Ref.TH.2784-CERN

ARBITRARY PARTON CROSS-SECTIONS - UNIQUE ASYMPTOTIC FREEDOM PREDICTIONS

B. Humpert and W.L. van Neerven

CERN - Geneva

A B S T R A C T

In the framework of asymptotic free (AF) field theories (mainly ϕ^3_6) we expose by simple examples the arbitrariness of perturbative parton cross-sections due to the ultra-violet (UV), mass (M) and the infra-red (IR) divergences and their regularization. Establishing a one-to-one correspondence between the renormalization group (RG) approach and the Feynman diagram calculations we gain insight into the importance of M factorization to arrive at unique AF predictions. We give details of the parton cross-sections for deep inelastic (DI) scattering and Drell-Yan (DY) massive lepton pair production.

Lectures given at the 9th International Summer
Institute on Theoretical Physics,
Kaiserslautern, F.R.G.,
13 - 24 August 1979

Ref.TH.2784-CERN
23 November 1979

The theoretical justification of perturbative calculations in the framework of Quantum Chromodynamics¹⁾ triggered in the recent past a plethora of theoretical investigations and exploratory analyses which determined the integrated and differential distributions of the quark-gluon sub-processes in the framework of the parton model²⁾. In a consistent calculation M factorization³⁾ would have to be carried out with all M singularities⁴⁾, originating from the initial partons, being absorbed in the momentum distribution. Once we go beyond a leading-log approximation this last step becomes essential since the quark-gluon cross-sections as they are determined from the Feynman graphs involve a high degree of arbitrariness which in these phenomenological calculations is often ignored. Although the quark-gluon cross-sections are arbitrary the AF predictions⁵⁾ resulting from a correct application of the M factorization program are unique.

With this short write-up we aim to demonstrate the dependence of the parton cross-sections on the regularization procedures to prevent the M , UV and the IR singularities. In its second part we focus on the AF program and expose a general and elegant method to extract from the perturbative parton cross-sections the M factorized functions of the RG approach. In order to keep all unnecessary technicalities apart from the discussion we present our reasoning in the framework of ϕ^3 theories (mainly ϕ_6^3 since it is asymptotically free and has no IR divergences⁶⁾).

In this first part our strategy is as follows: we present the QCD Feynman diagrams, analyse their singularity structure and explain how the appearing divergences are correctly removed by pointing to the crucial assumptions.

Considering DI scattering with the lowest order QCD diagrams exposed in Fig. 1 we distinguish the virtual gluon corrections due to the self-energy and the vertex graphs, the radiative gluon graphs and the initial gluon graphs. The analogous Feynman diagrams contribution to the DY process are shown in Fig. 2.

Feynman diagrams are well known to give rise to UV, M and IR singularities. We analyse their origin by using simple methods which permit immediate recognition of the singularity structure of each graph. Whilst attempting to carry out the loop integration in the virtual gluon terms we notice a logarithmic singularity as $k^\mu \rightarrow \pm\infty$ - the UV divergence. It is removed in two steps: UV regularization and renormalization. UV regularization can be achieved by several different techniques where one of them, dimensional regularization⁷⁾, is particularly attractive for gauge theories. It has the advantage of maintaining the Ward identities, respects unitarity and causality, and it is simple in its practical applications.

We apply it on the self-energy contribution and find

$$1 - Z_2^{-1} \equiv \left. \frac{\partial \Sigma}{\partial p^2} \right|_{p^2=m^2} \stackrel{n \rightarrow 6}{=} \frac{\alpha_s}{6\pi} \left[\frac{2}{n-6} + \ln \frac{m^2}{M_{\text{reg}}^2} + \text{constants} \right] \quad (1)$$

This result was obtained for ϕ_6^3 . There is an $(n-6)$ pole which reflects the UV divergence and it exhibits a logarithmic dependence on the quark mass m . The regulator mass M_{reg} was introduced to maintain a dimensionless action. Once we have technically prevented the singularities we can proceed to the renormalization step. Denoting $\mathcal{L}_0$ as the bare, unrenormalized Lagrangian we introduce the subtraction term $\Delta \mathcal{L}$ leading to additional terms which subtract out the UV divergences. There is a certain freedom in its definition which reflects itself in the fact that in Eq. (1) the pole term alone or additional constant pieces can be removed. The minimal subtraction scheme⁷⁾ chooses the $1/(n-6)$ pole term alone to disappear whereas renormalization at the propagator mass imposes that for $p^2 \rightarrow m^2$ the propagator is given by

$$\frac{Z_2}{p^2 - m^2} \quad \text{imposing} \quad \Sigma^R(p^2 = m^2) = 0 \quad (2)$$

Of course, any other choice would have been equally valid although the latter two alternatives are particularly convenient. We could go through the analogous analysis of the vertex diagram with precisely the same insight. We therefore come to our first conclusion: the perturbatively calculated parton cross-sections depend on the particularly chosen subtraction scheme whilst performing renormalization.

