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THE CONFINEMENT OF MASSLESS NON-ABELIAN GLUONS AND
RISING TOTAL CROSS SECTIONS

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ABSTRACT

It is argued that the critical Pomeron solution of the Reggeon Field Theory (RFT) occurs in massless Yang-Mills theories with an $SU(N)$ gauge group. A proviso is that gluon confinement must allow a principal-value regularization of the Regge limit infra-red divergences. The super-critical RFT phase occurs when the gauge symmetry is spontaneously broken down to a $U(1)$ symmetry. $SU(3)$ seems to be the minimum structure for the (broken) gauge group in the super-critical phase.

The method used is to extend the multi-Regge dispersion relation calculations of Fadin, Kuraev and Lipatov to a full RFT of the Pomeron applicable to Yang-Mills. The presence of a Pomeron satisfying RFT is closely related to the regularization procedure used.

Since the critical Pomeron is the only known unitary (s and t channel) description of rising cross-sections the results imply that their experimental observation is direct evidence for a non-Abelian gauge symmetry underlying strong interactions. The results also imply that diffraction scattering is precisely calculable in QCD.

When the experimental evidence which compels us towards QCD as a theory of the strong interaction is reviewed¹⁾ it is normal to discuss deep inelastic scaling phenomena (asymptotic freedom), the quark model of hadrons (SU(3) colour), the string model of the resonance spectrum (confined colour flux tubes) etc. The central property of diffraction scattering (the Pomeron), which rising cross-sections show dominates strong interactions at high energy, is usually given little or no mention. The strongest comment that has been made so far is that a Pomeron resulting from the exchange of confined gluons would have many desirable phenomenological properties^{2),3)}. This is certainly consistent with the string model picture of the Pomeron as formed from closed strings and we shall return to this point when comparing our work with recent work on dual strings⁴⁾. However, since the Pomeron has clearly emerged at ISR energies, with the resonance structure having disappeared (the pp differential cross-section can be interpreted as almost pure diffraction⁵⁾), it can be argued that a correct theory of the strong interaction should be able to describe the Pomeron without a detailed knowledge of the dynamics of the resonance spectrum, that is string formation. In this paper we shall argue that this is indeed the case for QCD and that the experimental observation of rising cross-sections is further direct evidence for the presence of confined, massless, non-Abelian gluons (and therefore a non-Abelian gauge symmetry) in nature. Our claim will be that a non-Abelian gauge symmetry is a sufficient condition to obtain the so-called critical Pomeron^{6),7)} (with SU(3) being possibly the minimum structure for the gauge group). We have previously argued⁸⁾ that the critical Pomeron is the only solution of the Reggeon Field Theory^{5),9)-11)} (RFT) consistent with both t - and s -channel unitarity giving rising total cross-sections.

The basis of our argument will be the extensive calculations of Fadin, Kuraev and Lipatov¹²⁾ on multi-Regge behaviour in spontaneously broken Yang-Mills theories, which we develop into the full RFT of the Pomeron. However, we make a fundamental modification of this work in that we replace the usual $i\epsilon$ rule by a principal-value prescription to regulate the t channel infra-red divergences in the zero-mass limit. If the gluons are indeed confined, then any infra-red regulation, including ours, should give the correct answer. However, since this regulation is in effect our input of the confinement and since it is so important for producing a Pomeron describable by RFT we immediately make several comments on it. The regularization is comparable to taking the real part of the Feynman propagator except that since our method of calculation involves only S matrix dispersion relations only gauge invariant quantities appear. We note nevertheless that:

- a) The ambiguity in the quantization of massless non-Abelian Yang-Mills theories found by Gribov¹³⁾ is an ambiguity of the resolution of the Yang-Mills field into positive and negative frequency parts due to the possibility of classical ghost production. If we wish to retain asymptotic freedom then we can hope that this ambiguity can be confined to the imaginary part of the propagator at high energy.
- b) The failure of the spontaneously broken theory to give the massless Yang-Mills theory in the zero-mass limit is due to an incorrect description of ghost loops¹⁴⁾.

By effectively basing our arguments on the real-part we can therefore hope to have avoided the full problems of the quantization and the zero-mass limit. We further note that:

- c) Our prescription has the effect that the t channel gluon particle thresholds (for non-zero mass) are replaced by Regge singularities which, in the zero-mass limit, generate the Pomeron - this seems consistent with the general picture of string formation.
- d) Our principal-value prescription is reminiscent of the principal-value prescription¹⁵⁾⁻¹⁷⁾ used to obtain confinement in two-dimensional QCD.

Our prescription may therefore have direct connections with other descriptions of confinement. Finally, we comment that

- e) The calculations of Lipatov et al. explicitly show that in the high-energy limit in the vacuum channel there is a cancellation of the t channel gluon singularities in the leading logarithmic approximation. As a result our prescription gives agreement with perturbation theory, even for the massive theory, in the vacuum channel. The work of Lipatov et al. also shows (what our results confirm) that the high-energy limit in the vacuum channel is controlled by the transversely polarized gluons so that the details of the Higgs effect and the behaviour of the longitudinal gluons do not enter our arguments.

The second ingredient in our arguments is the application of general RFT phase-transition analysis to Yang-Mills. Until now RFT has been related to specific underlying strong interaction theories only in a rather general way¹⁸⁾. That is we have understood how we expect RFT to emerge from an underlying theory without knowing a precise connection between the parameters of RFT and those of any underlying theory. We remark, however, that since we can justify⁵⁾ RFT simply on the basis of multiparticle unitarity and general multiparticle dispersion relations¹⁹⁾ it can even be argued that its status is better than any candidate strong interaction theory whose unitarity remains unproved. In fact, it is a vital part of our arguments that the specific form of RFT in Yang-Mills can be explicitly derived from the same multiparticle dispersion relations that prove its general status.

