Ref.TH.2834-CERN

QCD CORRECTIONS AT THE Z_0

B. Humpert and W.L. van Neerven
CERN -- Geneva

ABSTRACT

All QCD corrections in hadron-initiated lepton-pair production are questionable for $Q^2 \lesssim 100 \text{ GeV}^2$ since a perturbative expansion is not justified in this kinematical range. Application of the renormalization group (RG) program at Z_0 energies is meaningful. We exhibit the size of the non-log correction terms, introduce experimental tests, and point to the assumptions inherent in Drell-Yan (DY) type analyses.

Ref.TH.2834-CERN

17 March 1980

Recent analyses of the next-to-leading QCD corrections in proton initiated massive lepton-pair production have revealed corrective contributions which, for $Q^2 \lesssim 100 \text{ GeV}^2$, are a factor $\lesssim 3$ bigger than the leading log approximation ^{1),2)}.

The validity of perturbative QCD in this energy range must therefore be considered with reservations. Tests of QCD at energies of the Z_0 boson ($M_{Z_0} \approx 90 \text{ GeV}$) ³⁾, however, are meaningful since the order α_s, α_s^2 corrections become moderate. We illustrate in the following this insight by consistently evaluating the correction terms, and we discuss the assumptions and uncertainties hidden in our results.

The corrections to the leading log approximation (LL) of Drell-Yan lepton pair production result from $q\bar{q}$ annihilation, from qg scattering and from qq scattering. They may be written in the form

$$\frac{d\sigma}{dq^2} = LL + \alpha_s \cdot q\bar{q} + \alpha_s^2 \cdot q\bar{q} + \alpha_s \cdot qq + \alpha_s^2 \cdot qq \quad (1)$$

The order α_s corrections of $q\bar{q}$ annihilation and of qg scattering have been extensively analyzed ¹⁾. The $q\bar{q}$ corrections are large but gradually reduce below the LL approximation at large Q^2 , whereas the qg corrections are not significant even at moderate Q^2 . The large size of the $q\bar{q}$ contribution raises doubts on the validity of an RG approach in the lower Q^2 region. One begins to wonder about the size of the order α_s^2 corrections, particularly at larger Q^2 values where the order α_s terms are indeed small. There exists no complete analysis in this order yet. Estimates of the size of the higher order contributions to $q\bar{q}$ annihilation have been made ⁴⁾. To get a size estimate for $q\bar{q}$ annihilation we square the order α_s moments and carry out an inverse Mellin transformation. The result has the structure

$$\alpha_s^2 \cdot q\bar{q} \sim \left(\frac{2\alpha_s}{3\pi} \right)^2 \left[\sum_{n=0}^3 a_n(z) \left\{ \frac{\ln^n(1-z)}{1-z} \right\}_+ + \delta(1-z) \left\{ \frac{78}{45} \zeta(2) + \frac{7}{6} \zeta(2) + 24 \zeta(3) + 1 \right\} \right] \quad (2)$$

where $\zeta(n)$ is the zeta function ⁵⁾. The order α_s^2 contributions to qg scattering are almost certainly irrelevant, and similarly the order α_s^2 diagrams of qq scattering ^{6),7)} are of little influence in the experimentally accessible

τ range. In order to illustrate the size of, and the scale violation due to these contributions we indicate in Fig. 1, the Q^2 evolution (τ fixed) of γ^* and Z_0 muon pair production in proton-antiproton annihilation.

The numerical analysis has been carried out with the usual DY formula using LL parametrizations for the scale-dependent parton distributions ⁸⁾ and using the order α_s , α_s^2 corrections given in the literature ^{1),2),6)}. Without any QCD corrections $Q^4(d\sigma/dQ^2)$ will be Q^2 independent with a horizontal line passing through the input point at $Q^2 = Q_0^2 = 1.8 \text{ GeV}^2$. If the asymptotic freedom corrections (AF) are taken into account the leading log curve decreases. At lower Q^2 values the $q\bar{q}$ non-leading corrections rise substantially above the leading-log curve and the validity of the RG approach must be questioned.

