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SENSITIVE TESTS OF QCD IN LARGE p_T CALORIMETER MEASUREMENTS

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ABSTRACT

It is shown that in large p_T hadronic collisions one can measure some effective moments of quark and gluon fragmentation functions. This fact together with the expected presence of both gluon and quark jets produced with similar (Born) cross-sections underlies the possibility of very sensitive QCD tests in such collisions. An efficient method for testing the theory (and in particular for minimizing the non-perturbative contamination) is proposed which can be used even at energies as low as the ISR energy range. We suggest to measure a quantity which can be interpreted as an "effective radiation strength" of the jet and which, if QCD is correct, should interpolate between the values of the colour group factors $C_G = 3$ and $C_F = 4/3$ when x_T is varied in the range $0.05 \leq x_T \leq 0.4$.

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1. INTRODUCTION

Recent progress in perturbative calculations in QCD has not, as yet, brought any major breakthrough in the unambiguous experimental verification of the theory. Perturbative predictions apply at present machine energies to tiny effects, usually hidden behind dominating non-perturbative phenomena which are outside any theoretical control. This is, for instance, the case with the study of the jet structure in e^+e^- annihilation as discussed in detail in Ref. 1). The main experimental support for the theory (and in particular for its non-Abelian character) still comes from the measurements of the deep inelastic structure functions, where the observed breaking of Bjorken scaling, is, at large $x = Q^2/2M\nu$, in agreement with asymptotic freedom predicted by QCD. However, also in this case, the uniqueness of the QCD-based explanation is repeatedly disputed²⁾. It is, therefore, an important task to look carefully for effects and measurements where the basic properties of QCD manifest themselves in the strongest way.

In this paper we show that sensitive tests of QCD are likely to be provided by certain, conceptually very simple, one-arm calorimeter measurements in large p_T hadronic collisions (for similar suggestions, see Refs. 3)). We believe that interest in and importance of such experiments should be particularly stressed in view of new hadronic beams which are to operate in the reasonably near future ($p\bar{p}$ beams at CERN and pp beams at ISABELLE) and which will offer higher energies than planned lepton machines.

The exceptional role of hard hadron-hadron collisions with respect to QCD tests is based on the expectation that, according to our present views on hadron structure, they are the source of gluon and quark jets both produced with similar (Born) cross-sections. Therefore we are provided with the unique, among all hard processes, laboratory for the efficient study of high-energy gluon jets and their comparison with quark jets. The importance of such a study is due to the fact that according to QCD the evolution of gluon jets exhibits much stronger perturbative effects than that of quark jets⁴⁾.

In the first part of the paper we formulate the general idea of the proposed QCD tests. The second part is devoted to several numerical estimates of various effects (including non-perturbative ones) relevant to the proposed method. This analysis gives also some feeling on the most suitable choice of kinematical variables for actual experiments.

2. QUARK AND GLUON JETS

Our considerations are based on the common expectation that high p_T ($p_T \gg \gg 1$ GeV) hadronic collisions are dominated by hard collisions of quarks and gluons from the initial hadrons^{5),6)}. In such collisions, parent partons (quarks and gluons which develop into quark and gluon jets) are produced with some cross-section $d\sigma_0/dx d\Omega$ where $x = p/E_0$ is the fraction of the initial hadron energy E_0 carried by the parent parton in the c.m. frame (p is also the total energy of the jet). For simplicity, we shall consider the case $\theta = 90^\circ$ in hadron-hadron c.m. frame and then

$$x = x_T = p_T/E_0 \cong \sqrt{x_1 x_2} \quad (1)$$

where $x_{1,2}$ are Feynman x 's of the initial partons. The cross-section $d\sigma_0/dx d\Omega$ can also be called the inclusive jet cross-section.

