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SU(5) MONOPOLES, MAGNETIC SYMMETRY AND CONFINEMENT

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A B S T R A C T

The monopoles of the unified SU(5) gauge theory broken down to $H_E = SU(3)_C \otimes U(1)_{EM}$ [or to $K_E = SU(3)_C \otimes SU(2) \otimes U(1)_Y$], are classified. They belong to representations of a magnetic group $H_M(K_M)$, which is found to be isomorphic to $H_E(K_E)$. For SU(5) broken down to H_E , there exists a regular and stable monopole which is a colour magnetic triplet, and carries a non-zero Abelian magnetic charge. It is suggested that composite operators made out of this monopole and its antiparticle fields develop a non-zero vacuum expectation value, and so lead to a squeezing of the colour electric flux. Finally, we comment on the cosmological production of SU(5) monopoles.

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1. - INTRODUCTION

The embedding of the electroweak ¹⁾ $[SU(2) \times U(1)_Y]$ and the strong ²⁾ $[SU(3)_c]$ interactions in a unified gauge theory, such as $SU(5)$ ³⁾, has several important consequences. Some of these, such as the quantization of the electric charge, the predictions of the weak mixing angle ⁴⁾ and the proton lifetime ⁵⁾ have been extensively discussed in the literature. The purpose of this paper is to discuss some non-perturbative aspects of the unification which are related to the existence of monopoles in unified theories.

The presence of monopoles in unified theories [from now on we restrict ourselves to $SU(5)$] can be argued for on quite general grounds, even though explicit solutions are hard to construct. From the work of 't Hooft and Polyakov ⁶⁾, the monopoles are expected to be superheavy (typically, their mass is about two orders of magnitude heavier than the superheavy X bosons that mediate proton decay). Hence, their relevance for present energy physics is not immediately obvious. It was recently shown ⁷⁾, however, that the superheavy particles (the X bosons and the monopoles) and their bound states do play an essential rôle in the non-perturbative dynamics of the $SU(5)$ gauge theory. These superheavy particles play also an important rôle in the early Universe.

In this note we attempt a more systematic study of monopoles in the $SU(5)$ theory. These monopoles may have particular relevance for the problem of confinement in QCD. Let us explain this point.

It has been suggested ⁸⁾ that electric quark confinement in QCD is caused by a squeezing of the chromo-electric flux, in analogy to the Meissner effect in superconductivity. According to Mandelstam ⁹⁾, the analogues of the Cooper pairs in superconductivity are the Wu-Yang monopoles ¹⁰⁾ of QCD. A coherent plasma of such monopoles in the vacuum would cause a squeezing of the electric flux. The difficulty with this idea is that the Wu-Yang monopoles are irregular objects, so that the above idea is difficult, if not impossible, to implement.

The embedding of QCD in an $SU(5)$ theory provides us, however, with regular, stable monopoles which carry, in addition to $SU(3)_c$ magnetic quantum numbers, an Abelian magnetic charge. It is possible that the strong Abelian magnetic coupling leads, loosely speaking, to the formation of

monopole-antimonopole bound pairs. Condensation of such bound states in the vacuum would lead to a squeezing of the chromo-electric flux. Implementation of these ideas requires in practice a more detailed understanding of monopole dynamics than is available at present.

The plan of the paper is as follows. In Section 2 we discuss the question of the existence of topological magnetic charge in $SU(5)$, assumed to be broken down, either to $H_E = SU(3)_C \otimes U(1)_{EM}$, or to $K_E = SU(3)_C \otimes SU(2) \otimes U(1)_Y$. Some care is required, since the symbol $\otimes$ indicates a direct product only locally, and not globally. The topological charge of the monopole is found, in both cases, to be characterized by an integer. The monopoles form representations of a magnetic group ¹¹⁾, which is shown in Section 3 to be isomorphic to $H_E(K_E)$. In Section 4, the possible relevance of "elementary" monopoles for the problem of electric confinement is discussed. The possibility that the magnetic symmetry is a gauge symmetry in a partial Higgs phase is explored. Quantized electric vortices then appear, which carry finite energy per unit length, and the right quantum numbers to confine those $SU(5)$ particles which transform non-trivially under the centre of the $SU(3)_C$ subgroup. Finally, we comment on the cosmological production of $SU(5)$ monopoles.