Reggeon field theory can be thought of as an effective field theory (isolating the relevant degrees of freedom) for the study of a phase-transition phenomenon at high energy. From our recent study⁸⁾ of the nature of the possible phase transitions we have argued that we can obtain a Pomeron satisfying all unitarity constraints (s and t channel) and giving rising total cross-sections only precisely at the strong-coupling critical point (in the RFT parameter space). At this point we obtain the beautiful, calculable with no parameters, critical Pomeron^{6),7)}. In fact it cannot be emphasized strongly enough that if we can prove that the critical Pomeron occurs in any underlying theory (as we shall argue for QCD) then for the first time we can calculate a theory precisely in the high-energy limit.

When the critical Pomeron was first discovered only the sub-critical phase was known and it seemed that without a precise understanding of the relationship between the RFT parameters and those of an underlying theory, it would be impossible to determine whether or not the Pomeron was at the critical point. However, our subsequent study of the super-critical phase⁸⁾ has shown that it corresponds to the emergence of an odd-signature degenerate partner to the Pomeron. We also noted that if this partner could be identified with the Reggeized gluon of a non-Abelian gauge theory the critical Pomeron would be obtained in a limit in which the gluons became both massless and confined. Thus, the unknown RFT parameter, the Pomeron intercept, would become related to a specific parameter of the underlying theory -- namely the gluon mass. On a deeper level the criticality of the Pomeron would be seen to be related to exact gauge invariance of the underlying theory. The purpose of this paper is, therefore, to show that with the proposed regularization of massless gluons this conjectured connection between super-critical RFT and Yang-Mills theory with massive Reggeons can indeed be made.

In fact, our results can also be compared directly with recent work on dual strings⁴⁾. The super-critical RFT phase can be described by the occurrence of a vacuum expectation value for the Pomeron field and our result will be that this corresponds to Yang-Mills with all of the non-Abelian symmetry spontaneously broken, leaving just an Abelian symmetry. Consequently the RFT phase transition can be seen as the appearance of a (confined) Abelian gluon field. The corresponding result in dual string theory is that when (closed) dual strings condense, that is the Regge slope goes to infinity (as is the case for the critical Pomeron), an Abelian Yang-Mills field appears describing the vacuum expectation value of the dual field (defined on closed strings). Since in RFT we lose one transverse dimension it is consistent that the Pomeron expectation value is the high-energy equivalent of the dual string expectation value. Note that our claim that the non-Abelian symmetry appears only at the critical point is impossible at present to see in dual string

theory, since the Pomeron intercept depends very critically on the detailed quantization of the theory.

We now outline in a little more detail our results on supercritical RFT. We shall then go on to show how the structure we find emerges from Yang-Mills. The RFT Lagrangian is

$$\begin{aligned} \mathcal{L}(\bar{\psi}, \psi) = & \frac{1}{2} \bar{\psi} \overleftrightarrow{\frac{\partial}{\partial y}} \psi + \alpha'_0 \nabla \bar{\psi} \cdot \nabla \psi + \Delta_0 \bar{\psi} \psi \\ & + i \frac{r_0}{2} [\bar{\psi}^2 \psi + \bar{\psi} \psi^2] + \dots \end{aligned} \quad (1)$$

where $\bar{\psi}$ is a Pomeron creation generator, ψ a Pomeron destruction generator, $\alpha_0 = (1 - \Delta_0)$ is the bare Pomeron intercept, α'_0 the bare Pomeron slope and r_0 the bare triple Pomeron interaction. The fields $\bar{\psi}$ and ψ are functions of rapidity y and impact parameter $\underline{x}$. Higher order interactions not shown explicitly in (1) are unimportant near the strong-coupling critical point which is approached as Δ_0 decreases to Δ_{0c} where²⁰⁾

$$\Delta_{0c} = \frac{r_0^2}{16\pi \alpha'_0} \ln \left[\frac{r_0^2}{\alpha'^2 \Lambda} \right] + O \left(\frac{r_0^2}{\alpha'_0} \right) \quad (2)$$

Λ is a cut-off in the transverse momentum $\underline{k}$ (which is conjugate to $\underline{x}$). Beyond the critical point the fields $\bar{\psi}$ and ψ acquire rapidity-dependent expectation values which can be taken to be

$$\langle \bar{\psi} \rangle = \frac{2i[\Delta_0 - \Delta_{0c}]}{r_0} \Theta(y - y_0) \quad (3)$$

$$\langle \psi \rangle = \frac{2i[\Delta_0 - \Delta_{0c}]}{r_0} \Theta(y_0 - y) \quad (4)$$

if y_0 and $\bar{y}_0$ are averaged over⁸⁾. $\langle \bar{\psi} \rangle$ must vanish for $y = -\infty$ and $\langle \psi \rangle$ must vanish for $y = +\infty$ to maintain the creation, destruction property, for these operators. The modified RFT perturbation expansion obtained in the presence of these expectation values can be described as follows.

A) There is a shifted bare propagator

$$\left[E - |\Delta_o - \Delta_{oc}| - \alpha' k^2 \right]^{-1} \quad (5)$$

where $E = 1 - j$ (j = complex angular momentum) is the "energy" variable conjugate to y .

B) There are new Pomeron interaction vertices which are illustrated diagrammatically in Fig. 1. In these diagrams the circles are the usual RFT Green's functions (with the new bare propagator) evaluated in old-fashioned perturbation theory with the time-ordering implied by the diagrams.