In general, the $d\sigma_0(x, \theta, E_0)/dx d\Omega$ is a sum of gluon and quark jet cross-sections:

$$\frac{d\sigma_0}{dx d\Omega} = \frac{d\sigma_0^G}{dx d\Omega} + \frac{d\sigma_0^F}{dx d\Omega}$$

whose relative importance is a priori unknown. The fact that any measurement at large p_T includes contribution from both gluon and quark jets may, at first sight, seem to be only a strong disadvantage. It is so indeed as long as we study hadron distributions which do not suffice for an easy separation and analysis of both contributions. However, if we go to jet study, the presence of both contributions becomes the basic virtue of large p_T experiments. Firstly, it is important to realize that the relative contribution of quark and gluon jets to the $d\sigma_0(x_T, \theta = 90^\circ, E_0)$ varies strongly with x_T so that at sufficiently high energy E_0

$$d\sigma_0(x_T, \theta = 90^\circ, E_0) \cong \begin{cases} d\sigma_0^G(x_T, \theta = 90^\circ, E_0) & \text{small } x_T \\ d\sigma_0^F(x_T, \theta = 90^\circ, E_0) & \text{large } x_T \end{cases} \quad (2)$$

if our theoretical prejudice is correct. Indeed the jet cross-section can be calculated using the standard parton model approach from all possible Born contributions⁶⁾ and in terms of the initial parton momentum distribution functions $F_F(x_i, p_T^2)$ and $F_G(x_i, p_T^2)$. Our qualitative conclusion (2) follows from formula (1) if the gluon momentum distribution is steeper in x than that of quarks. In this case, since quarks and gluons carry about equal fraction

of hadron momentum, the density of gluons at small x is much larger than the density of quarks. In addition, although in the gluon-gluon scattering both large p_T quarks and gluons can be produced, the latter are strongly enhanced by the Born cross-section. Consequently at low x_T gluon jets dominate. On the other hand, at large x_T we probe the large x region of the proton structure functions and the $qq \rightarrow qq$ should dominate^{*)}. If QCD is correct then, due to the scale breaking effects, the difference between the quark and gluon structure functions discussed above must be true at large enough p_T irrespectively of the input "non-perturbative" parton distributions in the initial hadrons. Qualitatively speaking (we shall discuss this point quantitatively later on), a hard parton is slowed down before the collision since it may radiate soft gluons (and reach a virtuality up to large p_T^2). This effect is stronger for gluons than for quarks as the density of gluons radiated by a hard gluon is larger than the density of gluons radiated by a hard quark. The former is proportional to $C_G = 3$ and the latter to $C_F = 4/3$, C_G and C_F being the colour group factors. Secondly, due to the same physical reason as explained above, the jets developing after hard collisions from parent gluons are markedly different in their internal transverse momentum (relative to the jet axis) structure from those originating from parent quarks. Qualitatively again, gluon jets are distinctly broader than analogous quark jets.

Let us imagine first to have a mono-energetic "beam" of same jets (originated from collinear parent partons with virtuality up to some value p_T^2 determined by the momentum transfer in the hard jet production process). Then, their transverse structure could be measured in calorimeter experiments by measuring the inclusive distribution $dJ/dz(z, \delta)$ of signals in which a fraction z of the total jet (parent parton) energy is deposited in a solid angle 2δ around the jet axis, as a function of δ in a range of δ values such that $\delta \ll 1$. The QCD predictions for the $dJ/dz(z, \delta)$ in the leading log δ approximation read⁷⁾:

for parent quark (quark jet)

$$\frac{dJ_F(z, \delta)}{dz} = \frac{1}{\Gamma(2C_F\xi)} e^{\xi a_F} (1-z)^{2C_F\xi-1} [1 + O(1-z)] \quad (3)$$

*) In $p\bar{p}$ scattering there is an additional source of gluons at large x_T : $q\bar{q} \rightarrow gg$. As will be seen in the following, numerically it gives 20% contamination of large x_T gluon jets.

