Ref.TH.2838-CERN

HEAVY BARYON SPECTROSCOPY IN THE QCD BAG MODEL

P. Hasenfratz, R.R. Horgan, J. Kuti and J.M. Richard
CERN -- Geneva

ABSTRACT

The spectroscopy of heavy baryons with naked charm and/or beauty is developed. The potential energy of three heavy colour sources is determined from the QCD motivated bag model and the Schrödinger equation is solved by using the hyperspherical formalism.

Ref.TH.2838-CERN

27 March 1980

THE THREE-QUARK PROBLEM

In our previous paper¹⁾ we developed a QCD motivated bag model^{2,3)} with applications to heavy $Q\bar{Q}$ -systems. It was assumed that a bubble with well-defined surface is created around the heavy quarks. Perturbation theory is applied inside the bag where the chromoelectric permeability ϵ of the vacuum is approximately one. Across the surface ϵ crosses over from one to zero. The outside vacuum phase is pictured as a perfect dielectric and paramagnetic medium.

To calculate the bound state of three heavy quarks we proceed in two steps using the Born-Oppenheimer approximation¹⁾. In the first step we write the QCD Hamiltonian in Coulomb gauge for an external dielectric medium. In addition, there is the well-known bag energy $\Lambda_B^4 V + \sigma A$ where V is the volume of the region around the heavy quarks characterized by a chromoelectric permeability equal to one. The surface area is denoted by A , and the surface tension σ is set to zero in the first calculation. We shall later comment upon its interesting influence on the space-time structure of a baryon.

In the first step we minimize the bag energy for the fixed configuration of three point-like heavy quarks. This procedure determines the surface of the bag, the chromoelectric field configuration inside and the potential energy of the three quarks. This potential energy goes into the non-relativistic Schrödinger equation of the quark variables as the second step of our procedure.

The static energy we calculate is given by

$$H_{3Q}^{st}(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \Lambda_B^4 V + \frac{1}{2} g^2 \int d^3\vec{r} \int d^3\vec{r}' j_0^a(\vec{r}) G^{ab}(\vec{r}, \vec{r}') j_0^b(\vec{r}') \quad (1)$$

where $G^{ab}(\vec{r}, \vec{r}')$ is the Green's function of the confined chromoelectric field. G^{ab} is known analytically for spherical shape and is calculable numerically for general surfaces¹⁾. The integration in Eq. (1) is taken over the volume of the bag. The colour charge distribution of the static quarks is described by the following expression:

$$j_0^a(\vec{r}) = \frac{1}{2} \lambda_1^a \delta(\vec{r}-\vec{r}_1) + \frac{1}{2} \lambda_2^a \delta(\vec{r}-\vec{r}_2) + \frac{1}{2} \lambda_3^a \delta(\vec{r}-\vec{r}_3) \quad (2)$$

The static Hamiltonian of Eq. (1) is sandwiched between the colour singlet 3Q states and the energy is minimized with respect to the bag's shape. The parameters from our description of heavy $Q\bar{Q}$ -spectroscopy are determined as $\Lambda_B = 235$ MeV and $\alpha = 0.38$ ($\alpha = g^2/4\pi$). We do not have any new free parameter when going to the baryon sector.

The evaluation of the matrix elements of H_{3Q}^{st} between colour singlet states can be effectively described in terms of two different Abelian charges as given in Table 1.

We know from our numerical calculation of the heavy $Q\bar{Q}$ -system that the spherical approximation is sufficient for quark-quark separations smaller than Λ_B^{-1} . In this approximation the static energy of the 3Q system is given by

$$\begin{aligned} E_{3Q}^{st} = & \frac{4\pi}{3} R^3 \Lambda_B^4 - \frac{2}{3} \alpha \sum_{i < k} \left\{ \frac{1}{|\vec{r}_i - \vec{r}_k|} + \frac{R}{(R^4 + \tau_i^2 \tau_k^2 - 2R^2 \tau_i \tau_k \cos \theta_{ik})^{1/2}} \right. \\ & \left. - \frac{1}{R} \ln \left[\frac{1}{2} - \frac{\tau_i \tau_k \cos \theta_{ik}}{2R^2} + \frac{1}{2R^2} (R^4 + \tau_i^2 \tau_k^2 - 2R^2 \tau_i \tau_k \cos \theta_{ik})^{1/2} \right] \right\} + \\ & + \frac{2\alpha}{3} \sum_{i=1}^3 \left\{ \frac{R}{R^2 - \tau_i^2} - \frac{1}{R} \ln \left(1 - \frac{\tau_i^2}{R^2} \right) \right\}, \end{aligned} \quad (3)$$

where the quark position vectors $\vec{r}_i$, for $i = 1, 2, 3$, are measured from the centre of the sphere and $\vec{r}_i \cdot \vec{r}_k = r_i r_k \cos \theta_{ik}$. The static energy in Eq. (3) has to be minimized with respect to the radius R and the position of the centre.