We focus on the M divergence⁴⁾ and analyse for this purpose the Feynman diagrams describing gluon radiation (Fig. 1 (A) and (B)). The partons are given the following masses: the intermediate propagator mass is denoted by m_x^2 , p^2 stands for the initial quark mass and k^2 is the mass of the final gluon which for demonstration purposes was chosen non-vanishing. q^2 is the off-shell photon mass. Straightforward integration reveals (in ϕ_6^3) for the parton structure function the form

$$\hat{T}_2^R = \frac{\alpha_s}{\pi} \left[P(z) \ln \frac{q^2}{z^2 M^2(z)} + h(z) + \frac{1}{6(1-z)} \right] \quad (3)$$

where

$$P(z) = z(1-z) \quad h(z) = \frac{11}{3}(1-z)^2 - 3(1-z) - \frac{1}{2} \quad (4)$$

$$M^2(z) = \frac{z(1-z)p^2 - z k^2 - (1-z)m_x^2}{z(1-z)}$$

For the definitions and conventions we refer to Eqs (15) and (19) later in the text. This result exposes part of the mass dependences of the parton cross-section. If all masses vanish the cross-section diverges logarithmically due to its first term. The third term exhibits a pole at $z = 1$ which also gives rise to a mass divergence manifesting itself, however, in a slightly different form. Whilst doing QCD calculations in the parton model one assumes that if the infinity of the contributing Feynman diagrams is summed up the parton masses may be ignored as compared to q^2 and the only remaining mass is the renormalization point q_0^2 . Using the operator product expansion and renormalization group techniques this picture is indeed analytically verifiable. In our Feynman diagram calculations we see that the parton cross-sections diverge if all masses are assumed to vanish. We thus begin to realize that there are M singularities which technically are prevented by giving the partons on - or off-shell masses in order to arrive at finite mathematical expressions; this is called M regularization. As an alternative one also can use dimensional regularization assuming that all parton masses vanish giving

$$\hat{F}_2^R = \frac{\alpha_s}{\pi} \left[P(z) \left\{ \frac{z}{n-6} + \ln \frac{-q^2}{M_{reg}^2} \cdot \frac{1-z}{z} + \text{constants} \right\} + h(z) + \frac{1}{6(1-z)} \right]. \quad (5)$$

The M divergence explicitly appears as an $(n-6)$ pole which, however, cannot be distinguished from the $(n-6)$ pole of UV origin. The above regularization procedures introduce an arbitrariness in the cross-section since there is a priori no preferred mass assignment apart from reasons of simplicity and elegance. The first expression in Eq. (4) demonstrates clearly this point. What about the $(1-z)$ singularity? It is spurious and only appears since z integration and subsequent Bj limit with $q^2 \rightarrow -\infty$ were interchanged. In order to prevent all difficulties we split the z integration in a "soft" gluon part with $z \approx 1$ (which corresponds to the threshold region) and a "hard" gluon part. In the latter contribution the Bj limit may be determined first, giving a simple form of the parton cross-section without corrective masses. In the "soft" part all masses are kept finite in order to prevent the M singularities and z integration is carried out first; subsequently the scaling limit is imposed.

One immediately might ask how we then can dispose of these M singularities and arrive at unique predictions? The answer to this question comes from M factorization which will be the subject of the second part of this write-up. We come to our second conclusion: perturbative parton cross-sections are M singular being prevented by giving the partons on/off-shell masses or by using dimensional regularization in the totally massless case. These regularization procedures lead to results which suffer from a high degree of arbitrariness.

Whilst analyzing the perturbative QCD diagrams we come across a third type of divergence - the IR singularities⁴⁾. They are absent in ϕ_6^3 but reappear as we lower the dimension of space-time to $n = 4$. ϕ_6^3 is asymptotically free but not ϕ_4^3 which is IR singular instead. We therefore find it here suitable to keep for demonstration purposes the space-time dimension n open. One might criticize this simplified analysis as academic since QCD involves spin with $n = 4$. ϕ^3 theories with variable n , however, are convenient to demonstrate all the difficulties of QCD in their most simple form. In order to exhibit the origin of the IR singularities we use time ordered perturbation theory⁸⁾ and analyse all graphs with this calculation technique. The negative energy denominators are ignored since they cannot give rise to M and IR divergences. All contributions can be cast in the general form

$$d\sigma = \int d^n p_s \frac{N(\theta_i, \dots)}{[k(1-\cos\theta_1)]^A [k(1-\cos\theta_2)]^B} D(q_0) \quad (6)$$

k stands for the loop-momentum and θ_i is the angle between the loop-momentum and the initial or final state partons. All factors which contain a dependence on the photon energy q_0 are included in $D(q_0, \dots)$ which in specific cases involves a δ -function due to energy conservation. The remaining two denominators result from the propagator

$$\frac{1}{E_{\text{ext}} - E_{\text{int}} + i\epsilon} \Rightarrow \frac{1}{p - k - |\vec{p} - \vec{k}|} \xrightarrow{k \rightarrow 0} \frac{1}{k(1 - \cos\theta)} \quad (7)$$

$N(\theta_i, \dots)$ in the numerator is due to spin and the phase space factor, generalized to n dimensions, reads

$$d^n p_s \equiv |\vec{k}|^{n-3} dk (1 - \cos^2\theta_i)^{\frac{n-4}{2}} d\cos\theta_i \quad (8)$$

A simple calculation permits us to establish for each diagram the powers α and β , in Table 1 we give an example, and to decide about the $(1/k)$ divergence which gives rise to the IR singularity. In a completely analogous manner we then also determine the angular singularity which is at the origin of the M singular behaviour of the Feynman diagrams. Since we know where and how IR singularities occur we wonder how they are removed. We first are concerned with their regularization.