and for parent gluon (gluon jet)

$$\frac{dJ_G(z, \delta)}{dz} = \frac{1}{\Gamma(2C_G \xi)} e^{\xi Q_G} (1-z)^{2C_G \xi - 1} [1 + O(1-z)] \quad (4)$$

where

$$Q_F = C_F \left(\frac{3}{2} - 2\gamma \right), \quad \gamma = 0.57722... \text{ (Euler constant)}$$

$$Q_G = C_G \left(\frac{11}{6} - \frac{2}{3} \frac{f T_F}{C_G} \right), \quad f = \text{number of flavours, } T_F = 1/2$$

and

$$\xi = \int_{\delta}^1 \frac{d\theta}{\theta} \frac{\alpha(p_T \theta)}{\pi} = \int_{\delta}^1 \frac{d\theta}{\theta} \frac{\alpha(p_T)/\pi}{1 + (\alpha(p_T)/\pi) 2b \ln \theta} =$$

$$= - \frac{1}{2b} \ln \left(1 - \frac{\alpha(p_T)}{\pi} 2b \ln \frac{1}{\delta} \right), \quad (5)$$

$$b = \frac{25}{12} \text{ for } f = 4.$$

[Using the commonly used parametrization

$$\frac{\alpha(p_T)}{\pi} = \frac{1}{2b \ln \frac{p_T}{\Lambda}}, \quad \Lambda = 0.5 \text{ GeV}$$

we can also write

$$\xi = \frac{1}{2b} \ln \left(\frac{\ln \frac{p_T}{\Lambda}}{\ln \frac{p_T \delta}{\Lambda}} \right). \quad (5')$$

In the actual large p_T experiments we cannot directly measure the distributions $dJ_G(z, \delta)/dz$ ^{*)} and $dJ_F(z, \delta)/dz$ since we do not have a mono-energetic

*) Here we want to make one technical remark concerning our notation: in all cases in which the energy (p_T) dependence due to non-scaling effects is irrelevant for our discussion it will not be explicitly written.

collinear beam of the same jets. Nevertheless, as we shall see in the next section, we can get important information about those distributions, and, thus test the theory.

3. "EFFECTIVE RADIATION STRENGTH"

The quantity which we propose to measure is the inclusive cross-section

$$d\sigma(x_T, E_0, \theta, \delta) / dx_T d\Omega \Big|_{\theta = \theta_0 = 90^\circ}$$

for signals in which the fraction x_T of the initial hadron energy E_0 is deposited in a solid angle 2δ in the direction $\theta_0 = 90^\circ$, as a function of δ in a range such that $\delta \ll 1$. This cross-section can be measured by a set of calorimeters of aperture 2δ covering certain solid angle $\Delta\Omega$ around the direction θ_0 *). We note also that with increasing δ at fixed E_0 the cross-section should reach the δ independent limit which is the inclusive jet cross-section $d\sigma_0(x_T, \theta_0) / dx_T d\Omega$:

$$\frac{d\sigma_0(x_T, \theta_0)}{dx_T d\Omega} = \lim_{\delta \rightarrow 0(1)} \frac{d\sigma(x_T, E_0, \theta_0, \delta)}{dx_T d\Omega}.$$

The considered cross-section is given by a convolution of the parton (jet) cross-sections, $d\sigma_0^G$ and $d\sigma_0^F$, and the fragmentation distributions $dJ^G(z, \delta)$ and $dJ^F(z, \delta)$

$$\begin{aligned} \frac{d\sigma(x_T, E_0, \theta_0, \delta)}{dx_T d\Omega} = & \int_0^1 dz dy_T \left[\frac{d\sigma_0^F(y_T, E_0, \theta_0)}{dy_T d\Omega} \frac{dJ_F(z, \delta)}{dz} + \right. \\ & \left. + \frac{d\sigma_0^G(y_T, E_0, \theta_0)}{dy_T d\Omega} \frac{dJ_G(z, \delta)}{dz} \right] \delta(x_T - zy_T). \end{aligned} \quad (8)$$

Qualitatively speaking, in the case of steeply decreasing cross-sections $d\sigma_0^{G,F}(y_T, E_0, \theta_0)$ with increasing y_T the dominant contribution to the integral in formula (8) comes from the region $y_T \approx x_T$, i.e., $z \approx 1$ ("trigger bias" effect). Taking into account the varying with y_T relative contribution of quark and gluon cross-sections to the full cross-section and, in particular, Eq. (2),