The vacuum production and absorption vertices are given by

$$\langle \text{diagram} \rangle = \text{diagram} = |\Delta_o - \Delta_{oc}| \delta(E) \delta^2(k^2) \quad (6)$$

and are directly due to the expectation values (3) and (4). The chief characteristic of the new production vertices are that they are singular (outside the physical region) as functions of momentum transfer. For example, the simplest new addition to the triple Pomeron vertex is the diagram of Fig. 2 which is given by

$$\frac{i r_o (\Delta_o - \Delta_{oc})}{\alpha' t_1 - (\Delta_{oc} - \Delta_o)} \quad (7)$$

The super-critical RFT defined by the vertices of Fig. 1 satisfies Reggeon unitarity (and therefore t channel unitarity) but the origin of the singular momentum transfer dependence is at first sight obscure. An answer is suggested by asking for the s channel unitarity properties of the theory. To this end we consider the "cut RFT"^{18),21)} which incorporates the AGK cutting rules²²⁾ and defines RFT for all inclusive amplitudes in such a way that the inclusive sum rules are satisfied.

In the cut RFT formalism the Lagrangian (1) is extended to include three sets of fields $\bar{\psi}_+, \psi_+ (\equiv \bar{\psi}, \psi)$, $\bar{\psi}_-, \psi_-, \bar{\psi}_c, \psi_c$ with the interactions illustrated in Fig. 3. The Green's functions $\langle \bar{\psi}_c, \psi_c \rangle$, $\langle \bar{\psi}_+, \psi_+ \rangle$, $\langle \bar{\psi}_-, \psi_- \rangle$ give respectively the cut amplitude, the amplitude above the cut and the amplitude below the cut, and for an even signature

amplitude these Green's functions are, of course, equal and pure imaginary. In the super-critical phase all three fields acquire expectation values proportional to $|\Delta_0 - \Delta_{0c}|$. As a result, transitions of the form $\langle \bar{\psi}_c, \psi_+ \rangle$, given, for example, by the diagram shown in Fig. 4 occur and it is necessary to define new diagonal amplitudes. The new cut and physical amplitudes are defined uniquely by the requirement that they are equal for all Δ_0 and continuously equal to their sub-critical equivalents at the critical point. Examining the Reggeon unitarity content of these amplitudes at the one-loop level we find that in addition to the negative two Pomeron cut contribution corresponding to the triple Pomeron vertex shown in Fig. 1, there is a positive sign two-Reggeon cut term, involving just singular vertices, corresponding to a degenerate set (at least two) of odd-signature Reggeons with a trajectory degenerate with that of the Pomeron.

It follows then that s channel unitarity requires odd-signature Reggeons to accompany the Pomeron in the super-critical phase. This immediately leads us to note that the singularity structure of the super-critical Pomeron vertices - including (7) - could indeed be produced by the Pomeron interacting through vector particles whose mass went to zero with $(\Delta_0 - \Delta_{0c})$. These observations gave rise to the above conjecture even without a detailed consideration of an underlying theory containing Reggeizing vector particles⁸⁾.

We now wish to show that just the above structure of our super-critical RFT does indeed emerge from Yang-Mills when both confined, massless gluons and massive Reggeized gluons are present. The essential observation (already made above) is that an expectation value for the Pomeron fields corresponds to a (Reggeon) vacuum which can produce arbitrary numbers of zero energy (spin one), zero momentum (zero mass) Pomeron pairs. As we shall see this is exactly the property of a vacuum of confined, massless Abelian gluons when the Pomeron is itself formed from the combination of a massive Reggeized gluon and an arbitrary number of confined gluons.

We now outline the results of Lipatov and collaborators¹²⁾ on Yang-Mills theory with an $SU(N)$ gauge group, when the Higgs mechanism is applied to give all gauge vector particles mass M . The results we quote have been derived by dispersion relation methods and their consistency with unitarity has been explicitly checked both in Ref. 12) and more extensively by Bartels²³⁾. The results are also in agreement with explicit Feynman diagram calculations^{24), 25)}.

In the leading logarithmic approximation, multiparticle production in the multi-Regge limit is zero except where quantum numbers allow the exchange of (Reggeized) vector particles leading to

$$A_{2 \rightarrow 2+n} = s \Gamma(q_1^2) \frac{(s_1/M^2)^{\alpha(q_1^2)}}{(q_1^2 - M^2)} \gamma(q_1, q_2) \frac{(s_2/M^2)^{\alpha(q_2^2)}}{(q_2^2 - M^2)} \dots \gamma(q_n, q_{n+1}) \frac{(s_{n+1}/M^2)^{\alpha(q_{n+1}^2)}}{(q_{n+1}^2 - M^2)} \Gamma(q_{n+1}^2) \quad (8)$$

where the notation is explained in Fig. 5. The trajectory function is

$$\alpha(t) = 1 + \frac{q^2}{(2\pi)^3} (t - M^2) \left(\frac{d^2 k}{(k^2 - M^2)((q-k)^2 - M^2)} \right) \quad t = q^2 \quad (9)$$

The two-particle Reggeon vertex $\Gamma(q_i^2)$ and the two Reggeon/particle vertex $\gamma(q_i, q_{i+1})$ carry group quantum numbers and also a Lorentz index for the produced vector particle. All we need to know about γ is that it has the general form

$$\gamma(q_i, q_{i+1}) = 2 \left(\frac{p_A (q_i^2 - M^2)}{p_A \cdot p_i} - \frac{p_B (q_{i+1}^2 - M^2)}{p_B \cdot p_{i+1}} \right) + R(q_i, q_{i+1}) \quad (10)$$

