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MAGNETIC SUPERCONDUCTORS AND THE MIT BAG MODEL

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ABSTRACT

The system of a non-Abelian superconductor is discussed. The superconductor is treated as a relativistic extended object possessing a magnetic current. We examine the case where the object is a four-volume occupying all of the Minkowski space-time. It is shown that if a cavity should develop in the superconducting four-volume, the dynamics is identical to that of a glueball in an MIT bag. Upon including quark fields in the above system we find that the quark colour density must vanish everywhere outside the bag, but the fields themselves need not. Colour singlets may therefore propagate in the vacuum defined by a superconducting four volume.

There have been some speculations recently that the QCD vacuum has features similar to that of a magnetic superconductor, whereby magnetic supercurrents build up an electric Meissner effect¹⁾. As was suggested by Mandelstam²⁾ such a mechanism could be responsible for quark confinement. The physical picture of the corresponding vacuum is a "dual" version of a vacuum which exhibits magnetic flux confinement³⁾.

The MIT bag model⁴⁾ is believed to be a phenomenological approximation of the corresponding fundamental QCD theory⁵⁾. The bag is viewed as the result of a bubble (or cavity) formation in the physical vacuum of QCD. In the present short note we will present some recent work on a Lagrangian formalism for magnetic superconductors, and verify that a cavity in a four-dimensional superconductor obeys the MIT bag equations of motion.

We begin with a discussion of magnetically charged extended objects. By extended objects we mean relativistic strings, shells, and four volume. An n dimensional object, Σ , is parametrized by a set of n parameters $\sigma = (\sigma^0, \sigma^1, \dots, \sigma^{n-1})$. If we let $z^\mu(\sigma)$ denote the co-ordinate of the object, the standard action for the free system can be written⁶⁾

$$\begin{aligned} \mathcal{S}_0 &= -T \int_{\Sigma} d\sigma \sqrt{-\det M} \\ M_{ab} &\equiv \partial_a z^\mu \partial_b z_\mu \end{aligned} \quad (1)$$

Here $\partial_a \equiv \partial/\partial\sigma^a$ and T is a fundamental constant of our theory, which in general depends on the dimension of the extended object. Now we wish to couple this system to a Yang-Mills field. That is, given a field tensor $\mathcal{G}^{\mu\nu}(x)$, we require that

$$\mathcal{D}_\mu^* \mathcal{G}^{\mu\nu}(x) = k^\nu(x) \quad (2)$$

where $*\mathcal{G}^{\mu\nu}$ denotes the dual of $\mathcal{G}^{\mu\nu}$, $\mathcal{D}_\mu$ is the covariant derivative, and $k^\nu(x)$ is the magnetic source defined on the extended object⁷⁾. Note that we cannot write $\mathcal{G}_{\mu\nu}(x) = [\mathcal{D}_\mu, \mathcal{D}_\nu]$ since the Bianchi identity would imply that k^ν vanishes

everywhere^{*)}. Instead, we may apply a technique similar to that used by Dirac⁸⁾ in his description of magnetic monopoles. In Dirac's work the magnetic monopoles appeared at the boundaries of an open string. The string, however, was electrically neutral (and unphysical). In the following we shall generalize Dirac's scheme so that magnetic sources can be distributed along the interior of an extended object, as well as on its boundary.

We first define the field tensor $g^{\mu\nu}(x)$ according to

$$\begin{aligned} g^{\mu\nu}(x) &= F^{\mu\nu}(x) + {}^*M^{\mu\nu}(x) \\ F^{\mu\nu}(x) &= [D^\mu, D^\nu](x) \\ M^{\mu\nu}(x) &= -g \int_{\Sigma} d^n\sigma \delta^4(x - z(\sigma)) \partial_a z^\mu \partial_b z^\nu \chi^{ab} \end{aligned} \quad (3)$$

Here g is the magnetic charge. The variable χ^{ab} is antisymmetric in the indices a and b . It has values in the Lie-algebra associated with the gauge group, and transforms under the adjoint action of the group. Under a reparametrization $\sigma \rightarrow \sigma' = \varphi(\sigma)$, χ^{ab} transforms as a tensor density. Now let us compute the right-hand side of Eq. (2).