*) Even sizeable $\Delta\Omega$ are allowed since, experimentally, the large p_T inclusive cross-sections are very weakly θ -dependent.

we can expect to measure at small and large x_T some features of dJ_G and dJ_F distributions, respectively. More formally, denoting for abbreviation

$$\bar{\sigma}(x_T, \delta) = \frac{d\bar{\sigma}(x_T, E_0, \theta_0, \delta)}{dx_T d\Omega}$$

we can write $\sigma(x_T, \delta)$ in terms of its Mellin transform $\sigma(s, \delta)$:

$$\bar{\sigma}(x_T, \delta) = \int \frac{ds}{2\pi i} e^{ys} \bar{\sigma}(s, \delta) \quad , \quad y = \ln \frac{1}{x_T} \quad (9)$$

where

$$\bar{\sigma}(s, \delta) = \int_0^1 dx_T x_T^{s-1} \bar{\sigma}(x_T, \delta) = \sum_I \bar{\sigma}_0^I(s) J_I(s, \delta) \quad (10)$$

(we have introduced index $I = F, G$), and

$$\begin{aligned} \bar{\sigma}_0^I(s) &= \int_0^1 dy_T y_T^{s-1} \bar{\sigma}_0^I(y_T), \\ J_I(s, \delta) &= \int_0^1 dz z^{s-1} \frac{dJ_I(z, \delta)}{dz} = \left(\frac{1}{s}\right)^{2\alpha_I \xi} e^{\alpha_I \xi} \left[1 + O\left(\frac{1}{s}\right)\right]. \end{aligned} \quad (11)$$

Writing now

$$\bar{\sigma}(x_T, \delta) = \int \frac{ds}{2\pi i} \sum_I e^{ys + \ln \bar{\sigma}_0^I(s)} J_I(s, \delta) \quad (12)$$

we can apply the saddle-point approximation to each of the two terms in the integral since $J_I(s, \delta)$ are slowly varying functions of s . In this approximation we get the following result:

$$\bar{\sigma}(x_T, \delta) = \bar{\sigma}_0(x_T) \sum_I \frac{\bar{\sigma}_0^I(x_T)}{\bar{\sigma}_0(x_T)} J_I(s_0^I(x_T), \delta). \quad (13)$$

The saddle-points $s_0^I(x_T)$ are determined by the equations

$$\frac{d\bar{\sigma}_0^I(s_0)}{ds} + y \bar{\sigma}_0^I(s_0) = 0$$

or

$$\int_0^1 dy_T y_T^{s_0^I(x_T)-1} \bar{\sigma}_0^I(y_T) \ln \frac{y_T}{x_T} = 0$$

and for functions $\sigma_0^I(y_T)$ steeply decreasing with increase of y_T the values $s_0^I(x_T)$ are positive and large. For instance, adopting the form

$$\sigma_0(x_T) = \frac{1}{x_T^M} (1-x_T)^N$$

we get from an explicit calculation

$$S_0(x_T) = M + \frac{x_T}{1-x_T} N + O\left(\frac{1}{N}\right).$$

In general, it is convenient to have a numerical method for finding the saddle points $s_0^I(x_T)$ given the cross-sections $\sigma_0^I(x_T)$ in a tabulated form (such a method can then be also used in the experimental analysis). For this reason, let us consider a convolution $G^I(x)$ of the cross-section $\sigma_0^I(y)$ with an arbitrary function $F(z)$ with slowly varying Mellin transform:

$$G^I(x) = \int_0^1 dz \int_0^1 dy \sigma_0^I(y) F(z) \delta(x-yz).$$

Applying the Mellin transformation and in the saddle point approximation we can write (following the steps which lead to the result (13)):

$$G^I(x) = F(S_0^I(x)) \sigma_0^I(x)$$

where $F(s)$ is the Mellin transform of function $F(z)$. Taking, for example, $F(z) = \text{const}$ we get finally

$$G^I(x_T) = \int_{x_T}^1 \frac{dy_T}{y_T} \sigma_0^I(y_T) = \frac{1}{S_0^I(x_T)} \sigma_0^I(x_T) \quad (14)$$

and this is the desired result.