The rewriting of these expressions in terms of Toller variables shows that (in the notation of the general Sommerfeld-Watson formulae of Ref. 5)) the first term corresponds to a (Froissart-Gribov) partial wave amplitude in which the helicity n_i at the i^{th} vertex is coupled (in this case equal) to the angular momentum j_{i+1} of the $(i+1)$ Reggeon and the $(t \text{ channel})$ helicity of the produced particle is $+1$. The killing of the particle pole at $q_i^2 = M^2$ is a reflection of a general nonsense zero at $j_i = n_i + 1 - 1 = n_i$ with $+1$ being the helicity of the produced particle. That is the partial-wave amplitude of the first term in $\gamma(q_i, q_{i+1})$ is

$$\frac{(j_i - n_i) R(\dots j_i, j_{i+1}, \dots n_i, \dots)}{(n_i - \alpha_{i+1})(j_i - \alpha_i) \sin \pi n_i \sin \pi(j_i - n_i)} \sim \frac{R(\dots, \alpha_i, \alpha_{i+1}, \dots \alpha_{i+1}, \dots)}{\alpha_i (j_{i+1} - \alpha_{i+1})(j_i - \alpha_i)} \quad (11)$$

That is the right-signature zero at $j_i = n_i$ cancels the signature factor $\{\sin \pi(j_i - n_i)\}^{-1}$ which would otherwise give a pole at $\alpha_i = \alpha_{i+1}$ (note the pole would not be at $\alpha_i = 1$ for $\alpha_{i+1} \neq 1$). The second term on the left similarly corresponds to a partial wave of the same form as (11) but with i and $i+1$ interchanged. The remainder in (10) is associated with the production of the i^{th} particle in a $(t \text{ channel})$ "longitudinal" polarization state. The general form of (10) follows therefore from the multiparticle Sommerfeld-Watson representation for the production of vector particles by Reggeized vectors.

To develop our formalism it will be convenient to introduce a diagrammatic notation to represent (8) and (10). This is shown in Fig. 6. A wiggly line represents an odd-signature Reggeon complete with pole signature factor. The nonsense zeros in γ and Γ which appear when all particles and reggeons involved have maximum helicity (that is are "transversely polarized") are indicated by a solid black line. The presence of such a line attached to a produced particle therefore indicates that the particle is produced "transversely".

Inserting (8) into the unitarity equation leads to an integral equation for the two-particle/two-Reggeon partial-wave amplitude (see Fig. 7 for notation and illustration of the equation)

$$f_j^T(k, q-k) = \frac{g^2(j-1)}{A_T(q^2)} + \frac{g^2}{8\pi^3} \int \frac{d^2k' K^T(k, k', q) f_j^T(k', q-k')}{(k'^2 - M^2)((q-k')^2 - M^2)(j+1 - \alpha(k') - \alpha(q-k'))} \quad (12)$$

where T is a (t channel) $SU(N)$ representation label and the kernel K^T results from squaring $\gamma(q_i, q_{i+1})$ as illustrated in Fig. 8. The vertical solid line in Fig. 8 and subsequent figures represents a transverse momentum pole which results from the cross-product of two vertices for transverse vector production. This pole can be thought of as due to a kinematic singularity resulting from Toller angle dependence of the vertices which cancels in the direct product but not in the cross-product. The remainder R^T in Fig. 8 results from all cross-products not explicitly represented. R^T is analytic as a function of the transverse momenta and when the production of Higgs particles is included has the simple forms in the vacuum ($T = 0$) and vector ($T = 1$) channels shown in Fig. 9.

In the vector channel the solution of (12) is

$$f_j^1 = (j-1) g^2 [(t-M^2)(j-\alpha(t))]^{-1} \quad (13)$$

Note that the nonsense pole at $j = \alpha(k^2) + \alpha((q-k)^2) - 1$ is cancelled (as is necessary from the signature rules for Regge cuts). In the vacuum channel the solution is much more complicated. It is solved by extending the equation to one involving all external Reggeons with the transverse momentum notation shown in Fig. 10. Writing $k_i^2 = x_i^2$, averaging over the angles involved, neglecting q^2 and M^2 , and significantly rearranging the terms leads to

$$(j-1) f_j^{\circ}(x_1, x_2) = \frac{4 x_1 x_2}{(x_1^2 + 4 x_2^2)^{\frac{1}{2}}} + \frac{g^2}{2\pi^2} x_1 \int_0^{\infty} dx \left[\frac{f_j^{\circ}(x, x_2)}{x |x - x_1|} - \left(\frac{1}{x |x - x_1|} - \frac{1}{x (x_1^2 + 4 x^2)^{\frac{1}{2}}} \right) f_j^{\circ}(x_1, x_2) \right] \quad (14)$$

The solution of this equation is a fixed cut at

$$j = 1 + \frac{g^2 N \ln 2}{\pi^2}$$

which is due to the divergence of the equation as $x \rightarrow \infty$ with $x \sim x_1$. Note therefore that the dominant region of (12) for $T = 0$ is $|\underline{k}'^2| \gg q^2, M^2$. Bronzan and Sugar have noted²⁶⁾ that (12) is in agreement with the leading logarithmic calculations of Refs. 27) and 28) in the limit $q^2 \gg M^2$. However, because of the dominance of $\underline{k}'^2 \sim \infty$ the limits involved are sufficiently complicated that the large q^2 leading logarithms do not see the fixed cut.