$$\begin{aligned} D_\mu {}^*g^{\mu\nu}(x) &= -D_\mu M^{\mu\nu}(x) = \\ &= -g \int_{\Sigma} d^n\sigma \left\{ \partial_a \delta^4(x - z(\sigma)) \chi^{ab} + \right. \\ &\quad \left. + ie \delta^4(x - z(\sigma)) [A_a(\sigma), \chi^{ab}] \right\} \partial_b z^\nu \end{aligned} \quad (4)$$

^{*)} We assume that the Yang-Mills potentials are globally well defined.

where $A_a = \mathcal{A}_\mu(z) \partial_a z^\mu$, $\mathcal{A}_\mu(x)$ being the Yang-Mills potentials. (Here we have used $\mathcal{D}_\mu = \partial_\mu - ie[\mathcal{A}_\mu, \cdot]$.) Integrating by parts and applying Stokes theorem we find

$$\begin{aligned} k^\nu(x) = & g \int_{\Sigma} d\sigma \delta^4(x-z(\sigma)) k_{(i)}^a(\sigma) \partial_a z^\nu + \\ & + g \int_{\partial\Sigma} d^{n-1}\sigma \delta^4(x-z(\sigma)) k_{(s)}^a(\sigma) \partial_a z^\nu \end{aligned} \quad (5)$$

$$k_{(i)}^a(\sigma) = D_b \chi^{ba}(\sigma) \quad (6)$$

$$k_{(s)}^a(\sigma) = n_b \chi^{ab}(\sigma) \quad (7)$$

Here $D_b = \partial_b - ie[A_b, \cdot]$ and n_b a unit vector normal to the boundary $\partial\Sigma$. $k_{(i)}^a$ and $k_{(s)}^a$ thus represent magnetic charges flowing in the interior and at the boundary of Σ , respectively^{*)}.

Let us now study the dynamics of the system. A natural choice for the action is^{**) (8)}

$$\begin{aligned} \mathcal{S} &= \mathcal{S}_0 + \mathcal{S}_{YM} \\ \mathcal{S}_{YM} &= -\frac{1}{4} \int d^4x \text{Tr} G_{\mu\nu}(x) G^{\mu\nu}(x) \end{aligned} \quad (8)$$

*) In Ref. 8) the restriction $\chi^{ab} = \epsilon^{ab}$ was made (ϵ^{ab} is the antisymmetric symbol in two dimensions). Since the gauge group is $U(1)$ Eq. (6) implies $k_{(i)}^a = 0$. Consequently, Dirac's string is neutral.

**) It should be pointed out that our system is similar to that of Ref. 1). We believe, however, that we have a more complete description of the classical dynamics. [See Ref. 9).]

The dynamical variables in the theory are $\mathcal{A}_\mu$, z^μ and χ^{ab} . Here we shall examine the equations of motion resulting from variations of $\mathcal{A}_\mu$ and χ^{ab} . (The complete system of equations will be studied elsewhere⁹.)

Upon varying $\mathcal{A}_\mu$ in (8) we find the field equation

$$\partial_\mu G^{\mu\nu}(x) = 0 \quad (9)$$

Equation (9) states that no electric sources are present.

Upon varying χ^{ab} in (8) we find

$$\begin{aligned} \delta\mathcal{S} &= -\frac{1}{2} \int d^4x \operatorname{Tr}^* G_{\mu\nu}(x) \delta M^{\mu\nu}(x) = \\ &= \frac{1}{2} g \int_\Sigma d\sigma \operatorname{Tr}^* G_{ab}(\sigma) \delta \chi^{ab}(\sigma) \end{aligned} \quad (10)$$

where

$$^*G_{ab}(\sigma) \equiv ^*G_{\mu\nu}(z(\sigma)) \partial_a z^\mu(\sigma) \partial_b z^\nu(\sigma) \quad (11)$$

The resulting equation of motion is

$$^*G_{ab}(\sigma) = 0 \quad (12)$$

In order to understand the physical meaning of Eq. (12), let us first consider the case of a relativistic string, i.e., $n = 2$. In this case (12) reduces to a single equation $^*G_{01} = 0$. Upon identifying σ^0 as the time parameter; $z^0 = \sigma^0$; we have

$$^*G_{01}(\sigma) = ^*G_{0i}(z) \partial_1 z^i + ^*G_{ij}(z) \partial_0 z^i \partial_2 z^j = 0 \quad (13a)$$

Equation (13a) implies that in the rest frame of any infinitesimal segment of the string (i.e., $\partial_0 z^i = 0$) there is no tangential magnetic field. Nevertheless, a magnetic current is allowed to flow along the string. Consequently, the magnetic conductivity is infinite.