Coming back to our main equation (13) we want to stress that, for QCD Born cross-sections $\sigma_0^I(x_T)$ and QCD distributions $dJ_I(z, \delta)$ given in Eqs (3) and (4), it provides very good approximation to the exact cross-section $\sigma(x_T, \delta)$ given by formula (8). We have checked numerically that for $\delta < 0.5$ and calculating the saddle point positions according to (14) the accuracy of approximation (14) is better than 10%. Consequently we see that by measuring $\sigma(x_T, \delta)/\sigma_0(x_T)$ for small and large x_T we measure (if QCD is correct) some effective moments $J_G(s^G, \delta)$ and $J_F(s_0^F, \delta)$, respectively.

For the actual experimental analysis it is very convenient to define the function $C(x_T)$ as follows:

$$\bar{\sigma}(x_T, \delta) = \bar{\sigma}_0(x_T) \left(\frac{1}{S_0(x_T)} \right)^{2\xi C(x_T)} \quad (15)$$

which can be determined in a simple way from the data by performing calorimeter measurements at two different values of δ :

$$C(x_T) = \frac{\ln [\bar{\sigma}(x_T, \delta_1) / \bar{\sigma}(x_T, \delta_2)]}{2 [\xi(\delta_2) - \xi(\delta_1)] \ln S_0(x_T)} \quad (15')$$

where

$$S_0(x_T) = \frac{1}{\bar{\sigma}_0(x_T)} \int_{x_T}^1 \frac{dy_T}{y_T} \bar{\sigma}_0(y_T)$$

and $\sigma(x_T, \delta)$ and $\sigma_0(x_T)$ are measured cross-sections. As seen from formulae (3), (4), (11) and (15) the quantity $C(x_T)$ can be interpreted as an "effective radiation strength" of the jet. It is obvious from our previous discussion that in the small and large x_T limits the function $C(x_T)$ has a well-defined physical sense (in the QCD framework):

$$C(x_T) \rightarrow \begin{cases} C_G - \frac{a_G}{\ln S_0} \approx C_G & \text{small } x_T \\ C_F - \frac{a_F}{\ln S_0} \approx C_F & \text{large } x_T \end{cases} \quad (16)$$

To estimate quantitatively the expected relative contributions of gluon and quark jets to $\sigma_0(x_T)$ we have calculated the probabilities $P_I = \sigma_0^I(x_T) / \sigma_0(x_T)$ using all QCD Born terms⁶⁾ and the initial parton momentum distribution functions $F_I(x, p_T)$ as parametrized by Buras and Gaemers⁸⁾. In the gluon distribution of Ref. 8):

$$F_G(x, p_T^2) = \frac{A(p_T^2)}{x} (1-x)^{N_G(p_T^2)} \quad (17)$$

the value of N_G at $p_T^2 = 1.8 \text{ GeV}^2$ has been considered by us as a free parameter. As shown in Fig. 1, parametrization (17) taken with $1 \leq N_G (1.8 \text{ GeV}^2) \leq 2$ reproduces the values of the average gluon momentum $\langle x \rangle_G$

$$\langle x_G \rangle = \frac{\int_0^1 dx x^2 \frac{dN}{dx}}{\int_0^1 dx x \frac{dN}{dx}}, \quad \frac{dN}{dx} = F_G(x, Q^2) \quad (18)$$

recently extracted from neutrino data⁹⁾ in the range $5 \text{ GeV}^2 \leq Q^2 \leq 20 \text{ GeV}^2$. It should be noted that the average $\langle x \rangle$ defined by Eq. (18) characterizes the shape of the distribution: the steeper the distribution, the smaller the value of $\langle x \rangle$. Comparison of $\langle x \rangle_G$ with $\langle x \rangle_{Q-\bar{Q}}$ for valence quarks reported in Ref. 9) and also shown in Fig. 1 clearly shows the difference between the two distributions. Furthermore, we show in Fig. 2 that in the interesting range of Q^2 , parametrization Eq. (17) gives qualitatively a very similar difference between valence quark and gluon distributions practically for any input value of $N_G (1.8 \text{ GeV}^2)$. The calculated probabilities $P_I(x_T)$ are given in Fig. 3 for three different incoming energies (at $E_0 = 270 \text{ GeV}$ the calculation has been done for $p\bar{p}$ collisions) and for $N_G = 1.5$ and $N_G = 5$. We see that, as expected, in all cases there are well-distinguished regions in x_T where, to a good approximation, only one type of jet contributes to the cross-section $\sigma_0(x_T)$. We note that the transition from gluon to quark jets occurs for $x_T \leq 0.3 - 0.4$ so we do not need to go near to the phase space boundary to study both types of jets.