The IR singularities can be regularized by different techniques. Either we introduce an infinitesimal gluon regulator mass λ leaving the external quark masses on-shell^{17),18)} or we assume that all fermion masses slightly differ. This latter possibility was earlier referred to as "off-shell" regularization⁹⁾ leading to (intuitively motivated) confusion and incorrect results. For a clarification see Ref. 11),12). We also could have used dimensional regularization to prevent this divergence type¹⁰⁾. IR singularities cannot be argued away but may only be removed by their proper cancellation between different contributions (of the same order) to the cross-section⁴⁾. In many phenomenological calculations these cancellations were not properly worked out and this divergence was prevented by "natural" cut-offs. Predictions based on such manipulations bear a high degree of arbitrariness; it is to our mind doubtful whether they are even of qualitative value as their proponents claim. Going back to the cancellations we give in the following an example how they take place. We consider the Feynman graphs for the DI cross-section as shown in Fig. 3 and specialize for demonstration purposes to Figs 3b₂, 3b₃. The contributions of the two graphs read

$$d\sigma_1 = (2\pi i)^2 \int d^n p_5 \frac{1}{p_2^2 - m^2 + i\epsilon} \delta^+(p_5^2) \delta^+(p_3^2 - m^2) \frac{1}{(p_3 + p_5)^2 - m^2 + i\epsilon} \quad (9)$$

$$d\sigma_2 = (-2\pi i) \int d^n p_5 \frac{1}{p_2^2 - m^2 + i\epsilon} \frac{1}{p_5^2 + i\epsilon} \frac{1}{p_3^2 - m^2 + i\epsilon} \delta^+((p_3 + p_5)^2 - m^2) \quad (10)$$

Since the singular behaviour of the integrands in Eqs (9) and (10) originates only from the δ -function part of the propagators we use

$$\frac{1}{p^2 - m^2 + i\epsilon} = -i\pi \delta(p^2 - m^2) + P \frac{1}{p^2 - m^2} = -2\pi i \delta^+(p^2 - m^2) + P \frac{1}{p^2 - m^2} \quad (11)$$

The second equality in Eq. (11) is only possible since the positive and negative frequency parts contribute equally in the $d^n p_5$ integration. Eq. (10) is then changed to

$$d\sigma_2 = (2\pi i)^2 \int d^n p_5 \frac{1}{p_5^2 - m^2 + i\epsilon} \left\{ \frac{\delta^+(p_5^2)}{p_5^2 - m^2} + \frac{\delta^+(p_3^2 - m^2)}{p_5^2} \right\} \delta^+((p_3 + p_5)^2 - m^2). \quad (12)$$

The omitted pieces do not contribute to the singular behaviour since they emerge from the principal value parts which account for the off-shell contribution. We drop all masses and take the δ constraint into account which permits us to write

$$d\sigma_1 + d\sigma_2 = (2\pi i)^2 \int d^n p_5 \frac{1}{\delta p_3 p_5} \left[\frac{\delta(p_{30} - |\vec{p}_3|) \delta(p_{50} - |\vec{p}_5|)}{|\vec{p}_3| |\vec{p}_5|} \right. \\ \left. \frac{\delta(p_{30} + p_{50} - |\vec{p}_3 + \vec{p}_5|) \delta(p_{50} - |\vec{p}_5|)}{|\vec{p}_3 + \vec{p}_5| |\vec{p}_5|} - \frac{\delta(p_{30} + p_{50} - |\vec{p}_3 + \vec{p}_5|) \delta(p_{30} - |\vec{p}_3|)}{|\vec{p}_3 + \vec{p}_5| |\vec{p}_3|} \right] \frac{1}{p_5^2 - m^2 + i\epsilon} \quad (13)$$

If the angle θ between $\vec{p}_3$ and $\vec{p}_5$ goes to zero Eq. (13) can be simplified to

$$d\sigma_1 + d\sigma_2 = (2\pi i)^2 \int d^n p_5 \frac{\delta(p_{50} - |\vec{p}_5|) \delta(p_{30} - |\vec{p}_3|)}{\delta p_3 p_5} \left[\frac{1}{|\vec{p}_3| |\vec{p}_5|} - \frac{1}{|\vec{p}_5| (|\vec{p}_3| + |\vec{p}_5|)} - \frac{1}{|\vec{p}_3| (|\vec{p}_3| + |\vec{p}_5|)} \right] \frac{1}{p_5^2 - m^2 + i\epsilon} \quad (14)$$

We observe that the IR singularity is cancelled between the first and second term ($|\vec{p}_5| \rightarrow 0$) whereas we need all three terms to cancel the M singularity connected with the final quark lines 3,4. We come to our third conclusion: All IR divergences of the Feynman diagrams, contributing to a given order in perturbative QCD, cancel. The form of the remaining finite cross-section depends on how the IR singularities were regularized. If the cancellation is not properly worked out the resulting parton cross-sections strongly depend on the regularization procedure.