As we go to higher Q^2 values the large constant terms are damped by $\alpha_s(Q^2)$ and the leading-log approximation becomes more and more valid. The QCD corrections are less pronounced for smaller τ values; they amplify if τ is increased and the validity of the leading-log approximation sets in later. The corrections following from qq and qg scattering are small. At Q^2 values around the Z_0 mass the next-to-leading corrections no longer exceed the leading-log results, and we consequently have reached the appropriate region of perturbative QCD. $\alpha_s \cdot q\bar{q}$ lies a factor 2.5 below the leading-log approximation and $\alpha_s^2 \cdot q\bar{q}$ reduces to about 15%. This latter term is only an indication of the size of the order α_s^2 $q\bar{q}$ corrections and must be added and subtracted to estimate the error on the prediction through $O(\alpha_s)$. We emphasize the suppression of the $\alpha_s^2 \cdot q\bar{q}$ term. Its small size indicates that the particular choice of the mass factorization, influencing substantially the order α_s $q\bar{q}$ term, will not introduce important changes due to higher order contributions.

The $1/Q^4$ decrease of muon pair production results in cross-section values which exclude experimental tests in an extended Q^2 range. However, the production of Z_0 with the large counting rate at its peak gives the possibility to test the RG approach in its appropriate kinematical region. We have for this reason indicated in Fig. 1 the γ^* and Z_0 rates assuming for the mass $M_{Z_0} = 88.66 \text{ GeV}$ and the width $\Gamma_{Z_0} = 2.49 \text{ GeV}$. This latter value is derived assuming six quark flavours and three leptons plus their neutrinos. The value of the Weinberg angle $\sin^2\theta_W = 0.23$ leads to a negligible $\gamma^* - Z_0$ interference term.

Writing for γ and Z_0 exchange

$$\left(\frac{d\sigma}{dQ^2}\right)_\gamma + \left(\frac{d\sigma}{dQ^2}\right)_{Z_0} \equiv a \frac{A(\tau, Q^2)}{Q^4} + b \frac{B(\tau, Q^2)}{(Q^2 - M_{Z_0}^2)^2 + \Gamma_{Z_0}^2 M_{Z_0}^2} \quad (3)$$

with

$$a \equiv \frac{4\pi\alpha^2}{9}, \quad b \equiv \frac{G^2 M_{Z_0}^2}{288\pi} \{ (4\sin^2\theta_W - 1)^2 + 1 \} \quad (4)$$

the strongest Q^2 dependence has been explicitly exposed. $A(\dots)$ and $B(\dots)$ are convolutions over the parton momentum distributions multiplied with the corresponding electromagnetic/weak charges.

Once the Z_0 boson is found and its mass and width are experimentally determined one is able to carry out tests on the RG approach. Q^2 is fixed at the Z_0 peak-height and if the initial $p\bar{p}$ energy is varied different τ values result. The entire τ dependence of the muon pair production cross-section is contained in $B(\tau, M_{Z_0}^2)$. In Fig. 2a we show its anticipated shape using scale-independent (Curve 1) and scale-dependent (Curves 2-4) parton distributions. The size of the leading-log contribution and of the order α_s, α_s^2 $q\bar{q}$ correction terms is indicated. The scale-independent calculations reveal a moderately changing τ dependence for $\tau < 0.1$, whereas strong τ variation is observed if AF corrections are taken into account. In the $\tau \sim 0.1$ region the AF curves are almost an order of magnitude below the predictions with scale-independent parton distributions.

We conclude that measurement at the Z_0 boson mass of two maximally separated τ points provides us with a rapid and clean test of the RG approach to gluon radiation.

Can a similar test be carried out at lower Q^2 values, at $Q^2 = 100 \text{ GeV}^2$, say? The result of our numerical analysis is presented in Fig. 2b. We notice here that the size of the non-leading terms is still considerable although the leading-log curve dominates. We notice qualitatively the same characteristics as were found at the Z_0 boson but the results for scale-independent and -dependent momentum distributions appear less distinct. At this lower Q^2 value one might be able to reach much smaller τ values with the $p\bar{p}$ (and pp) accelerators at present in construction or being planned.