4. NON-PERTURBATIVE EFFECTS AND NUMERICAL ESTIMATES

At this stage we can summarize our discussion as follows: In an asymptotic situation (when the non-perturbative, finite p_T , effects are limited to the region $\delta = 0$) a measurement of the proposed quantity $C(x_T)$ provides a sensitive test of QCD both on the qualitative and quantitative level. Qualitatively, if there are different types of jets in large p_T collisions their presence will be seen in a single arm experiment as a variation with x_T of the quantity $C(x_T)$ in the range $0.05 \leq x_T \leq 0.5$, say. Quantitatively, the proposed method allows for the direct experimental determination of the values of the colour group factors C_G and C_F .

As we have already mentioned, a difficult problem for any QCD test at non-asymptotic energies is the contamination of non-perturbative effects. We shall now discuss this point in detail for the measurements proposed in this paper.

Non-perturbative transition from quarks and gluons contained in a solid angle 2δ and carrying fraction z of the parent parton energy to final hadrons distributed in a cylinder (of radius given by the average transverse momentum characteristic for non-perturbative parton $\rightarrow$ hadron transitions) causes some additional loss of energy contained in the angle 2δ . This is due to an obvious geometrical fact that part of the cylinder always remains outside a solid angle $2\delta \ll 180^\circ$ around the cylinder axis. The correction can easily be taken into account in an analytic form if we adopt a simple uncorrelated jet model for the non-perturbative cylinder¹⁾. Let us introduce the inclusive distribution $dN(z, \delta, P)/dz$ of signals in which a fraction z of the total energy P contained in a non-perturbative cylindrical jet is deposited in a solid angle 2δ around the jet axis. Then, the "corrected" formula (13) takes the following form^{*)}:

$$\bar{\sigma}(x_T, \delta) = \bar{\sigma}_0(x_T) \sum_{I=F,G} \frac{\bar{\sigma}_0^I(x_T)}{\bar{\sigma}_0(x_T)} J_I(s_0^I(x_T), \delta) N_I(s_0^I(x_T), \delta, x_T E_0) \quad (19)$$

where $N_I(s, \delta, P)$ is the Mellin transform of the distribution $dN_I(z, \delta, P)$:

$$N_I(s, \delta, P) = \int_0^1 dz z^{s-1} \frac{dN_I(z, \delta, P)}{dz}$$

and in the uncorrelated jet model calculation it reads¹⁾:

$$N_I(s, \delta, P) = \left(\frac{1}{1+s\tau} \right)^{\lambda_I} e^{-\lambda_I \frac{s\tau}{1+s\tau}}, \quad \tau = \frac{\langle p_T^{NP} \rangle}{2P\delta} \quad (20)$$

The parameters λ_I are the hadron densities in rapidity in the central region for the gluon and quark non-perturbative jets. We have studied numerically the expected role of non-perturbative effects at finite energies by calculating the function $C(x_T)$ from formula (15):

- a) assuming the absence of non-perturbative effects (i.e., calculating $\sigma(x_T, \delta)$ from Eq. (13));
- b) assuming only non-perturbative evolution of a jet (i.e., calculating $\sigma(x_T, \delta)$ from Eq. (19) with $J_I(s_0, \delta) = 1$, $I = F, G$);

*) Formula (19) is not exact because of the non-scaling explicit P dependence in the $dN_I(z, \delta, P)$. However, due to the trigger bias effect we can take $P \approx p_T = x_T E_0$ and then Eq. (19) follows.

c) including both effects according to formula (17).

