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THE FATE OF PRIMORDIAL MAGNETIC MONOPOLES

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ABSTRACT

Unified gauge theories, in which the $SU(2)_L$ and $U(1)_Y$ subgroups do not have non-trivial intersections, possess an in-built mechanism for a complete extinction of the primordial magnetic monopoles. The potential conflict with the standard hot big bang cosmology is therefore avoided in such theories. The usual gauge theories, based on $SU(5)$, $SO(10)$ and E_6 do not have this property.

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Attention has recently been drawn by several authors¹⁾⁻³⁾ to a potential conflict between unified gauge theories of strong, weak and electromagnetic interactions (UGTS) and the standard hot big bang cosmology. In particular, the density of superheavy magnetic monopoles⁴⁾ of UGTS^{5),6)} at the time of helium synthesis (and at present) has been estimated to be about 10 (15) orders of magnitude larger than the value permitted by cosmological observations.

We wish to point out that the problem can be avoided in UGTS in which the $SU(2)_L$ and the $U(1)_Y$ subgroups do not have non-trivial intersections with each other. This requirement is not fulfilled by the usual unified theories based on $SU(5)$ ⁶⁾, $SO(10)$ ⁷⁾ and E_6 ⁸⁾.

The magnetic quantum numbers of the monopoles produced in the very early universe are determined by the global structure of the unbroken subgroup, in particular by its electroweak part. These quantum numbers, in turn, determine the subsequent fate of the primordial monopoles. The following phenomenon occurs in UGTS in which $SU(2)_L$ and $U(1)_Y$ do not have non-trivial elements in common. After the breaking of $SU(2)_L \times U(1)_Y$ down to $U(1)_{EM}$, each monopole gets connected to an antimonopole by a magnetic flux string of the Z^0 field. Rapid annihilation then takes place and the potential conflict with the standard hot big bang cosmology is avoided.

Let us first consider the $SU(5)$ gauge theory which undergoes the standard symmetry breaking^{*)}:

$$SU(5) \xrightarrow{\langle \Phi \rangle} (SU(3)_c \times SU(2)_L) \otimes U(1)_Y \xrightarrow{\langle H \rangle} SU(3)_c \otimes U(1)_{EM}^{(1)}$$

The expectation values of the Higgs 24 plet Φ and the Higgs 5 plet H are of order 10^{15} GeV and 300 GeV, respectively. Suppose now that very early in the history of the universe, the temperature T exceeds a critical temperature $T_{c1} \sim \langle \Phi \rangle$. The $SU(5)$ gauge symmetry is then fully restored and no monopoles are present. When the universe cools down below T_{c1} , $SU(5)$ breaks down to $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$, and superheavy magnetic monopoles are produced. These monopoles are topologically stable and carry $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ magnetic fields simultaneously⁹⁾. Their topological quantum number corresponds to their $U(1)_Y$ magnetic charge.

*) The symbols $\times$ and $\otimes$ mean direct product and local direct product, respectively.

For our purposes, it is sufficient to consider only the "elementary" monopole (and its antimonopole) with mass of the order of 10^{16} GeV, which carries the minimal $U(1)_Y$ magnetic charge^{*}). This monopole forms an $SU(3)_C$ magnetic triplet, an $SU(2)_L$ magnetic doublet, and carries $y = -1/2 U(1)_Y$ magnetic charge (for details see the first paper in Ref. 9)). This charge is the smallest $U(1)_Y$ magnetic charge allowed by the Dirac quantization condition in the presence of particles which are singlets under $SU(3)_C$ and $SU(2)_L$. It is important to note that the $SU(3)_C$ and $SU(2)_L$ subgroups of $SU(5)$ have non-trivial intersections with $U(1)_Y$. This feature is responsible for the fact that the elementary monopole carries $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ magnetic charges simultaneously.

When the universe cools below a critical temperature $T_{c2} \sim \langle H \rangle$, the electroweak subgroup $SU(2)_L \otimes U(1)_Y$ breaks down to $U(1)_{EM}$. We consider the fate of the primordial monopoles under this phase transition. The $SU(3)_C$ magnetic fields remain unaffected. The $SU(2)_L$ magnetic field of the monopole can be brought, by a suitable gauge transformation, into the third direction of $SU(2)_L$. Its strength is characterized by the magnetic charges $t_3 = \pm 1/\sqrt{2}$ (see the first paper in Ref. 9)). It is instructive to re-express the $SU(2)_L \otimes U(1)_Y$ magnetic charges of the monopole in an alternative orthogonal basis:

$$Q = -\sqrt{2} T_3^L + Y = \left(\begin{array}{c|cc} -\frac{1}{3} 1_3 & 0 \\ \hline 0 & 0 & 0 \\ & 0 & 1 \end{array} \right) \quad (2)$$

$$Z = \frac{10}{3\sqrt{2}} T_3^L + Y = \left(\begin{array}{c|cc} -\frac{1}{3} 1_3 & 0 \\ \hline 0 & \frac{4}{3} & 0 \\ & 0 & -\frac{1}{3} \end{array} \right)$$

where

$$T_3^L = \frac{1}{2\sqrt{2}} \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \\ & & & & -1 \end{pmatrix} \quad (3)$$