The results presented in Fig. 2 emerge from a summation over several $q\bar{q}$ combinations. In symbolic notation they are indicated by

$$\begin{aligned} A, B = & e_u^2 (u_v \bar{u}_v + s u_v + \bar{u}_v s) \\ & + e_d^2 (s d_v + \bar{d}_v s) + (2e_u^2 + 4e_d^2) (ss) \end{aligned} \quad (5)$$

where $u \equiv u_v + s$, $d \equiv d_v + s$ ($\bar{d} \equiv \bar{d}_v + s$, etc.) are the quark distributions in the proton (antiproton) and s represents the SU(3) symmetric sea. The coefficients $e_u^2 = 4/9$, $e_d^2 = 1/9$ indicate the product of quark charges if electromagnetic muon pair production is considered. Similarly in Z_0 production they stand for the weak charges with

$$e_u^2 \equiv (a_u^2 + b_u^2) \quad , \quad e_d^2 \equiv (a_d^2 + b_d^2) \quad (6)$$

where, in the standard Weinberg-Salam model ⁶⁾

$$a_u \equiv 1 - \frac{8}{3} \sin^2 \theta_W \approx 0.387 \quad , \quad b_u \equiv -1 \quad (7)$$

$$a_d \equiv \frac{4}{3} \sin^2 \theta_W - 1 \approx -0.693 \quad , \quad b_d \equiv +1 \quad (8)$$

In Fig. 3a we exhibit the percentage of the different contributions with changing τ values keeping Q^2 fixed at the Z_0 mass. The analogous results for the electromagnetic process at $Q^2 = 100 \text{ GeV}^2$ are shown in Fig. 3b. When comparing Fig. 3 with the analogous calculations assuming no strong interaction corrections one notices the strong rise of the ss contribution at small τ values which is a dominating factor in the considerable AF changes seen in Fig. 2.

One immediately notices the considerable influence of the combination

$$e_u^2 (u_v \bar{u}_v + s u_v + \bar{u}_v s) \quad (9)$$

which, in the region $\tau \sim 0.1$, dominates for both γ^* and Z_0 production. The Z_0 calculation indicates $\sim 20\%$ error due to the $d_v \bar{d}_v$ quark combination which reduces to 5% for the analogous γ^* process. Apart from the different e_q^2 factor for the two processes one therefore is roughly measuring the same quark combinations at two different Q^2 values.

The product of quark charges is $e_u^2 = 4/9$ (for γ^*), 1.15 (for Z_0) giving $B/A = 2.6$ (for $\tau \sim 0.1 - 0.2$ and $Q^2 = M_Z^2$).

The error due to the other quark contributions is ignored. In Fig. 4 we have plotted the τ dependence of A, B (Curves 1 and 2) and find the anticipated discrepancy (a factor of 2.6) due to the difference in the electromagnetic and weak quark charges. Curve 3 indicates that τ dependence of $A(\tau, Q^2)$ if Q^2 is reduced from 88.66^2 to 10^2 GeV^2 . We notice the change of a factor 2 (for $\tau \sim 0.1$) which originates in the AF corrections.

We have carried out the analogous analysis for proton-proton scattering and indicate the changes. The γ^* and Z_0 production rate is smaller, of course. The τ dependence, analogous to Fig. 2, increases steadily but there is no plateau characteristic of the scale-independent curves at small τ values. The AF corrections reinforce these tendencies, reaching at the lowest τ values the proton-antiproton predictions since the ss quark combination dominates. Around $\tau \sim 0.1$ the $(su_v + \bar{u}_v s) [(sd_v + \bar{d}_v s)]$ combination reaches 60% [30%].

The above results involve a few uncertainties which we would like to point out.

- (i) The scale parameter Λ is not precisely known. In the above results we have used $\Lambda = 0.5 \text{ GeV}$. Using instead $\Lambda = 0.3 \text{ GeV}$ slightly flattens the τ dependence in Fig. 2 lowering the AF curves by a factor ~ 1.5 in the lower τ region and lifting them by the same amount in the region of maximal τ values.
- (ii) The input momentum distributions have been chosen at $Q^2 = 1.8 \text{ GeV}^2$ in the deep inelastic region. This choice fixes the normalization of the Drell-Yan cross-section in terms of the deep inelastic data.