The Born (jet) cross-section $\sigma_0(x_T) = \sigma_0^G(x_T) + \sigma_0^F(x_T)$ has been calculated as explained before in the context of our discussion of probabilities $P_I(x_T)$, with $N_G(1.8 \text{ GeV}^2) = 1.5$. The non-perturbative parameters λ_I have been fixed to be $\lambda_G = \lambda_F = 3$. Some comments are worth while on the values of parameters λ_F and λ_G . The value of λ_F can be considered as known experimentally from the e^+e^- annihilation into hadrons (it is given by the height of the hadronic plateau). We take it in the range $1.5 \leq \lambda_F \leq 3$. The value of λ_G is completely unknown. In particular one can imagine even the situation when $\lambda_G \rightarrow \infty$ or in more physical terms when the total energy of the perturbative gluon is converted non-perturbatively into soft hadrons. Then the direction of the perturbative parent is not remembered by the final hadronic state and the very concept of the gluon jet makes no sense. The popular assumption that the gluon jets are well defined (it means measurable) physical quantities means in terms of non-perturbative effects that λ_G cannot be much larger than λ_F . The simplest assumption is to take $\lambda_G = \lambda_F = 3$. The results for function $C(x_T)$ in each case (a)-(c) are shown in Fig. 4 for three different energies. Plotted values represent the average (by means of the least square method) over the results obtained applying formula (15) for several values of δ in the range indicated in Fig. 4.

From Fig. 4 we observe that non-perturbative effects imitate the perturbative QCD predictions for the dependence of $C(x_T)$ on x_T . Numerically, both effects give about equal contribution to $C(x_T)$ for $E = 270 \text{ GeV}$ and 400 GeV . Therefore, even at such energies the qualitative behaviour of the function $C(x_T)$ does not provide by itself a QCD test but attention has to be paid to the actual numerical values of this function.

One can also remark at this point that the importance of non-perturbative effects even for such energetic jets put in doubt the often advocated possibility of studying the perturbative properties of gluon jets originating from the decay of heavy quark bound states.

So far, we have considered fixed incoming energy E_0 . The situation can be considerably improved if one extends the measurements over some range of incoming energy. It follows from formulae (19) and (20) that if the positions $s^I(x_T, E_0)$ of saddle points are only weakly dependent on E_0 , the contribution of non-perturbative effects remains constant under the transformation:

$$E'_0 = E_0 \eta, \quad \delta' = \delta / \eta.$$

Consequently, the non-perturbative effects factorize out^{*)} if we measure $C(x_T)$ using formula (15') in the following way:

$$C(x_T) = \frac{\ln\left(\frac{\sigma(x_T, \delta, E_0)}{\sigma_0(x_T, E_0)} / \frac{\sigma(x_T, \delta/\eta, \eta E_0)}{\sigma_0(x_T, \eta E_0)}\right)}{-2\ln S_0(x_T) [\xi(\delta, x_T E_0) - \xi(\delta/\eta, x_T \eta E_0)]} \quad (21)$$

It is interesting to note that by using (5') we get:

$$\xi(\delta, x_T E_0) - \xi(\delta/\eta, \eta x_T E_0) = \frac{12}{50} \ln\left(\frac{\ln \frac{x_T E_0}{\Lambda}}{\ln \frac{\eta x_T E_0}{\Lambda}}\right) \quad (22)$$

so this difference depends only on η but not on δ and moreover it can be written as $\xi(\eta, x_T E_0)$. Consequently, the η dependence of the ratio

$$J(\eta x_T E_0, s_0, \delta/\eta) / J(x_T E_0, s_0, \delta)$$

expected from perturbative QCD is the same as δ dependence of the function $J(x_T E_0, s_0, \delta)$.

We have repeated our numerical exercise now using the improved method based on Eq. (20), again for three different energies E_0 , for several values of δ and varying η in the range $0.25 < \eta < 1$. The results for $\lambda_G = \lambda_F = 3$ and $N_G(1.8 \text{ GeV}^2) = 1.5$ are summarized in Fig. 5.