^{*}) Monopoles with non-minimal $U(1)_Y$ magnetic charge, if produced, may be regarded, at least as far as their quantum numbers are concerned, as composite states made out of the "elementary" monopole and its antimonopole.

and

$$Y = \left(\begin{array}{c|c} -\frac{1}{3} 1_3 & 0 \\ \hline 0 & \frac{1}{2} 1_2 \end{array} \right) \quad (3) \text{ cont.}$$

is the generator of $U(1)_Y$. Q generates $U(1)_{EM}$ and Z is the generator in the Z^0 boson direction. The $SU(2)_L \otimes U(1)_Y$ magnetic field strength of the monopole

$$B = t_3 T_3^L + y Y = \pm \frac{1}{\sqrt{2}} T_3^L - \frac{1}{2} Y, \quad (4)$$

expressed in the new basis, becomes

$$B = q Q + z Z$$

where

$$(q, z) = \left(-\frac{1}{2}, 0 \right) \text{ or } \left(-\frac{1}{8}, -\frac{3}{8} \right). \quad (5)$$

The fate of the primordial monopoles, after the breaking of $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{EM}$, depends on their magnetic charges q and z . The monopoles with $(q, z) = (-1/2, 0)$ develop into pure $U(1)_{EM}$ magnetic monopoles carrying a Coulombic field. However, the monopoles with $(q, z) = (-1/8, -3/8)$ develop a magnetic flux tube of the Z^0 boson field because of the partial Higgs breaking of the electroweak subgroup^{*)}. They carry, in addition, a Coulombic $U(1)_{EM}$ magnetic field. Clearly, the latter monopoles become bound to their antimonopoles by their magnetic flux strings and get rapidly annihilated. However, the pure $U(1)_{EM}$ monopoles do survive, and apparently lead to a conflict between the $SU(5)$ gauge theory and the hot big bang cosmology.

Before proceeding to gauge groups larger than $SU(5)$, let us show that the flux of the Z^0 field through the tube of the $(q, z) = (-1/8, -3/8)$ monopole corresponds to the minimal flux allowed by the quantization condition. Consider a small circle C about a Z^0 field flux tube. The Z^0 field on this circle is a pure gauge transformation

$$A_i^Z = \frac{1}{i} e^{-i\theta Z} \partial_i e^{i\theta Z} = \partial_i \theta Z \quad (i=1,2,3). \quad (6)$$

Consequently, the Z^0 flux through C is

*) Monopoles carrying a flux tube of Z^0 field have first been considered in Ref. 10) in the context of $SU(2)_L \times U(1)_Y$ model.

$$\oint_C \mathbf{A}_i^Z dx_i = \Delta\theta Z \quad (7)$$

where $\Delta\theta$ is the change in θ in going once around C . The quantization condition for $\oint_C \mathbf{A}_i^Z$ is obtained by noting that

$$e^{i\Delta\theta Z} \in SU(3)_c \otimes U(1)_{EM} \quad (8)$$

where $SU(3)_c \otimes U(1)_{EM}$ is the unbroken subgroup of $SU(5)$. Equation (8) implies

$$\oint_C \mathbf{A}_i^Z = \Delta\theta Z = \frac{3\pi}{2} n Z, \quad n = 0, \pm 1, \pm 2, \dots \quad (9)$$

The magnetic flux carried by the Z^0 field tube of the monopole with charge $(q, z) = (-1/8, -3/8)$ is $4\pi z Z = (-3\pi/2)Z$. Comparison with Eq. (9) shows that this flux corresponds to the minimal flux allowed by the quantization condition.

We now turn to a discussion of unified groups larger than $SU(5)$. For definiteness, consider the gauge group $SO(10)$ which undergoes the following chain of symmetry breakings^{*)}:

$$\begin{array}{c} SO(10) \\ \downarrow M_X \\ SU(4)_c \otimes (SU(2)_L \times SU(2)_R) \\ \downarrow M_c \\ (SU(3)_c \times SU(2)_L) \otimes U(1)_Y \\ \downarrow M_W \\ SU(3)_c \otimes U(1)_{EM} \end{array} \quad (10)$$

^{*)} By $SO(10)$ we really mean its universal covering group $Spin(10)$. The first stage of the symmetry breaking is then $Spin(10) \rightarrow Spin(6) \otimes Spin(4)$, where $Spin(6) = SU(4)_c$ and $Spin(4) = SU(2)_L \times SU(2)_R$.

The mass scales M_x , M_c and M_w are of order 10^{15} GeV, 8×10^{13} GeV and 10^2 GeV, respectively¹¹⁾. Note that the -1_4 element of $SU(4)_c$ is identified with the $(-1_2, -1_2)$ element of $SU(2)_L \times SU(2)_R$, as can be checked by their actions on the fundamental spinor. The generator Y of $U(1)_Y$ is a linear combination of the fifteenth generator C_{15} of $SU(4)_c$ and the third generator T_3^R of $SU(2)_R$:

$$Y = T_3^R + \sqrt{\frac{2}{3}} C_{15} \quad (11)$$

It follows that $U(1)_Y$ has non-trivial intersections with $SU(3)_c$. Their common points form the centre of $SU(3)_c$ ^{*)}. Moreover, as in the $SU(5)$ case, $U(1)_Y$ does have non-trivial intersections with $SU(2)_L$.