For a shell ($n = 3$), in addition to (13a) we have

$${}^*G_{02}(\sigma) = {}^*G_{0i}(z)\partial_2 z^i + {}^*G_{ij}(z)\partial_0 z^i\partial_2 z^j = 0 \quad (13b)$$

$${}^*G_{12}(\sigma) = {}^*G_{ij}(z)\partial_1 z^i\partial_2 z^j = 0 \quad (13c)$$

Equations (13a) and (13b) imply that in the rest frame of any infinitesimal segment of the shell there is no tangential magnetic field. Thus again the magnetic conductivity is infinite. Now Eq. (13c) implies that the electric field normal to the shell surface must vanish; i.e., electric flux cannot penetrate the shell. This is the "dual" Meissner effect. It is clear that an extended object satisfying (12) has all the characteristic of a magnetic superconductor (with zero penetration depth).

We now discuss the case of a superconducting four-volume. Here, the expression for $\mathcal{M}^{\mu\nu}$ [Cf., Eq. (3)] can be simplified. By performing the reparametrization $\sigma \rightarrow z$,

$$\mathcal{M}^{\mu\nu}(x) = -g \int_R d^4 z \delta^4(x-z) \chi^{\mu\nu}(z) \quad (14)$$

R being the region occupied by the superconductor^{*)}. By varying $\chi^{\mu\nu}$ in (8) we again obtain the superconductivity condition. Only here it implies that all the fields $\mathcal{G}_{\mu\nu}$ must vanish in R . This in turn implies that no magnetic currents are present within R . (However, they may be present at the boundary ∂R .) $k_{(i)}^\mu = 0$ follows from the superconductivity condition

^{*)} In terms of four-dimensional magnetostatics⁵⁾ we see from (3) and (14) that $\chi^{\mu\nu}$ corresponds to a bulk magnetization tensor.

$$*F_{\mu\nu}(x) + g\chi_{\mu\nu}(x) = 0 \quad (15)$$

and the Bianchi identity.

Let us now consider the case where $R = \mathbb{R}^4$; i.e., the superconductor occupies all of the Minkowski space. Note that no derivatives of $\chi^{\mu\nu}$ appear in the action $\mathcal{S}_{YM}$. Thus we can substitute (15) into the expression for the action, and thereby eliminating $\chi^{\mu\nu}$. The result is $\mathcal{S}_{YM} = 0$. Consequently, the action describing the "superconducting vacuum" is given by

$$\mathcal{S}_{VAC} = \mathcal{S}_0 = -\gamma \int_{\mathbb{R}^4} d^4x \quad (16)$$

which is of course divergent.

If a bubble (or a cavity) occupying a region V should develop in the superconductor, we would then have

$$\mathcal{S} = -\gamma \int_{\mathbb{R}^4 - V} d^4x - \frac{1}{4} \int_V d^4x \text{Tr} G_{\mu\nu}(x) G^{\mu\nu}(x) \quad (17)$$

In Equation (17) we have again used (15) to remove the field kinetic energy in the superconducting region. In order to make the expression (17) finite we can subtract off the vacuum action Eq. (16). We therefore obtain

$$\mathcal{S} - \mathcal{S}_{VAC} = \int_V d^4x \left\{ \gamma - \frac{1}{4} \text{Tr} F_{\mu\nu}(x) F^{\mu\nu}(x) \right\} \quad (18)$$

In (18) we have used $g^{\mu\nu}(x) = \mathcal{F}^{\mu\nu}(x)$, $x \in V$. Now the original dynamical variables in the theory were $\chi^{\mu\nu}$, $\mathcal{A}_\mu$ and z^μ . Many of these degrees of freedom have been eliminated. $\chi^{\mu\nu}$ was eliminated through the application of (15). The potentials $\mathcal{A}_\mu$ were eliminated outside V . The only remaining degrees of freedom z^μ correspond to the freedom to deform the surface ∂V . In conclusion we are left with the following dynamical variables (a) $\mathcal{A}_\mu$ in V , (b) $\mathcal{A}_\mu$ at the boundary ∂V and (c) the boundary ∂V .