The last term in Eq. (3) describes the finite part of the static chromoelectric self energy of heavy quarks inside the sphere. Of course, we expect deformations

of the approximate spherical shape when the interquark separation becomes comparable or larger than Λ_B^{-1} .

The potential energy $V_{3Q}(r)$ of the three-quark system is shown in Fig. 1 after the minimization of Eq. (3) with respect to R . An equilateral triangle configuration is chosen for simplicity whose side r is the interquark distance. Our precise numerical solution which allows for deformations is also shown for comparison. At short distances, for $r \ll \Lambda_B^{-1}$, the Coulomb term dominates the potential energy. At about $r \sim \Lambda_B^{-1}$ there is a rapid crossover into the confinement regime. For the equilateral configuration our potential V_{3Q} is well approximated by

$$V_{3Q}(r) \sim -2 \frac{\alpha}{r} + \sqrt{3} \lambda r, \quad (4)$$

where r is the interquark distance.

For large interquark separations two distinct geometric configurations become degenerate after the energy minimization. They are shown in Fig. 2 and labelled as Y and Δ . The junction of the Y-shape is determined from the minimum of the energy. It is a point in the plane of the three quarks from which the sides of the arbitrarily chosen triangle are seen under equal angles of $2\pi/3$. The tension λ along the legs of the Y-shape is the same as in the vortex tube of the $Q\bar{Q}$ -system. The radius of the leg (vortex tube) is 0.64 fermi asymptotically. The distribution of the flux lines is shown in Fig. 2.

For the equivalent Δ -configuration the energy minimum is the same as for the Y-geometry, though the tension along each side of the Δ becomes a complicated function of the shape of the whole triangle. This means that even for the Δ -shape the interaction cannot be expressed in terms of two-body potentials.

It is interesting to note that the Δ -shape is lifted upwards in energy compared with the Y-shape when surface energy is switched on in the model. The Y-shape becomes the absolute minimum then.

Our preliminary computer calculations confirm the above discussion. For large interquark separation we find a genuine three-body force whose potential energy can always be calculated from the Y-configuration. For arbitrary quark positions the approximate three-quark potential

$$V_{3Q} \sim -\frac{2}{3} \alpha \sum_{i < k} \frac{1}{|\vec{r}_i - \vec{r}_k|} + \lambda \sum_{i=1}^3 d_i, \quad (5)$$

turns out to be close to our numerical results. In Eq. (5) d_i designates the distance of the i^{th} quark from the junction. The location of the junction is determined from the energy minimum.

With the potential energy of the three quarks accurately determined we are faced with the problem of solving the 3-body Schrödinger equation. We choose the method of hyperspherical expansion which is reviewed elsewhere⁴⁾. This formalism has been already applied to the 3-nucleon problem with an efficiency comparable to the use of the Faddeev equations. In atomic physics this method also works, although standard variational methods are more competitive. Our investigation has shown that the hyperspherical expansion is very well suited for the 3-quark problem, in particular so in the case of symmetric spatial configuration of the three quarks. We believe that baryon spectroscopy with our three-quark potential is one of the most appropriate fields to use the hyperspherical formalism.

We briefly outline the spirit of the method and refer the reader to review articles⁴⁾ and to a forthcoming article⁵⁾ for more details and bibliography. The wave function is written as

$$\Psi(\vec{r}_1, \vec{r}_2, \vec{r}_3) = \sum_{[L]} \frac{u_{[L]}(\xi)}{\xi^{5/2}} \psi_{[L]}(\Omega) \quad (6)$$

where the system is described by a single over-all distance

$$\xi \propto \left\{ \sum_{i < j} (\vec{r}_i - \vec{r}_j)^2 \right\}^{1/2}$$

and a set of 5 angles denoted by Ω . $[L]$ consists of a generalization of the usual angular momentum, namely the "grand" orbital $L = 0, 1, 2 \dots$ supplemented by appropriate magnetic quantum numbers. The angular wave functions $\psi_{[L]}(\Omega)$ are the associated orthonormal eigenfunctions called hyperspherical harmonics. The "grand" angular momentum, unlike the *total* angular momentum, is not an exactly conserved quantity. Therefore, the problem consists, in general, of an infinite set of coupled equations.

The miracle here is that our QQQ potential is almost hypercentral, i.e. practically diagonal and scalar in the hyperspherical basis. This implicates that each quantum state is described by a single hyperradial wave function with the equation

$$u''_{[L]}(\xi) - \frac{(L+\frac{3}{2})(L+\frac{5}{2})}{\xi^2} u_{[L]}(\xi) + \mu(E - V_{[L][L]}(\xi)) u_{[L]}(\xi) = 0^{(7)}$$

very similar to the familiar radial Schrödinger equation in the 2-body problem.