So far we have not mentioned that any mass assignment to prevent the M singularities is subject to consistency constraints. This insight emerges from considering the lowest order perturbative diagrams contributing to the virtual

Compton amplitude $\gamma^* q \rightarrow \gamma^* q$. Its discontinuity gives the parton cross-sections contributing to the lowest order QCD corrections in DI scattering such as $\gamma^* q \rightarrow qg$. We consider a set of illustrative graphs shown in Fig. 3. The virtual Compton graph is not M divergent in the intermediate quark lines 234 whereas the corresponding DI graphs are M divergent. All M singularity related to the lines 234 therefore must cancel in their sum. This can only be achieved if the M singularities are regularized in a consistent way. Unitarity imposes that the regulator masses m_2, m_3, m_4 , which slightly may differ, must be chosen as indicated in Figs 3. It clearly implies that the final state fermion mass is not necessarily the same in all diagrams. The mass assignment was fixed in the M finite graphs and then is consistently carried over to the cut-graphs. Explicit evaluation of all DI diagrams shows that in their entire sum all final state M singularities indeed cancel whereas those of the initial parton lines persist.

In this example of DI scattering we immediately understand and accept the need of this constraint, but its extension to other processes was not so obvious. The DY sub-processes as $q\bar{q} \rightarrow \gamma^* g$ have of course no analogue to the virtual Compton amplitude in DI scattering and still all final state M singularities are absent or cancel. The remaining initial state M singularities of this process are, however, absorbed in the scale dependent momentum distributions; they therefore are replaced in the M factorization program by the analogous ones of the DI structure function. It therefore is of utmost importance to realize that the M singularities in both type of processes must be regularized in precisely the same way. The essential point is to start from one and the same M finite Feynman graph - the vacuum bubble - and to derive the corresponding DI and DY diagrams through appropriate cuttings (in the sense of Cutkosky)^{11),4)}. We give a simple example in Fig. 4. The mass assignment is chosen in the vacuum bubble and then consistently carried over to the two parton processes. We come to our fourth conclusion: the (auxiliary) mass assignment must be chosen on an M finite amplitude which via cuttings leads to the parton processes under consideration.

We have determined the DI structure function and the DY cross-section on the parton level and give in the following some of the intermediate and final results for ϕ_6^3 theory¹²⁾. We have selected the on-shell and the completely massless cases which satisfy the above constraints and used for the latter mass assignment n dimensional regularization. The DI structure function is

$$\frac{\nu W_2}{x} \equiv F_2(x, q^2) = \int_x^1 \frac{dz}{z} f\left(\frac{x}{z}\right) \hat{F}_2(z, q^2) \quad (15)$$

with the scaling variables defined as

$$x = \frac{-q^2}{2Pq}, \quad z = \frac{-q^2}{2pq}, \quad p = \frac{x}{z} P \quad (16)$$

The quark part of the parton structure function is parametrized as

$$\hat{F}_2(z, q^2) = \delta(1-z) + \alpha_s \left\{ P(z) \ln \frac{q^2}{p^2} + f^{DI}(z) \right\} \quad (17)$$

with $P(z)$ and $f^{DI}(z)$ given in Table 2. The DY cross-section reads

$$s Q^2 \frac{d\sigma}{dQ^2} = \frac{4\pi\alpha^2}{3} \int dz \int \frac{dx_1}{x_1} \frac{dx_2}{x_2} \left[f(x_1) \bar{f}(x_2) + \bar{f}(x_1) f(x_2) \right] \hat{W}(z, Q^2) \quad (18)$$

where $z = \tau/x_1 x_2$, $\tau = Q^2/s$ with the analogous parton cross-section parametrized as

$$\hat{W}(z, Q^2) = \delta(1-z) + \alpha_s \left\{ \sum_{j=1}^2 P(z) \ln \frac{Q^2}{p_j^2} + f^{DY}(z) \right\} \quad (19)$$

and $P(z)$ and $f^{DY}(z)$ listed in Table 2.

We come to the second part of our presentation which concerns the connection between the RG approach and the lowest order perturbative calculations¹³⁾. In order to expose clearly the essential reasoning we ignore all mixing complications. Our arguments can, of course, be extended to the singlet case where mixing may not be ignored anymore. The AF approach⁵⁾ to the DI structure functions is based on

- the operator product expansion¹⁴⁾
- the solution to the RG equation for the Wilson coefficients¹⁵⁾
- and analyticity establishing the connection¹⁶⁾

$$M_i^{(n)} = f^{(n)} A^{(n)} c_i^{(n)} \quad (20)$$

$M_i^{(n)}$, $f^{(n)}$, $A^{(n)}$, $c_i^{(n)}$ stand for the moments of the structure function i , the parton momentum distribution, the reduced operator matrix element and the Wilson coefficient function. All dependence on the parton momentum p^2 is contained in $A^{(n)}$, whereas $c_i^{(n)}$ only depends on q^2 . The renormalization point q_0^2 appears in both.