The above system is identical to that of the MIT bag model in the absence of matter^{*)}. The bag is seen to occupy the cavity in a magnetic superconductor. Magnetic currents are freely flowing at the boundary ∂V and are the origin of the field strength discontinuities at ∂V . The equation of motion resulting from a variation of $\mathcal{A}_\mu$ at the boundary ∂V is

$$n_\mu \mathcal{F}^{\mu\nu}(x) = 0 \quad (19)$$

where n_μ is a unit vector normal to the surface ∂V . From (19) the electric fields normal to the surface ∂V vanish. This result can again be interpreted as a "dual" Meissner effect. In (18) we have to identify T with $-B$ where B is the MIT bag constant. By (16) this means that the energy density of the vacuum is negative. We notice that a ferromagnetic state of QCD has lower energy density than the conventional perturbative vacuum^{**) as is discussed in the literature¹⁰⁾.}

Here we have given a discussion of magnetic confinement of a pure Yang-Mills field in terms of a superconducting vacuum which exhibits a "dual" Meissner effect. The reverse situation of electric confinement and a vacuum which exhibits the conventional Meissner effect can also be discussed. This is done by making use of a slight modification of the formalism developed in the present paper [see Ref. 9) for a detailed account]. We find thereby that it is impossible to have a situation with both magnetic and electric confinement due to the incompatibility of boundary conditions. This fact is in accordance with a recent result of 't Hooft's¹²⁾.

Let us now consider matter fields ψ coupled to the above system of magnetic superconducting four volume. The additional contribution to the action (18) is in general given by

*) In Ref. 9) a more complete derivation is given in terms of equations of motion. The presentation here is aimed to be pedagogical and economical in view of the space needed.

**) A simple and heuristic argument in favour of a QCD vacuum in the negative energy density has been given by S. Coleman and K. Johnson. We refer to the presentation by V.F. Weisskopf¹¹⁾.

$$\mathcal{S}_M = \int_{\mathbb{R}^4} d^4x \left\{ \mathcal{L}_F(\Psi, \partial\Psi) + e \text{Tr} j_\mu(\Psi, \partial\Psi) \mathcal{A}^\mu(x) \right\} \quad (20)$$

where $\mathcal{L}_F$ is the free field Lagrangian density, and j_μ is the (electric) current density of the matter fields. In order to reproduce the standard bag model it is necessary to impose the restriction

$$\Psi(x) = 0 \quad ; \quad x \in R \quad (21)$$

R being the superconducting region. Note, however, that unlike the case with condition (15), our theory does not require that we make the above restriction. Let us therefore examine the consequences of dropping Eq. (21). We find that the potentials $\mathcal{A}_\mu(x)$, $x \in R$, appear in the action without the presence of a kinematic energy term. $\mathcal{A}_\mu(x)$, $x \in R$, is thus effectively a Lagrange multiplier. Variations of $\mathcal{A}_\mu$ in the superconducting region then leads to^{*})

$$j_\mu(\Psi, \partial\Psi) = 0 \quad (22)$$

Equation (22) replaces the stronger condition (21). Thus we find that the (electric) current density of the matter fields must vanish everywhere in the superconducting region. Theories of this type are of current interest because they exhibit the feature of (quark) confinement¹³⁾. It should be pointed out that in the case of quark fields, non-trivial solutions satisfying (22) are known to exist¹⁴⁾. It actually turns that out these solutions can be used to break the chiral invariance (the quark mass matrix being zero) on the classical level. We will return to this interesting question elsewhere. Thus to conclude we have shown that quark fields may be present in the superconducting region outside the bag provided they form colour singlets (locally).

^{*}) Equation (22) is also seen to follow from $\mathcal{D}_\mu \mathcal{G}^{\mu\nu} = j^\nu$ and $\mathcal{G}^{\mu\nu}(x) = 0$, $x \in R$.

Needless to say most of the physical arguments appearing in the present work are not new or original and our results are in line with some existing ideas on a description of the confinement mechanism in gauge theories. It occurs to us relevant, however, that one can describe some aspects of the deep question of confinement in terms of a classical Lagrangian which explicitly exhibits the dual Meissner effect and the presence of magnetic supercurrents. Having a Lagrangian formalism one can study quantum corrections. Actual calculations may suggest that the gluons can be generated by quantum fluctuations in a manner not very different from what is known to happen in some two dimensional models¹⁵⁾. We intend to discuss this question elsewhere.

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