2. - TOPOLOGICAL MAGNETIC CHARGE IN $SU(5)$

We consider the unified $SU(5)$ gauge theory spontaneously broken down to $H_E = SU(3)_C \otimes U(1)_{EM}$. This symmetry breaking is achieved by employing a 24-plet and a 5-plet of Higgs fields. The theory admits magnetic monopoles as regular, finite energy classical solutions. The topological magnetic charges associated with the monopoles correspond to the elements of the second homotopy group $\pi_2(SU(5)/H_E)$ ¹²⁾. Since $SU(5)$ is simply connected,

$$\pi_2(SU(5)/H_E) = \pi_1(H_E) \quad , \quad (2.1)$$

where $\pi_1(H_E)$ is the first homotopy group of the unbroken subgroup H_E .

In order to compute $\pi_1(H_E)$, let us first give the definitions of the $SU(3)_C$ and $U(1)_{EM}$ subgroups of $SU(5)$:

$$\left(\begin{array}{c|c} U & 0 \\ \hline 0 & 1_2 \end{array} \right) \in SU(3)_c, \quad e^{i\phi Q_E} \in U(1)_{EM}, \quad (2.2)$$

where U is a unitary, unimodular 3×3 matrix and the electric charge generator Q_E is given by

$$Q_E = \begin{pmatrix} -\frac{1}{3} & & & \\ & -\frac{1}{3} & & \\ & & -\frac{1}{3} & \\ & & & 0 & \\ & & & & 1 \end{pmatrix}. \quad (2.3)$$

The electromagnetic group $U(1)_{EM}$, being a subgroup of $SU(5)$, is compact, and the parameter ϕ is an angular variable with period 6π . Moreover, it follows from (2.2) and (2.3) that $U(1)_{EM}$ and $SU(3)_c$ have three elements in common. That is,

$$e^{i\phi Q_E} = \left(\begin{array}{c|c} e^{-i\phi/3} 1_3 & 0 \\ \hline 0 & 1_2 \end{array} \right) \text{ for } \phi = 0, -2\pi \text{ and } -4\pi. \quad (2.4)$$

Clearly, the common elements, which we denote by $1, a, a^2$, respectively, coincide with the elements of the centre of $SU(3)_c$.

Because of the non-trivial intersections of $SU(3)_c$ and $U(1)_{EM}$, H_E is a direct product of these two groups locally, but not globally. The group H_E can be regarded as the set of pairs (g, h) , $g \in SU(3)_c$ and $h \in U(1)_{EM}$, where (g, h) is identified with (ga, ha^{-1}) . The unit element of H_E is therefore $(1, 1) = (a^2, a) = (a, a^2)$. In order to construct $\pi_1(H_E)$, we consider closed curves in H_E which start and end at the unit element. Denote by x the homotopy class of (those) curves $(g(t), h(t))$ ($0 \leq t \leq 1$), such that $g(1) = a^2 g(0)$, $h(1) = ah(0)$, and $h(t)$ does not wind around $U(1)_{EM}$. It is easily checked that $\pi_1(H_E)$ is generated by x . That is,

$$\pi_1(H_E) = \{ x^n, n \in \mathbb{Z} \}. \quad (2.5)$$

Observe that $x^2 = y$ is the homotopy class of curves $(g(t), h(t))$ ($0 \leq t \leq 1$), such that $g(1) = ag(0)$, $h(1) = a^2 h(0)$ and $h(t)$ does not wind around $U(1)_{EM}$. Moreover, $x^3 = z$ is the homotopy class of curves such that $h(t)$ winds around $U(1)_{EM}$ once, with $g(0) = g(1)$ and $h(0) = h(1)$. Note that $\pi_1(U(1)_{EM})$ is generated by z :

$$\pi_1(U(1)_{EM}) = \{ z^n, n \in \mathbb{Z} \} . \quad (2.6)$$

Clearly, the two groups (2.5) and (2.6) are isomorphic, but not identical. Since $\pi_1(H_E)$ is isomorphic to the set of integers $\mathbb{Z}$, the topological magnetic charge of any $SU(5)$ monopole can be characterized by an integer.