Instead we could have carried out our analysis in the parton model using QCD perturbation theory with

$$M_i^{(n)} = f^{(n)} d\sigma_i^{(n)} = f^{(n)} \Gamma^{(n)} d\bar{\sigma}_i^{(n)} \quad (21)$$

The Eqs (20) and (21) are distinct in that all parton momentum dependence on the one hand and the dependence on the off-shell photon mass are separated by the operator product expansion whereas this is not the case in a perturbative calculation. $d\sigma_i^{(n)}$ is a function of p^2, q^2 (and the renormalization point q_0^2 in higher orders). We can achieve their separation through M factorization as indicated by the second equality in Eq. (21). All parton mass dependence is absorbed in $\Gamma^{(n)}(p^2, q_0^2)$ whereas $d\bar{\sigma}_i^{(n)}(q^2, q_0^2)$ depends only on the dynamical variable q^2 ; the renormalization point q_0^2 appears in both even in a lowest order calculation. In phenomenological applications the product $f^{(n)}\Gamma^{(n)} \equiv u^{(n)}$ is treated as one function and considered as the input momentum distribution which is fixed by the experiment at a chosen value q_0^2 . Since we calculate perturbatively

$$d\sigma_i = \text{const.} [\bar{s}(1-z) + g^2 \Lambda] \quad (22)$$

the explicit form of $\Lambda(p^2, q^2)$ emerges from evaluation of all QCD Feynman diagrams to order g^2 as indicated above. It thus is possible to specify $d\sigma_i^{(n)}$ and to carry out M factorization.

The arbitrariness mentioned earlier due to the counter term $\Delta\mathcal{L}$ in the Lagrangian in order to achieve renormalization permits the formulation of the Callan-Symanzik (differential) equation¹⁵⁾ for $A^{(n)}$ (in the parton model) and for $e_i^{(n)}$, with the solution for the latter quantity given by

$$e_i^{(n)}(q) = c_i^{(n)}(\bar{g}) \cdot \exp \left[\int_{\bar{g}}^{\bar{g}} \frac{\gamma^{(n)}(x)}{\beta(x)} dx \right] \quad (23)$$

$\bar{g} \equiv \bar{g}(Q^2)$ is the running coupling constant and $\gamma^{(n)}(g), \beta(g)$ stand for the anomalous dimensions and the β function characterizing the field theory under consideration. By this calculation technique one is able to determine the influence of an infinity of Feynman diagrams as indicated by Eq. (20). We seek connection between these two calculation methods in lowest order of the strong coupling constant g using the general ansatz

$$\begin{aligned}
 \beta(g) &= [g^3 b_1 + g^5 b_2 + \dots] \\
 \gamma^{(n)}(g) &= [g^2 d_1 + g^4 d_2 + \dots]^{(n)} \\
 A^{(n)}(g) &= [1 + g^2 \{a_{11} \ln \frac{p^2}{\mu^2} + a_{10}\} + \dots]^{(n)} \\
 c_i^{(n)}(\bar{g}) &= [1 + c_i \bar{g}^2 + \dots]^{(n)}
 \end{aligned} \tag{24}$$

The running coupling constant is given by

$$\bar{g}^2(t) = \frac{g^2}{1 - 2 b_1 g^2 t}, \quad t = \frac{1}{2} \ln \frac{p^2}{p_0^2} \tag{25}$$

where $b_1 < 0$ is the necessary condition for asymptotically free field theories. We insert these expansions in Eqs (23) and (20) and find

$$\begin{aligned}
 M_i^{(n)} &= f^{(n)} [1 + g^2 \Lambda + \dots]^{(n)} \\
 \Lambda^{(n)} &= [\frac{1}{2} d_1 \ln \frac{p^2}{p_1^2} + c_1 + a_{10}]^{(n)}
 \end{aligned} \tag{26}$$

with

By this simple expansion in the fixed coupling constant g we have established the connection between the renormalization group result and a lowest order perturbative calculation. One-to-one identification therefore allow us to determine the expansion coefficients in Eq. (24) and in this way to specify the contribution of an infinity of Feynman diagrams via the RG equation in Eq. (20). We realize that all mass divergences of the perturbative cross-sections have been absorbed in $A^{(n)}$ (or $\Gamma^{(n)}$ since we ignore all mixing complications) and finally were shifted in the phenomenological input momentum distributions.

But what about the arbitrariness of the perturbative parton cross-section mentioned earlier due to UV renormalization and the M and IR divergences ? The RG equations are just a consequence of the subtraction ambiguity whilst performing renormalization. Since the moments of the virtual Compton amplitude $M_i^{(n)}$ satisfy a RG equation and similarly $A^{(n)}$ and $c_i^{(n)}$ all three depend on the subtraction choice. However, we have to distinguish between coupling constant renormalization and operator renormalization. All three quantities are calculated with the same renormalized coupling constant which also fixes β .

The operator renormalization thus concerns only $A^{(n)}$ and $e_i^{(n)}$ in Eq. (20). In practical calculations one determines perturbatively the virtual Compton amplitude Eq. (22) with a certain mass assignment to prevent the M singularities. Whilst calculating the right-hand side of Eq. (20) we first determine the reduced operator matrix element $A^{(n)}$ in Eq. (24) with a certain operator renormalization prescription. Since $M_i^{(n)}$ and $A^{(n)}$ are now fixed we have automatically determined $e_i^{(n)}$ in Eq. (23). This quantity in itself is a product of two terms, the coefficient function $C_i^{(n)}$ and the exponent. The latter is calculated via the anomalous dimensions of the operators leading us to the conclusion that $C_i^{(n)}$ is fixed. Changing the renormalization of the operator matrix element $A^{(n)}$ changes $C_i^{(n)}$. The renormalization arbitrariness of $A^{(n)}$ reflects itself in the mass factorization since we have identified $A^{(n)} \equiv \Gamma^{(n)}$ and $e_i^{(n)} = d\bar{\sigma}_i^{(n)}$, and influences the non-leading terms in $A^{(n)}$ and $C_i^{(n)}$ [see Eq. (24)]. The above procedure applies for the structure function $F_2(x, q^2) = u(x, q^2)$, being conventionally defined as the q^2 dependent momentum distribution with $C_2^{(n)}(\bar{g}) \equiv 1$ in Eq. (23), which imposes that there are no constant terms but only logs. Of course, any other choice could have been made, but this one is experimentally best known.