As the universe cools below a critical temperature $T_{c1} \sim M_x$, the $SO(10)$ symmetry breaks down to $SU(4)_c \otimes (SU(2)_L \times SU(2)_R)$. However, no stable magnetic monopoles are produced. Further cooling below $T_{c2} \sim M_c$ breaks the symmetry down to $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$. Topologically stable primordial monopoles, with mass $\sim 4 \times 10^{15}$ GeV, can then be created. Since the global structure of the unbroken subgroup $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$ is the same as in the $SU(5)$ case, the previous discussion still applies. Therefore, the conflict with the hot big bang cosmology is not avoided in extending $SU(5)$ to $SO(10)$ or E_6 ^{**) .}

One way out of the difficulty is now clear. One needs to embed $SU(2)_L$ and $U(1)_Y$ in a unifying group, so that they do not have non-trivial intersections. The primordial monopoles do not then possess $SU(2)_L$ magnetic quantum numbers^{***)}. When $SU(2)_L \times U(1)_Y$ breaks down to $U(1)_{EM}$, they unavoidably carry a magnetic flux tube of the Z^0 boson field, as well as a Coulombic $U(1)_{EM}$ field. It is clear that the Z^0 and the $U(1)_{EM}$ magnetic fluxes of these monopoles are strictly proportional. Consequently, in contrast to the previous cases, all primordial monopoles get connected to their antimonopoles by their magnetic flux strings, and then rapidly annihilate (see below). The potential conflict between unified gauge theories and the standard hot big bang cosmology can therefore be avoided.

*) The centre of $SU(3)_c$ is also contained in $U(1)_{EM}$.

**) See Ref. 12) for examples of larger groups.

***) Monopoles that carry non-zero $SU(2)_L$ charges are, in this case, dynamically unstable. This is because they form $SU(2)_L$ magnetic triplets, quintets, etc.

In order to gain some insight into the annihilation mechanism of the magnetic monopoles, let us consider the phase transition from $SU(2)_L \times U(1)_Y$ to $U(1)_{EM}$. This transition takes place at a cosmic time of order 10^{-10} sec. At that time, using the estimates given in Ref. 2), the mean monopole-antimonopole separation is $\sim 10^{-13}$ cm. The thickness of the Z^0 flux strings and the size of the monopoles^{*)} are $\sim 10^{-16}$ cm¹⁰⁾. Thus, a typical monopole-antimonopole pair is connected by a well-defined string with tension $\sim 10^6$ GeV². Since the monopoles are superheavy, their motion is non-relativistic. It turns out that the monopole-antimonopole bound system predominantly undergoes longitudinal vibrations^{**)} , with a period $\sim 10^{-19}$ sec. Therefore, it radiates away the energy contained in the flux tube by emitting Z^0 's, W 's, γ 's etc., and perhaps also flux loops of the Z^0 field. Let us say that one Z^0 boson is emitted per oscillation. The energy of the flux tube is then radiated away in about 5×10^{-15} sec. The monopole-antimonopole system now forms a magnetic Coulombic bound state, which annihilates almost instantaneously (i.e., in about 10^{-40} sec.). It is clear, therefore, that the monopole-antimonopole annihilation takes place well before the time of helium synthesis. Note that this annihilation heats the universe back to a temperature $\sim 10^3$ GeV.

To summarize, it is not unexpected that cosmological considerations may impose constraints on unified theories of strong, weak and electromagnetic interactions. In order to decide whether an extension of the known models is necessary, it is essential to refine¹³⁾ the previous estimates on the density of superheavy magnetic monopoles produced in the early universe. Nevertheless, it is important to realize that UGTS in which $SU(2)_L$ and $U(1)_Y$ do not have non-trivial intersections, possess an inbuilt magnetic confinement mechanism which leads to a complete extinction of the primordial monopoles. Such theories must contain new kinds of particles that violate the selection rules obeyed by the weak isospin and hypercharge of the known particles. The problem of constructing a realistic example of such a theory deserves further attention.

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*) Note that the monopoles possess a superheavy core of size $\sim 10^{-28}$ cm. This property is, of course, not possessed by the monopoles considered in Ref. 10).

**) The rotational modes of the system are essentially absent. This is so because the centrifugal force, for separation $\sim 10^{-13}$ cm and thermal velocity $\sim 1.7 \times 10^{-7}$, is $\sim (24 \text{ GeV})^2$, which is much smaller than the string tension ($\sim (1 \text{ TeV})^2$). The centrifugal force is $\sim (1 \text{ TeV})^2$ for separation $\sim 2 \times 10^{-16}$ cm. However, the linear force is then absent, and the magnetic Coulomb attraction dominates.

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