From our point of view the important feature of (14) is the cancellation of the singularities at $x = 0$ and $x = x_1$ which are the relic of the particle poles in (12) and the pole in K_T respectively. The singularity at $x = x_1$ cancels explicitly while the substitution $f = x_1 \tilde{f}(x_1, x_2)$ cancels that at $x = 0$. As a consequence both singularities are absent when (14) is solved by expansion in powers of g^2 . This cancellation, or infra-red finiteness²⁹⁾, (it implies the limit $M^2 \rightarrow 0$ exists) is a property of the vacuum channel only.

We note further that only the singular part of the kernel K^T contributes to (14) (that is R^T is not involved) and this involves only the production of transversely polarized vectors -- as we have emphasized. It is clear then that not only does the zero mass limit in the vacuum channel exist, but also it involves only terms that go smoothly into the corresponding terms in massless Yang-Mills. Of course, the fixed cut solution of (14) leads to a violation of the Froissart bound as well as a violation of t channel unitarity and so it does not provide a sensible approximation with which to begin a discussion of the zero-mass theory. The problem is therefore how to first calculate higher-order perturbation theory modifications of the leading logarithmic approximation and then organize the whole theory so that the zero-mass limit can be discussed.

The approximation in which all production and elastic amplitudes are self-consistently given, because of (13), by (8) and there is no vacuum scattering is a sensible one and it can be iterated to give a well-defined perturbation expansion for all amplitudes as follows.

The full unitarity equations for all multiparticle amplitudes are equivalent in the multi-Regge asymptotic region to a complete set of multiparticle dispersion relations whose derivation is described in Ref. 19) where a full graphical description of these equations is given. As an example, the 2-4 amplitude satisfies a triple dispersion relation involving as spectral functions the multiple discontinuities given by the unitarity-like integrals shown in Fig. 11. It is a general and very non-trivial result that all the complicated Landau singularity structure disappears asymptotically and all amplitudes can be reconstructed from their multiple normal threshold discontinuities. Since the integration regions in these formulae reduce to normal phase-space integrals the calculations of Lipatov et al. (and the extensions of these results obtained by Bartels) can immediately be iterated through these equations to obtain a full RFT structure. (Note that this procedure differs from that proposed by Bartels²³⁾ in that he proposed to iterate the full unitarity equations. We believe that such a procedure quickly encounters counting ambiguities which the dispersion relations were designed to solve.) Since the dispersion relations can be used⁵⁾ to define generalized Froissart-Gribov continuations which in turn can be used to show that multiparticle t channel unitarity requires an RFT structure it is no surprise that the direct iteration of these equations leads to an RFT structure. We shall illustrate the complete process only diagrammatically.

Let us summarize Fig. 8 by the first two equations of Fig. 12 where R^T is now represented as an (essentially) local four-Reggeon interaction. We add to this some further definitions involving the two-particle/Reggeon vertex $\Gamma(q^2)$. When the initial structure of Fig. 6 is iterated through the dispersion relations all the structures of Fig. 12 and many more will appear. The phase-space integrals will become transverse momentum integrals as in (12), with the integrations over sub-energies similarly replaced by the Reggeon (Rayleigh-Schrödinger) propagator²⁰⁾ $[E - \sum_i \alpha(q_i^2)]^{-1}$ (where $E = 1 - j$) for each vertical cut that can be made through a distinct set of Reggeons and the sum is over the Reggeon's cut. As Bronzan and Sugar originally emphasized²⁶⁾ the integral equation (12) simply sums the Reggeon diagrams shown in Fig. 13.

In fact, if we join together the couplings of Fig. 6 in all possible ways to form all possible Reggeon interactions, both local and non-local, using the rules of Fig. 12 then all such couplings will appear in our final RFT. The absolute magnitudes of these couplings will not concern us for the moment, only the general singularity structure. It might already be clear that our ultimate aim is to

relate the singular momentum dependence of our Reggeon couplings and the singular momentum dependence of the Pomeron interactions shown in Fig. 1. For the moment, however, we do not even have a Pomeron and the most obvious approximation in the vacuum channel leads to (14). In general, we have the full complexity of RFT with the additional complication that it is an RFT for an odd-signature Reggeon with accompanying signature factor pole. As is usual in the absence of a critical phenomenon our ability to calculate is essentially zero. In the phase-transition language the dominance of large transverse momentum in (14) is due to the dominance of a strongly momentum dependent Reggeon coupling (the non-local four-Reggeon coupling of Fig. 12) which would become irrelevant if we were approaching a strong-coupling critical point dominated by small momentum and energy.

At this point we introduce the modified $i\epsilon$ prescription for the gluon referred to in the Introduction. Our modifications will (initially) apply to the transverse momentum poles in (12) which are particular cases of the signature factors accompanying Reggeons all of which we modify and the transverse momentum poles in the Reggeon interactions. The modification is that we first regard all transverse momentum integrals as being over two-dimensional time-like vectors, which is what we obtain if we perform all the calculations for $t_i > 0, \forall t_i$. This is in fact the natural place to start the calculations of Lipatov et al. Alternatively it is straightforward to analytically continue all expressions to positive t 's. Writing $\underline{k} = (k_1, k_2)$ with the metric $\underline{k}^2 = k_1^2 - k_2^2$ then the usual $i\epsilon$ prescription is

$$\int \frac{dk_1 dk_2}{(\underline{k}^2 - M^2)} \equiv \int \frac{dk_1 dk_2}{(k_1^2 - k_2^2 - M^2 + i\epsilon)} \quad (15)$$

while our prescription is to introduce light-cone co-ordinates $k_{\pm} = k_1 \pm k_2$ and write

$$\int \frac{dk_1 dk_2}{(\underline{k}^2 - M^2)} = \frac{1}{2} \int \frac{dk_+ dk_-}{k_+ k_- - M^2} = \frac{1}{2} \left[\text{P.V.} \int \frac{dk_-}{k_-} \right] \left[\text{P.V.} \int \frac{dk_+}{k_+ - \frac{M^2}{k_-}} \right] \quad (16)$$

$$\xrightarrow{M^2 \rightarrow 0} \frac{1}{2} \left[\text{P.V.} \int \frac{dk_-}{k_-} \right] \left[\text{P.V.} \int \frac{dk_+}{k_+} \right] \quad (17)$$