Using the fact that the moments of all other structure functions $\hat{T}_i$ can be written as

$$M_i^{(n)} = u^{(n)}(q^2) [1 + \bar{g}^2 \Delta_i^{DI} + \dots]^{(n)} \quad (i \neq 2) \quad (27)$$

which is easily obtained with Eq. (20) using $C_i(\bar{g})$ in Eq. (24), we realize that all other structure functions are parametrized by $u(x, q^2)$ plus a correction term which is responsible for all next-to-leading corrections. In this way all M singularities of $d\sigma_i$ are subtracted (or divided out) by those of $d\sigma_2$ since the factorization matrix $\Gamma^{(n)}$ is universal and in particular is not i -dependent. The above form is particularly suitable for phenomenological applications¹⁷⁾. $u(x, q^2)$ is determined via Mellin inversion from $M_2^{(n)}$ with simple parametrizations being proposed for its practical use¹⁸⁾. $\Delta_i^{DI} \equiv f_i^{DI} - f_2^{DI}$ is available from the lowest order QCD calculations. This latter expression is independent of any regularization procedure !

We briefly indicate the completely analogous procedure for the DY process. The moments of its cross-section read

$$d\sigma^{(n)} = \text{const.} \cdot f^{(n)} \bar{f}^{(n)} \hat{W}^{(n)} \quad (28)$$

with the lowest order QCD result given in Eq. (19). Its mass singularities and part of its Q^2 dependence is absorbed in the scale dependent momentum distributions

$$d\sigma^{(n)} = \text{const.} \cdot u^{(n)}(q^2) \bar{u}^{(n)}(q^2) [1 + \bar{g}^2 \Delta^{DY} + \dots]^{(n)} \quad (29)$$

in complete analogy to Eq. (27) in DI scattering. The general form of the parton cross-section then is

$$\hat{W}^{(n)} = [1 + g^2 \{ \Lambda + \bar{\Lambda} + \Delta^{DY} \} + \dots]^{(n)} \quad (30)$$

The Λ 's stand for the order g^2 corrections in DI scattering [Eq. (22)]. Comparison between the Eqs (30) and (19) determines $\Delta^{DY} = f^{DY} - 2f_2^{DI}$ where $a_{10} = f_2^{DI}$ was identified earlier. Δ^{DY} is expected to be independent of any regularization procedure; this could indeed be verified for on/off-shell mass assignment and using n dimensional regularization in the case of all vanishing parton masses (see Table 2).

In this short note we exposed the main assumptions in calculating parton cross-sections in the framework of perturbative QCD. We pointed to the divergence problems and the arbitrariness emerging from technically preventing the UV, M and IR divergences. Subsequently the DI structure functions and the DY cross-sections on the parton level are determined in order g^2 . We use on-shell mass assignment and n dimensional regularization with all partons massless. Detailed results are given for ϕ_6^3 theory.

In the second part of this lecture we exposed the connection between the RG approach and perturbative evaluation. Establishing a one-to-one correspondence between these two calculation methods we demonstrate that regularization independent predictions still can be obtained. We outline the arguments for DI scattering and briefly indicate their extension to the DY process.

ACKNOWLEDGEMENTS

One of us (B. H.) thanks Profs W. Rühl and H.J.W. Müller-Kirsten for their invitation and kind hospitality during the NATO Summer Institute.

Table 1

Analysis of the DI graphs in Fig. 3 on their M- and IR-singular behaviour.

qY* → qg Graph	Fig.	(α,β)	ϕ ₆ ³ (n = 6)	Q.C.D. (n = 4)	
			M singul.	M singul.	IR singul.
$\hat{t}$ - graph	3a ₂	(2,0)	yes	yes	yes
$\hat{st}$ -, Vx - graph	3b ₂ , b ₃	(1,1)	no	yes	yes
$\hat{st}$ -, Vx - graph	3c ₂ , c ₃	(1,1)	no	yes	yes
$\hat{s}$ -, S ₂ - graph	3d ₂ , d ₃ , d ₄	(2,0)	yes	yes	yes
S ₁ - graph	3e ₂	(2,0)	yes	yes	yes
S ₁ - graph	3f ₂	(2,0)	yes	yes	yes

Table 2
Formulas for the parton structure function in DI scattering and for
the DY parton cross-section calculated in ϕ_6^3 field theory.

