	0.25
	0.009-0.018		normalized			1.0
	0.018-0.03		interpolated			1.0
	0.03 -0.075		normalized			1.0
	0.075-0.45		unnormalized			2.0
	0.45 -0.72		interpolated			3.5
	0.72 -1.27		unnormalized			5.0
	1.27 -3.0		normalized			5.0
62.5	0.0 -0.009	77A2 78B1 75K1 78N1	interpolated	1.010 0.980 1.000 1.000		0.25
	0.009-0.036		normalized			1.0
	0.036-0.10		normalized			1.0
	0.10 -0.265		interpolated			3.0
	0.265-0.8		normalized			5.0
	0.8 -3.0		normalized			5.0
	3.0 -6.0		extrapolated			10.0

Table 5
Elastic cross-sections

σ_{el} (mb) $\sqrt{s}$ (GeV)	From ref. 3 (1973)	From ref. 14 (1978)	Present analysis (1979)
23.5	6.8 ± 0.2	6.82 ± 0.08	6.73 ± 0.08
30.7	7.0 ± 0.2	7.39 ± 0.08	7.16 ± 0.09
44.7	7.5 ± 0.3	7.45 ± 0.08	7.17 ± 0.09
52.8	7.6 ± 0.3	7.56 ± 0.08	7.45 ± 0.09
62.5		7.77 ± 0.10	7.66 ± 0.11
Scale error	± 0.3	± 0.1 a)	± 0.09

a) This error is estimated on the basis of the error on the luminosity quoted in ref. 14.

Table 6
Inelastic cross-sections

σ_{inel} (mb) $\sqrt{s}$ (GeV)	From ref. 14 (1978)	Present analysis (1979)
23.5	32.40 ± 0.14	32.21 ± 0.14
30.7	32.79 ± 0.22	32.98 ± 0.14
44.7	34.55 ± 0.22	34.62 ± 0.14
52.8	35.07 ± 0.20	35.22 ± 0.16
62.5	35.79 ± 0.24	35.66 ± 0.21
Scale error	± 0.20 a)	± 0.20

a) This error is estimated on the basis of the error on σ_{tot} quoted in table 1.

Table 7
Ratios of elastic to total cross-sections

σ_{el}/σ_{tot} $\sqrt{s}$ (GeV)	From ref. 14 (1978)	Present analysis (1979)
23.5	0.174 ± 0.003	0.1728 ± 0.0016
30.7	0.184 ± 0.003	0.1784 ± 0.0017
44.7	0.177 ± 0.003	0.1716 ± 0.0018
52.8	0.177 ± 0.003	0.1746 ± 0.0016
62.5	0.178 ± 0.003	0.1768 ± 0.0021

Figure captions

- Fig. 1 : Selected set of differential cross-sections between 0 and 0.4 (GeV/c)². The selection and normalization procedure is described in the text. The errors shown are statistical only. The solid lines indicate the interpolations chosen in the regions where no data exist.
- Fig. 2 : Selected set of differential cross-sections between 0 and 4 (GeV/c)². The solid lines indicate interpolated values where no data exist.
- Fig. 3 : Ratio of the integrated elastic cross-section to the total cross-section as a function of energy. The errors are one-standard-deviation statistical errors. The dotted line indicates a χ^2 -fitted straight line for σ_{el}/σ_{tot} versus $\ln s$: its slope is compatible with zero. Imposing zero slope leads to the solid line with $\langle \sigma_{el}/\sigma_{tot} \rangle = 0.1747 \pm 0.0012$.
- Fig. 4 : Ratio of the real to the imaginary part of the pp elastic scattering amplitude for one energy as given by eqs. (8), (9), and (10) and used for evaluating $G_{in}(a)$ with the help of eqs. (4)-(7). The precise knowledge of the real part $R(t)$ is not essential since its effect on $G_{in}(a)$ is less than 1% at all values of a .
- Fig. 5 : The inelastic overlap functions as a function of the impact parameter at the five ISR energies. On the linear scale (a-e), the errors on G_{in} are smaller than the points drawn. The solid line on the logarithm scale (f) is a Gaussian adjusted to fit at $a = 0$ and $a = 1.6$ fm. A Gaussian adjusted between 0.6 and 1.6 fm would be higher at $a = 0$.
- Fig. 6 : Inelastic overlap function as a function of the energy at four fixed impact parameters. The solid lines are χ^2 -fitted straight lines of G_{in} versus $\ln s$. The dotted error bar for G_{in} (44 GeV, $a = 0$) is explained in the text.

Fig. 7 : a) Central opaqueness $G_{in}(0)$, and b) mean (inelastic) interaction radius $\sqrt{\langle a^2 \rangle}$, defined by eqs. (12) and (13), as a function of the energy. The solid points are from this analysis. The dotted error bar at 44 GeV is explained in the text. The open points are from ref. 29, and the triangle is from ref. 28.

Fig. 8 : Increase ΔG_{in} of the inelastic overlap function over the ISR energy range as a function of the impact parameter; ΔG_{in} is obtained by using eq. (14) and the results at all five energies.

Fig. 9 : Increase ΔG_{in} of the inelastic overlap function over the ISR energy range is compared with two theoretical models. a) Geometrical scaling predicts $\Delta G_{in}(a/r) = 0$, where $r = [\sigma_{tot}(s)/\sigma_{tot}(s_0)]^{1/2}$. Our results follow this expectation reasonably well. At fixed values of a (fig. 8), G_{in} reaches 0.04, whereas here, at fixed values of a/r , its absolute value is smaller than 0.006 everywhere. b) Factorizing eikonal models predict $\Delta \ln \Omega$ to be a constant independent of a [see eq. (15) for the definition of the eikonal Ω , and eqs. (18), (19)]. Our results contradict this expectation.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8

Fig. 9