Thus in the limit $M^2 \rightarrow 0$ the k_+ and k_- integrations become strictly independent principal-value integrals. Having discussed the significance of this change in the Introduction we repeat only comment (e), being more specific. Because of the cancellation of all the poles in (14) at least the perturbation expansion of this equation is unmodified by our change. Thus we have agreement with perturbation theory at large momentum transfer in the vacuum channel to a much greater extent than required by asymptotic freedom.

The non-vacuum channels will change drastically. However, from the example¹⁵⁾⁻¹⁷⁾ of two-dimensional QCD we are already familiar with the fact that in a confining theory the behaviour of the unphysical channels will be dramatically dependent on the method chosen to regularize the infra-red divergences. In particular the trajectory function (9) appears to lose all momentum dependence since we have defined the integral to be finite and yet it has dimension $1/t$. Since with $M^2 = 0$ there is no scale which would allow it to be finite at $t = 0$, the integral must be identically zero with the prescription (16). This result will be unchanged by higher-order perturbation theory terms and so it is essential at this point that we add the scale μ which we know is necessary to generate masses in massless Yang-Mills or QCD. This enters in a non-trivial way via the non-perturbative features of the theory¹⁾. Here we note only that this scale will enter the gluon propagator either through additional (presumably non-physical) singularities at finite $\underline{k}^2$ or more attractively appear in the behaviour at $\underline{k}^2 = \infty$ of an entire function propagator. In either case the $\underline{k}$ integration will pick up this scale and we will have (from dimensional arguments only) that (9) is replaced by

$$\alpha(t) = 1 + (t - M^2) \left[\tilde{\alpha}(q^2)/\mu^2 + O(M^2) \right] \quad (18)$$

which shows that the intercept $\alpha(0) \sim M^2$ still, but the slope is $\sim 1/\mu^2$.

In the vacuum channel the situation is different because of the non-cancellation of the nonsense pole (the two-Reggeon propagator) at $j = \alpha(\underline{k}^2) + \alpha((\underline{k} - \underline{q})^2) - 1$ as in the vector channel. The principal-value prescription (16) amounts to a removal of the phase-space contour integration around the poles in (12). When the two-Reggeon propagator collides with this "missing-phase space" it will generate singularities which the usual $i\epsilon$ prescription avoids. These include the famous Mandelstam^{30),31)} or Reggeon-particle cuts, which are a pair of complex-conjugate branch-points at (when the particles are vector particles)

$$j = \alpha ((\sqrt{t} - M)^2) \quad (19)$$

These cuts arise when the pole of the two-Reggeon propagator collides with a (signature factor) particle-pole and the boundary of phase-space and are usually³¹⁾⁻³³⁾ on the unphysical sheet of the two-Reggeon cut. In effect, we have traded the gluon threshold at $t = 4M^2$ for these cuts. That this is possible is illustrated by the plot of the trajectories of the singularities in Fig. 14. It seems attractive to think of this replacement of point-particle singularities by Regge singularities as a mathematical realization of the process of string formation. This situation will also generalize to all the higher gluon thresholds if (16) is used everywhere.

So far, we have discussed only the SU(N) theory studied by Lipatov et al. in which all vectors have the same mass M. The pattern of the Higgs symmetry breaking is chosen so that the gauge symmetry becomes an over-all global symmetry. Suppose now that we break the symmetry in such a way that one vector remains massless. That is we have a remaining Abelian gauge U(1) symmetry. (This is always possible³⁴⁾, for example SU(N) can be broken down to SU(2) by N-2 vector representations of Higgs scalars. The addition of further Higgs scalars in the adjoint representation of the remaining SU(2) will break the symmetry down to U(1) finally.) Since the Reggeization of the vector particles is not dependent on the pattern of the symmetry breaking³⁵⁾ the calculations will go through as before. (We have emphasized that the structure of the Reggeon vertices of Fig. 6 resulted from general arguments only.) Also we can define the theory by a limit in which the single Abelian "gluon" becomes massless and we use the prescription (16) for the poles corresponding to this vector only. In this case none of the trajectory functions for the Reggeons will be trivial because only the momentum integrations involving just Abelian gluon poles can possibly give zero.

The Reggeon-particle cuts formed from a massive Reggeon and an Abelian gluon will coalesce into a single pole degenerate with the Reggeon. In fact, it is easy to see now that the main effect of the replacement of (15) by (17) in all our RFT integrals will be as follows. When the theory is analytically continued to negative t (by rotating all the time-like $\underline{k}$ integrations to be space-like as usual) we will obtain the usual RFT space-like momentum integrations together with a contribution from each Abelian gluon of the form

$$2\pi^2 \int d^2 \underline{k} \, s^2(\underline{k}) \quad (20)$$

from the "missing" double-pole integration in (17), together with contributions of the form $\int dk_- \int dk_+ \delta(k_+)$ which are less singular at $k^2 = 0$. Whether or not the gluon Reggeizes it certainly has spin one at $k^2 = 0$. If we treat it as elementary in our RFT (leaving the possible Reggeization to the interactions) then we can use the assumption that it is confined to give it negative Reggeon energy $E = -\epsilon$ (that is spin $1 + \epsilon$). The (most singular) contribution of a single gluon to the RFT Rayleigh-Schrödinger expansion will then be the momentum integration (20) together with the propagator

$$\left[E - \sum_i \alpha_i + \epsilon \right]^{-1} \quad (21)$$

where the sum is over all Reggeons also present in the intermediate state. We now define the rapidity Fourier transform so that if there are no Reggeons in the intermediate state then the negative energy of the gluon implies no contribution to the large rapidity limit. (Note that (20) is directly due to our having engineered the finiteness of all Reggeon trajectories at zero gluon mass.)