	$V_{2F}^x(q^2)$	$\Sigma_1^2(m^2)$	$\Sigma_2^2(m^2)$
on-shell	$\frac{2\alpha_s}{6\pi} \left[\frac{2}{6-n} \frac{M_{reg}^2}{-q} - \gamma + \ell n 4\pi + \frac{11}{3} \right]$	$\frac{\alpha_s}{6\pi} \left[\frac{2}{6-n} \frac{M_{reg}^2}{m} - \gamma + \ell n 4\pi + \frac{5}{3} \right]$	$\frac{\alpha_s}{6\pi} \left[\frac{2}{6-n} \frac{M_{reg}^2}{m} - \gamma + \ell n 4\pi + \frac{5}{3} \right]$
massless		0	0

	$V_{2F}^v(q^2)$	$\hat{\Sigma}_2^{R-soft}$
on-shell	$\frac{2\alpha_s}{6\pi} \left[\frac{m^2}{\ell n \frac{m^2}{-q} + 2} \right]$	$\delta(1-z) \frac{\alpha_s}{6\pi} \left[\frac{2}{6-n} \frac{eq^2}{\ell n \frac{eq^2}{m} - 1} \right]$
massless	$\frac{2\alpha_s}{6\pi} \left[\frac{2}{6-n} \frac{M_{reg}^2}{-q} - \gamma + \ell n 4\pi + \frac{11}{3} \right]$ (5)	$\delta(1-z) \frac{\alpha_s}{6\pi} \left[\frac{2}{6-n} \frac{eq^2}{\ell n \frac{eq^2}{m} - 1} - \gamma + \ell n 4\pi + \frac{8}{3} \right]$ (6)

Table 2 (cont.)

	$\alpha_{S^P q}$	$\alpha_{S^f q}$ (on-shell)	$\alpha_{S^f q}$ (massless)	$\alpha_{S^{\bullet\Delta} q}$
$\hat{\mathcal{F}}_2$	$\alpha_S \left[\frac{1}{\pi} \left[z(1-z) - \frac{1}{6} \delta(1-z) \right] \right]$	$\alpha_S \left[\frac{1}{\pi} \left[-z(1-z) \ell n z (1-z) + \frac{11}{3} (1-z)^2 - 3(1-z) - \frac{1}{2} + \frac{1}{6(1-z)} + \delta(1-z) \right] \left\{ \frac{1}{6} \ell n \epsilon + \frac{1}{2} \right\} \right]$	$\alpha_S \left[\frac{1}{\pi} \left[z(1-z) - \frac{1}{6} \delta(1-z) \right] \left\{ \frac{2}{n-6} + \gamma - \ell n 4\pi \right\} + z(1-z) \ell n \frac{11}{z} + \frac{11}{3} (1-z)^2 - 3(1-z) - \frac{1}{2} + \frac{1}{6(1-z)} + \delta(1-z) \right] \left\{ \frac{1}{6} \ell n \epsilon + \frac{7}{9} \right\} \right]$	0
$\hat{W}$	$\alpha_S \left\{ \frac{1}{\pi} \left[z(1-z) - \frac{1}{6} \delta(1-z) \right] \right\}$	$\alpha_S \left[\frac{1}{\pi} \left[-z(1-z) \ell n z + (1-z)(2-3z) + \frac{1}{3} \delta(1-z) \right] \right]$	$\alpha_S \left[\frac{1}{\pi} \left[z(1-z) - \frac{1}{6} \delta(1-z) \right] \left\{ \frac{2}{n-6} + \gamma - \ell n 4\pi \right\} + z(1-z) \ell n \frac{(1-z)^2}{z} + (1-z)(2-3z) + \frac{11}{18} + \frac{1}{18} \delta(1-z) \right] \right]$	$\alpha_S \left[\frac{1}{\pi} \left[z(1-z) \ell n (1-z) - \frac{2}{3} (1-z)^2 + 2(1-z) + \frac{1}{2} - \frac{1}{6(1-z)} - \delta(1-z) \right] \left\{ \frac{1}{6} \ell n + \frac{1}{6} \right\} \right]$