In Eq. (7) μ is the reduced mass of the three quarks and

$$V_{[L][L]}(\xi) = \int d\Omega \, \gamma_{[L]}^*(\Omega) V(\vec{r}_1, \vec{r}_2, \vec{r}_3) \gamma_{[L]}(\Omega).$$

The ccc ground state with triple charm, for instance, has mainly grand orbital $L = 0$. The first correction corresponds to a coupling to $L = 4$. It changes the energy of the system by less than 1%. Also, the norm of the $L = 4$ impurity in the wave function is less than 1%.

For ccb, cbb, or cbt configurations the potential is slightly less hyperspherical, since some symmetry of the 3c system is lost. Keeping a single equation still remains a very good approximation to the solution of the problem, much better in any case than for atomic Helium. Details will be given elsewhere⁵⁾.

Our results are given in Table 2 where no spin corrections were applied.

A few comments are in order here:

- i) A hyperspherical potential has almost all the degeneracies of the harmonic oscillator model except the equality of the (L,n) and $(L+2,n-1)$ energies where the second integer denotes the number of nodes. Also, the various spacings are not necessarily equal.
- ii) The location of the ccc ground state is rather sensitive to the choice of the mass m_c of the charmed quark. Empirical charmonium models can actually accommodate values of m_c between 1.2 GeV and 1.8 GeV. Our value of $m_c = 1.35$ GeV was determined in the QCD bag model of $c\bar{c}$ -spectroscopy¹⁾. As shown in a very general framework⁶⁾ and illustrated in our previous calculation¹⁾, the $m_b - m_c$ mass difference is much more constrained by the quarkonium data if one assumes a flavour-independent potential. This implies here that differences like $M(bbb) - M(ccc)$ or $M(bbc) - M(ccb)$ are rather stable with respect to the initial choice of m_c .
- iii) In Ref. 5 we shall extend the present calculation to higher levels, include spin corrections and propose some production mechanisms and characteristic decay modes for the identification of these heavy baryons.

It did not escape our attention that it is rather tempting to apply the above potential picture to the spectroscopy of light baryons. The serious problem with regard to this attempt is the relativistic nature of quarks and the question mark on the validity of our Born-Oppenheimer approximation in this particular case. We shall return to the problem and to the discussion of QQq and $Q\bar{q}$ states in our forthcoming publication⁵⁾.

Acknowledgements

We are grateful to M. Fabre de la Ripelle for very useful correspondence on the three-body problem. We also acknowledge the kind hospitality of the Theory Division at CERN and valuable discussions with our colleagues, especially with A. Martin and Chan Hong-Mo.

REFERENCES

- 1) P. Hasenfratz, R.R. Horgan, J. Kuti and J.M. Richard, CERN TH.2387 (1980), preprint.
- 2) A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn and V.F. Weisskopf, Phys. Rev. D 9 (1974) 3471.
- 3) P. Hasenfratz and J. Kuti, Phys. Rep. 40C (1978) 75.
- 4) For a review, see, for example, M. Fabre de la Ripelle *in* the Proc. of International School on Nuclear Theory Physics, Predeal, Romania, September 1969.
Yu. Simonov *in* The nuclear many-body problem, (eds. F. Calogero and C. Ciofi degli Atti), 1972.
- 5) P. Hasenfratz, R.R. Horgan, J. Kuti and J.M. Richard, in preparation.
- 6) A. Martin, Phys. Lett. 88B (1979) 133.

Table 1

The two effective Abelian charges can be interpreted as being proportional to the third component of colour isospin and the colour hypercharge, respectively

Abelian charge	Q_1	Q_2	Q_3
$2I_3$	1	-1	0
$\sqrt{3}Y$	$\frac{1}{\sqrt{3}}$	$\frac{1}{\sqrt{3}}$	$-\frac{2}{\sqrt{3}}$

Table 2

Heavy baryon mass spectrum

Spin	Total ℓ parity	Grand orbital L	Number of nodes n	Masses of			
				ccc	bbb	ccb	cbb
$\frac{3}{2}$	0^+	0	0	4.79	14.30	8.03	11.20
$\frac{3}{2}$	0^+	0	1	5.30	14.75	8.52	11.66
$\frac{1}{2}$	1^-	1	0	5.14	14.66	8.37	11.54
$\frac{1}{2}$	$0^+, 1^+, 2^+$	2	0	5.42	14.91	8.64	11.92

Figure captions

- Fig. 1 : The potential energy of the 3Q-system is shown when the quarks form an equilateral triangle (Fig. 1a). The solid line is the spherical approximation and the dotted line corresponds to the numerical result with a multi-parameter bag shape. Ground state wave functions are also plotted. The shape of the bag is shown for an interquark separation of 3 fm.
- Fig. 2 : The distribution of colour flux lines for large interquark separation is shown. The Y-shape and Δ -shape are degenerate in energy for fixed quark configuration. The open circles represent the colour hypercharge of quarks while the black points correspond to the third component of colour isospin. The dashed and solid lines are the colour flux lines of the two different charges, respectively.

Fig. 1

Fig. 2