We observe the quantity $C(x_T)$ has the behaviour expected from perturbative QCD. Because of small contamination of quark jets at low x_T (see Fig. 3) the δ dependence does not cancel exactly as in Eq. (22). Nevertheless, the corrections are small and one can get quite good numerical estimation of the value of the radiation strength C_G for the gluon jet. (The structure of $C(x_T)$ in the transition region is due to the non-scaling effects in the jet cross-section $\sigma_0(x_T, E_0)$.)

To study the dependence on the value of λ_G we present also the results for $\lambda_G = 1.5$ and $\lambda_G = 6$ (Fig. 6). We observe that the δ dependence of the obtained curves for $C(x_T)$ is enhanced for large λ_G . However, there is still a very clear effect: the effective radiation strength $C(x_T)$ definitely rises at the low x_T region as compared with the constant value $C(x_T) \approx C_F$ at $x_T \gtrsim 0.4$. This effect

*) This is true in the whole range of x_T if $\lambda_F = \lambda_G$ and only in the two regions of dominance of the gluon or quark jets in the other case. However, only these two regions are important for our purpose.

is particularly striking when compared with the behaviour of $C(x_T)$ expected from purely non-perturbative, limited p_T jets (also shown in the Figures). We study also the dependence of the effect on the value of $N_G(1.8 \text{ GeV}^2)$ which, as seen from Fig. 7, is weak.

5. CONCLUSIONS

We have shown in this paper that the large p_T hadron-hadron collisions provide a very efficient framework for sensitive tests of QCD. The exceptional virtue of such collisions stems from the fact that they are the source of gluon and quark jets both produced with comparable (Born) cross-sections. In addition, as was shown by explicit calculation, gluon and quark jets are expected to be well separated in the x_T phase space, and hence, they can be separately analyzed and compared.

It has been shown that in the single arm calorimeter experiments one can measure some effective (large) moments of quark and gluon fragmentation functions. Based on this observation we propose to study a quantity which can be interpreted as an "effective radiation strength" of the jet and which we have found very sensitive to the existence of the two types of jets. According to perturbative QCD, this quantity should vary from $C_F = 4/3$ to $C_G = 3$ when x_T decreases from 1 to 0.

We discuss in detail the non-perturbative background which is sizeable even at Isabelle energies and makes the verification of the perturbative QCD predictions difficult. An efficient method is proposed to minimize those hadronic effects. The study of the "effective radiation strength" extracted by this method provides, in our opinion, the strongest QCD tests available in the reasonable future in the large Q^2 processes.

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Figure captions

Fig. 1 : Comparison of the CDHS result for

$$\langle x_G \rangle = \int dx x^2 \frac{dN}{dx} / \int dx x \frac{dN}{dx} , \quad \frac{dN}{dx} = F_G(x, Q^2)$$

with the Buras and Gaemers parametrization for several different values of $N_G(1.8 \text{ GeV}^2)$.

Fig. 2 : Comparison of $\langle x \rangle_Q$ and $\langle x \rangle_G$ versus Q^2 for several different values of N_G (Buras and Gaemers parametrization).

Fig. 3 : Probabilities $P_I(x_T)$ of quark and gluon jets versus x_T for ISR $p\bar{p}$ and ISABELLE.

Fig. 4 : "Effective radiation strength" $C(x_T)$ versus x_T for ISR, $p\bar{p}$ and ISABELLE (method I).

Fig. 5 : "Effective radiation strength" $C(x_T)$ versus x_T for ISR, $p\bar{p}$ and ISABELLE (method II). Dashed curve represents the pure perturbative prediction for $\delta = 0.10$ (case B).

Fig. 6 : "Effective radiation strength" $C(x_T)$ versus x_T (ISABELLE, method II) for some values of λ_Q, λ_G . (Dashed curve as in Fig. 5.)

Fig. 7 : "Effective radiation strength" $C(x_T)$ versus x_T (ISABELLE, method II) for some values of N_G . (Dashed curve as in Fig. 5.)

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG.7