REFERENCES

- 1) H. Fritzsch, M. Gell-Mann and H. Leutwyler, Phys. Letters 47B (1973) 365;
H. Fritzsch, CALT-68-524 (1975), Nordita Lectures (1975);
H.D. Politzer, Phys. Rep. 14C (1974) 129;
W. Marciano and H. Pagels, Phys. Rep. 36 (1978) 137;
S. Weinberg, Rev. Mod. Phys. 46 (1974) 255.
- 2) For reviews see:
Y.L. Dokshitzer, D.I. D'yakonov and S.I. Troyan, Lectures at Leningrad Winter School (1978); SLAC-TRANS-183 (1978);
Proc. of the EPS Internat. Conf. on High Energy Physics, Geneva (1979);
Proc. of the 19th Internat. Conf. on High Energy Physics, Tokyo (1978);
H. Fritzsch, 18. Internat. Universitätswochen für Kernphysik, Schladming, (1978);
C.H. Llewellyn Smith, 18. Internat. Universitätswochen für Kernphysik, Schladming (1978);
Cargèse Summer Institute (1977);
J. Ellis. SLAC-PUB-2121 (1978);
B. Humpert, Helv. Phys. Acta 51 (1978) 542, and Ref. 18) below.
- 3) H.D. Politzer, Nucl. Phys. B129 (1977) 301;
D. Amati, R. Petronzio and G. Veneziano, Nucl. Phys. B140 (1978) 54 and Nucl. Phys. B146 (1978) 29;
R.K. Ellis, H. Georgi, M. Machacek, H.D. Politzer and G.G. Ross, Phys. Letters 78B (1978) 281 and Nucl. Phys. B152 (1979) 285;
S. Libby and G. Sterman, Phys. Letters 78B (1978) 618;
C.T. Sachrajda, Phys. Letters 73B (1978) 185.
- 4) T. Kinoshita, J. Math. Phys. 3 (1962) 650;
T.D. Lee and M. Nauenberg, Phys. Rev. 133 (1964) B1549;
N. Nakanishi, Progr. Theor. Phys. 19 (1958) 159.
- 5) M. Glück and E. Reya, Phys. Letters 69B (1977) 77;
W.A. Bardeen, A.J. Buras, D.W. Duke and T. Muta, Phys. Rev. D18 (1978) 3998;
E.G. Floratos, D.A. Ross and C.T. Sachrajda, Nucl. Phys. B129 (1977) 66, Nucl. Phys. 139 (1978) 545, and Nucl. Phys. (1979) 493;
J. Ellis, Lectures at the Les Houches Summer School (1976);
A. Peterman, CERN-TH.2581 (1978), to appear in Physics Reports;
M.K. Gaillard, XII Rencontre de Moriond (1977), p. 485; and conference reports cited in Ref. 2).
- 6) G.C. Marquez, Phys. Rev. D9 (1974) 386;
P.M. Fishbane, C.S. Lam and T.M. Yan, Cornell University (1979) (and references cited therein).
- 7) G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189;
C.G. Bollini and J.J. Giambiagi, Nuovo Cimento 12B (1972) 20;
G. 't Hooft and M. Veltman, CERN-Report 73-9, "Diagrammar" (1977).
- 8) H. Heitler, "The Quantum Theory of Radiation", Oxford Clarendon Press (1959), International Series of Monographs on Physics;
J.J. Sakurai, "Advanced Quantum Mechanics", Reading, MA, Addison Wesley (1967), (Addison Wesley Series in Advanced Physics);
S.J. Brodsky, R. Roskies and R. Suaya, Phys. Rev. D8 (1973) 4574.

- 9) B. Humpert and W.L. van Neerven, Phys. Letters 84B (1979) 327, Phys. Letters 85B (1979) 471;
 G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. B143 (1978) 521;
 K. Harada, T. Kaneko and N. Sakai, Nucl. Phys. B155 (1979) 169 and details in CERN preprint TH.2619 Erratum (1979);
 J. Kripfganz, "Comments on Regularization Prescription Dependence of QCD Corrections to the Drell-Yan Formula", Mc Gill University (1979).
- 10) W.J. Marciano, Phys. Rev. D12 (1975) 3861 and references therein;
 G. Altarelli, R.K. Ellis and G. Martinelli, Nucl. Phys. B157 (1979) 461;
 B. Humpert and W.L. van Neerven, Ref. 9).
- 11) B. Humpert and W.L. van Neerven, Ref.TH.2838-CERN (1979), to appear in Phys. Letters.
- 12) B. Humpert and W.L. van Neerven, "IR and M regularization in AF field theories I: ϕ_6^3 ", preprint CERN-TH.2785 (1979).
- 13) We assume that the reader is familiar with the basics of these approaches.
- 14) Y. Frishman, Phys. Rep. 13C (1974) 1;
 J. Ellis and R.L. Jaffe, U.C. Santa Cruz Summer School Lectures SLAC-PUB-1253 (1973).
- 15) C.G. Callan, Phys. Rev. D2 (1970) 1541;
 K. Symanzik, Comm. Math. Phys. 18 (1970) 227;
 G. 't Hooft, Nucl. Phys. B61 (1973) 455;
 M.J. Holwerda, W.L. van Neerven and R.P. Van Royen, Nucl. Phys. B75 (1974) 303;
 R.G. Crewther, Cargèse Summer Institute Lectures, preprint CERN-TH.2119 (1976);
 S. Coleman, "Dilatations", Lectures given at the 1971 International Summer School of Physics "Ettore Majorana", (Ed. Compositori, Bologna 1973).
- 16) N. Christ, B. Hasslacher and A.H. Mueller, Phys. Rev. D6 (1972) 3543.
- 17) J. Abad, B. Humpert and W.L. van Neerven, Phys. Letters 83B (1979) 371;
 B. Humpert and W.L. van Neerven, Phys. Lett. 85B (1979) 293;
 B. Humpert, Zeitschr. f. Physik 2C (1979) 73;
 A.N. Schellekens and W.L. van Neerven, Univ. of Nijmegen, THEF-NYM-79.8 (1979);
 J. Kubar-André and F.E. Paige, Phys. Rev. D19 (1978) 221.
- 18) B. Humpert, Workshop on Lepton-Pair Production in Hadron-Hadron Collisions, Bielefeld, Germany (1978) preprint CERN-TH.2639 (and references therein).

FIGURES CAPTIONS

- Fig. 1 : Lowest order QCD diagrams contributing to DI scattering.
- Fig. 2 : Lowest order QCD diagrams contributing to the DY process.
- Fig. 3 : Lowest order QCD diagrams contributing to the virtual Compton amplitude (left). Their discontinuities (right) provide the graphs contributing to the DI cross-sections.
- Fig. 4 : M- and IR- singularity free vacuum bubble with the appropriate cuts generating the set of corresponding QCD diagrams which contribute in the DI, DY and the e^+e^- processes.

Fig.1

Fig.2

VIRTUAL COMPTON AMPLITUDE

DEEP INELASTIC CROSS SECTION

Fig. 3

Fig. 4