The above prescription also implies that the exchange of one Reggeon and an arbitrary number of gluons will be a pole degenerate with the Reggeon. In general the symmetry breaking will allow a contribution to the vacuum (t)-channel of the unbroken symmetry from some Reggeon and various numbers of gluons. In SU(3) we can take the "gluon" to be the $I=1, I_3=0, Y=0$ state. The $I=0, I_3=0, Y=0$, "Reggeon" can then combine with two or more gluons in an SU(3) singlet state. In this case we can define a Pomeron as a Reggeon "surrounded by a cloud of zero mass gluons" (that is combined with an arbitrary number of gluons) in such a way that we have vacuum quantum numbers -- again according to the unbroken symmetry. [Note that the signature factor for the Reggeon is always cancelled by a nonsense zero if we consider the gluons to have been produced only by interactions involving transverse vectors. A necessary condition for the signature factor to also be cancelled when non-singular interactions are included is that R^0 should have a zero at $t = M^2$ -- the Reggeon mass. From Fig.9 we see that this is the case as $N \rightarrow \infty$ which suggests that the most straightforward description of our proposed Pomeron might be in the $1/N$ expansion. This seems likely also in that in such an expansion the tendency of our asymmetric symmetry breaking to produce complicated numbers and destroy cancellations would be removed.] Similar Reggeons "surrounded by clouds of gluons" will appear in all quantum number channels. We therefore have a degenerate Pomeron and Reggeon as in the super-critical RFT.

Consider now how the Pomeron interacts. We represent the gluon by a dotted line and the Pomeron by a Reggeon surrounded by a cloud of gluons as in Fig. 15. A non-singular triple Pomeron interaction will involve no isolated gluon interaction, for example, the interactions of Fig. 16. However, if some gluons interact after the initial interaction and before the final Pomerons are formed, by exchange of a massive vector particle as in Fig. 17 then we have a singular triple Pomeron interaction. It is straightforward to see that when (20) and (21) are substituted for the gluons involved, the contribution to the triple Pomeron interaction is precisely that of the RFT diagram of Fig. 2. The t singularity of (7) is directly due to the presence of a transversely polarized massive vector particle produced in an s -channel intermediate state. As illustrated further in Fig. 18 the Abelian gluon acts exactly like the expectation value of the Pomeron field in the super-critical RFT. It can produce a pair of vectors, one Reggeized and the other not, which is exactly like the vacuum production of Pomerons once the accompanying gluons neutralize the quantum numbers of the vectors. It seems therefore that the formation of super critical RFT is complete and we can make the identification.

Pomeron vacuum expectation value $\leftrightarrow$ confined Abelian gluon field

Singular Pomeron interactions $\leftrightarrow$ s -channel production of massive
vector mesons.

The group structure needed to realize the super-critical phase is implied by the following requirements:

- i) an Abelian gluon field coupled to pairs of massive vectors is necessary for for the gluon to act as a Pomeron expectation value;
- ii) the same massive vectors must have a triple interaction to give a triple Pomeron interaction; and
- iii) the vacuum channel (of the unbroken symmetry) must contain a state with a massive Reggeon and some number of gluons.

It appears that $SU(3)$ is the minimum structure that achieves all of the above. $SU(2)$ fails on (ii), once (i) is required. $SU(2) \times SU(2)$ fails on (ii) once (iii) is required.

However, consider now how we take the limit in which all vectors become massless and the original symmetry is restored. If we keep one gluon asymmetrically massless but use the prescription (16) for the massive gluons the additional Reggeon-particle singularities generated will be difficult to deal with. An alternative which is certainly well-defined within the iteration of the dispersion relations is to restore the symmetry by making all Reggeons massive but set the mass appearing in all signature factor particle poles to zero using the prescription (17). It seems that then the limit will be approached symmetrically and

exactly as the super-critical RFT approaches the critical RFT. A Pomeron and a Reggeon can be formed, similarly to those above, from a Reggeon and an arbitrary number of zero-mass gluons but now the "gluons" will be the particle poles in the signature factors. The Pomeron and Reggeon will be degenerate and have finite slopes in the zero-mass limit. The parameter which measures the approach to the critical point can still be seen simultaneously as the mass associated with the Regge trajectory and the mass of s -channel produced transversely polarized vector particles. The critical point can therefore be identified with the restoration of local gauge invariance.

From this last point of view it is possible that the critical Pomeron could be obtained with $SU(2)$ as the gauge group since we have effectively separated one Reggeized gluon into two. That is a non-Reggeized massless gluon (the signature factor) which has no triple interaction because of the nonsense zero in (15) and a Reggeized gluon which has a triple interaction in that two Reggeized gluons can produce an s channel vector particle. Clearly only more work on a complete counting of the numbers involved in the various vertices will determine whether a complete matching with the critical RFT is possible in this case. In fact, it is possible that the dependence on the N of $SU(N)$ would force us to define a large number of Pomeron fields for large N to match this N dependence. The net result would be a dependence of the critical exponents of the critical Pomeron on N (as in $O(N)$ phase-transition theory, for example). However, we think the possibility mentioned above that the large N limit is instead simple and matches the simple RFT exactly is much more likely. In either case the criticality of the Pomeron and rising cross-sections will remain tied to local gauge invariance as a sufficient requirement for the vanishing of the Pomeron expectation value and the vanishing of massive vector production in the s -channel. In particular, we have demonstrated that $SU(3)$ and therefore QCD provides all the necessary structure to give rising total cross-sections.