- (iii) The input s quark (and gluon) distributions are not well-known. The counting rule powers in the different parametrizations differ significantly. Choosing a higher (lower) counting rule power results in a flatter (steeper) τ dependence of the AF corrected curves. In going from $n = 7$ to $n = 10$ considerable changes appear at small τ values (a factor 3-4 for $\tau \lesssim 10^{-3}$) whereas a moderate increase occurs (a factor 1.5 for $\tau \sim 0.1$). We have earlier pointed to this difficulty in proton-proton collisions where a blind Q^2 extrapolation will lead to considerable errors⁹⁾.
- (iv) The ambiguity following from mass factorization permits a redefinition of the scale-dependent parton distributions. The non-leading corrections of the DY process can by this freedom be defined to be small, but those of DI scattering then obviously increase. In the parametrizations used above the choice was fixed via $xu(x, Q^2) \equiv vW_2$ with all non-log terms defined to be absent.
- (v) The parametrizations for the scale-dependent parton distributions are limited to the leading-log approximation with the next-to-leading corrections in DI scattering being ignored.

ACKNOWLEDGEMENTS

One of us (B.H.) thanks J. Ellis, J.-M. Gaillard and J. Strauss for discussion and remarks.

REFERENCES

- 1) B. Humpert and W.L. van Neerven - Phys.Letters 85B, 293 (1979) ; CERN Preprints TH. 2785 (1979) and TH. 2805 (1980).
- 2) J. Kubar-André and F.E. Paige - Phys.Rev. D19, 221 (1978) ;
J. Abad and B. Humpert - Phys.Letters 80B, 286 (1979) ;
G. Altarelli, R.K. Ellis and G. Martinelli - Nuclear Phys. B157, 461 (1979).
- 3) R.F. Peierls, T.L. Trueman and L.L. Wang - Phys.Rev. D16, 1397 (1977) ;
C. Quigg - Rev.Mod.Phys. 49, 297 (1977) ;
I. Hinchliffe and C.H. Llewellyn Smith - Nuclear Phys. B128, 93 (1977) ;
J. Kogut and J. Shigemitsu - Nuclear Phys. B129, 461 (1977).
- 4) G. Curci and M. Greco - CERN Preprint TH. 2786 (1979) ;
G. Parisi - Phys.Letters 90B, 295 (1980).
- 5) M. Abramowitz and I.A. Stegun - "Handbook of Mathematical Functions",
Dover Publications Inc., N.Y., p. 804.
- 6) A.N. Schellekens and W.L. van Neerven - "Calculation of the Quark-Quark
Contribution to the Drell-Yan Process", Nijmegen Preprint, THEF-NYM-79.8,
to be published in Phys.Rev.D ;
J. Kripfganz and A.P. Contogouris - Phys.Letters B84, 473 (1979) ;
A.P. Contogouris and J. Kripfganz - Phys.Rev. D20, 2295 (1979).
- 7) The exchange contributions missing so far can be seen to contribute less.
- 8) J.F. Owens and E. Reya - Phys.Rev. D17, 3003 (1978) ;
A.J. Buras and K.J.F. Gaemers - Nuclear Phys. B132, 249 (1978) ;
For a review, see : B. Humpert - Proceedings of the Workshop on Lepton-Pair
Production in Hadron-Hadron Collisions, Bielefeld (1978).
- 9) B. Humpert - Z.Physik - 2C, 73 (1979).

FIGURE CAPTIONS

- Figure 1 Q^2 dependence of proton-antiproton initiated DY muon pair production at $\sqrt{\tau} = 0.17 = \text{fixed}$. The size of γ^* exchange (leading-log and order α_s and α_s^2 corrections) is indicated, and the position and shape of the Z_0 boson is shown.
- Figure 2a τ dependence of proton-antiproton initiated muon pair production via the Z_0 boson. $B(\tau, M_{Z_0}^2)$ exhibits the convolution of the parton distributions in the scaling parton model (Curve 1) and assuming QCD corrections (Curves 2-4). The weak charges are included.
- Figure 2b τ dependence of proton-antiproton initiated muon pair production via γ^* exchange at $Q^2 = 100 \text{ GeV}^2$. $A(\tau, Q^2)$ exhibits the convolution of the parton distributions in the scaling parton model (Curve 1) and assuming QCD corrections (Curves 2-4). The quark charges are included.
- Figure 3a Percentage of the convoluted parton distributions at the Z_0 boson.
- Figure 3b Percentage of the convoluted γ^* parton distributions at $Q^2 = 100 \text{ GeV}^2$.
- Figure 4 Comparison of the convoluted parton distributions for Z_0 and γ^* exchange at $Q^2 = 88.66^2$ and 10^2 GeV^2 .

FIG. 1

FIG. 2

FIG. 3

FIG. 4