Finally we remark on the relevance of asymptotic freedom. This is relevant to the worry that we might "overshoot" the critical point as we go to the massless limit. That is the situation might be similar to removing an external magnetic field in the Ising model. One can end up far from the critical point where there is spontaneous magnetization.

If we consider spontaneously broken Yang-Mills with at most only an Abelian vector massless as in the above then we have no asymptotic freedom. In the present state of ignorance of such theories we must (in view of the renormalon problem³⁶⁾ alone) impose a cut-off in the large transverse momentum. This cut-off can be taken arbitrarily large by taking the self-interactions of the Higgs scalars sufficiently small and we can arrange to have³⁷⁾ "temporary asymptotic

freedom" out to our cut-off Λ as indicated in Fig. 19. When the mass of the Reggeized gluons is non-zero so that we have no critical effect it is likely that the large transverse momentum effect found by Lipatov et al. will dominate. In fact, we can use Lipatov et al.'s Pomeron -- modified to a pole by our cut-off (it is a fixed cut only because of the transverse momentum divergence) to develop a bare RFT involving just the Pomeron. We can think of this as having integrated out all Reggeon degrees of freedom in the RFT Lagrangian. As a result, the expansion will be that of the bare sub-critical phase. All quantities will be dominated by large transverse momenta close to the cut-off and because of "temporary asymptotic freedom" can be estimated by their lowest order perturbation theory values. These are

$$-\Delta_0 \sim g^2, \alpha'_0 \sim g^2, \gamma_0 \sim g^4, \text{ higher couplings } \sim g^6, g^8, \dots \quad (22)$$

with

$$g^2 \sim \frac{g_0^2}{\ln \Lambda} \quad (23)$$

Substituting these expressions into (2) we see that we must be in the super-critical phase since

$$[-\Delta_0 \sim g^2] > [-\Delta_{oc} \sim \frac{\gamma_0^2}{\alpha'_0} \ln \Lambda \sim g^4] \quad (24)$$

Since these calculations would apply also in the massless theory if we were away from the critical point it follows that we cannot "overshoot" the critical point in the massless limit. In fact when this last result is combined with the above arguments it suggests that the critical Pomeron is obtained only in the double limit

$$M^2 \rightarrow 0, \Lambda \rightarrow \infty \quad (25)$$

Can the limit $\Lambda \rightarrow \infty$ be safely taken when $M^2 \rightarrow 0$? Isn't the formulation of RFT endangered by no transverse momentum cut-off? Bronzan and Sugar have argued that asymptotic freedom would not be a sufficient cut-off to turn Lipatov et al.'s fixed cut into moving Regge singularities. Their argument is based on the infra-red divergence of the Lipatov et al. kernel K^T which they show a renormalization group transformation mixes with the ultra-violet divergence. Fortunately, when the completely massless theory is regulated entirely by (17) this infra-red divergence is eliminated and the renormalization-group transformation of Bronzan and Sugar can be used to show that asymptotic freedom is a sufficient cut-off for the Lipatov et al. kernel to be guaranteed to produce only Regge singularities controllable by RFT. Hence the limit (25) can be taken and the result is the critical Pomeron.

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FIG.1

FIG.2

FIG.3

FIG.4

FIG.5

$$\frac{s^{a(q_i^2)}}{q_i^2 - M^2} = \text{wavy line} \rightarrow q_i$$

$$\gamma(q_i, q_{i+1}) = \text{wavy line with vertical line} + \text{wavy line with vertical line} + \text{wavy line with vertical line}$$

$\xrightarrow{q_i} \quad \quad \quad \xrightarrow{q_{i+1}}$

$$\Gamma(q_i^2) = \text{wavy line with vertical line} + \text{wavy line with vertical line} \xrightarrow{q_i}$$

FIG.6

$$\text{Diagram with vertex } f_j^T \text{ and external lines } k, q-k = \text{Diagram with vertex } K_0^T + \text{Diagram with vertices } f_j^T \text{ and } K^T$$

FIG.7

$$\begin{aligned}
 & \text{Diagram with } K^T \text{ vertex} = \left(\begin{array}{c} k \quad k' \\ \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \\ q-k \quad q-k' \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} \right) \times \\
 & \left(\begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \\ q-k \quad q-k' \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} \right) \\
 & = \begin{array}{c} k \quad k' \\ \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \\ q-k \quad q-k' \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} + \begin{array}{c} \text{---} \text{---} \text{---} \\ | \\ \text{---} \text{---} \text{---} \end{array} \text{ with } R^T \text{ vertex} \\
 & \quad \quad \quad = \left[(k-k')^2 - M^2 \right]^{-1}
 \end{aligned}$$

FIG. 8

$$\begin{aligned}
 & \text{Diagram with } R^0 \text{ vertex} = 2q^2 - 2 \frac{(N^2+1)}{N^2} M^2, \quad \text{Diagram with } R^1 \text{ vertex} = q^2 - M^2
 \end{aligned}$$

FIG. 9

FIG. 10

FIG. 11

FIG. 12

FIG. 13

FIG.14

FIG.15

FIG.16

FIG.17

FIG.18

FIG.19