The above considerations may be extended to the case when the unbroken subgroup of $SU(5)$ is $K_E = SU(3)_C \otimes SU(2) \otimes U(1)_Y$, where the $SU(2)$ and $U(1)_Y$ subgroups are defined by :

$$\left(\begin{array}{c|c} 1_3 & 0 \\ \hline 0 & V \end{array} \right) \in SU(4) , \quad e^{i\psi Y} \in U(1)_Y , \quad (2.7)$$

where V is a unitary, unimodular 2×2 matrix and

$$Y = \begin{pmatrix} -\frac{1}{3} & & & & \\ & -\frac{1}{3} & & & \\ & & -\frac{1}{3} & & \\ & & & \frac{1}{2} & \\ & & & & \frac{1}{2} \end{pmatrix} . \quad (2.8)$$

One obtains

$$\pi_1(K_E) = \{ u^n, n \in \mathbb{Z} \} , \quad (2.9)$$

where u is the homotopy class of closed curves $(g(t), k(t), h(t))$ ($0 \leq t \leq 1$) in $SU(3)_C \otimes SU(2) \otimes U(1)_Y$, such that $g(1) = ag(0)$, $k(1) = bk(0)$ and $h(1) = e^{-2\pi i Y} h(0)$ [$h(t)$ does not wind around $U(1)_Y$]. Here

$$b = \left(\begin{array}{c|c} 1_3 & 0 \\ \hline 0 & -1_2 \end{array} \right) . \quad (2.10)$$

Note that $\pi_1(U(1)_Y)$ is generated by $v = u^6$.

3. - SU(5) MONOPOLES AND THE MAGNETIC GROUP

The magnetic charges of SU(5) monopoles require a more detailed specification than the topological considerations given in the previous section. For SU(5) broken down to H_E , the monopoles are characterized by three charges β_i ($i=1,2,3$) which we may regard as components of a vector $\vec{\beta}$. The quantization condition ^{11),13)} for $\vec{\beta}$ is :

$$\exp(4\pi i \vec{\beta} \cdot \vec{T}) = 1 \quad , \quad (3.1)$$

where the components T_i ($i=1,2,3$) of $\vec{T}$ span the Cartan subalgebra of H_E . The aim of this section is to solve (3.1) for $\vec{\beta}$, and to find the corresponding magnetic group ¹¹⁾ H_M , with respect to which the monopoles form (irreducible) multiplets.

To this end, we first construct the weight lattice $\Lambda(H_E)$ of H_E . Our definition for the T_i is as follows :

$$T_1 = \frac{1}{2\sqrt{3}} \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix}, \quad T_2 = \frac{1}{6} \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -2 & \\ & & & 0 \end{pmatrix}, \quad T_3 = Q_E. \quad (3.2)$$

Consider

$$e^{i \vec{\theta} \cdot \vec{T}} \in C(H_E) \quad , \quad (3.3)$$

where $C(H_E)$ is the Cartan subgroup of H_E generated by $\{T_i\}$, and $\vec{\theta} = (\theta_1, \theta_2, \theta_3)$. Different values of $\vec{\theta}$ need not necessarily correspond to different elements of $C(H_E)$. Let us find the set Λ_θ of all those $\vec{\theta}$ which correspond to the unit element. That is,

$$e^{i \vec{\theta} \cdot \vec{T}} = 1 \quad \text{for} \quad \vec{\theta} \in \Lambda_\theta. \quad (3.4)$$

From (3.2), we see that (3.4) is equivalent to

$$\vec{\theta} \cdot \vec{e}^i = 2\pi k^i \quad , \quad k^i \in \mathbb{Z} \quad , \quad i=1,2,3 \quad , \quad (3.5)$$

where

$$\begin{aligned}\bar{e}^1 &= \left(\frac{1}{2\sqrt{3}} , \frac{1}{6} , -\frac{1}{3} \right) \\ \bar{e}^2 &= \left(-\frac{1}{2\sqrt{3}} , \frac{1}{6} , -\frac{1}{3} \right) \\ \bar{e}^3 &= (0 , 0 , 1) .\end{aligned}\tag{3.6}$$

The general solution of (3.5) is

$$\bar{\theta} = \sum_{i=1}^3 n^i \bar{\theta}^i , \quad n^i \in \mathbb{Z} ,\tag{3.7}$$

where the $\bar{\theta}^i$ satisfy

$$\bar{\theta}^i \cdot \bar{e}^j = 2\pi \delta^{ij} .\tag{3.8}$$

Explicitly,

$$\begin{aligned}\bar{\theta}^1 &= (2\sqrt{3}\pi, 6\pi, 0) \\ \bar{\theta}^2 &= (-2\sqrt{3}\pi, 6\pi, 0) \\ \bar{\theta}^3 &= (0, 4\pi, 2\pi) .\end{aligned}\tag{3.9}$$

Thus, Λ_{θ} is the lattice generated by the basis vectors $\bar{\theta}^i$ ($i=1,2,3$).

Let $|\bar{w}\rangle$ be an eigenstate of $\bar{T}$ with eigenvalue $\bar{w} \in \Lambda(H_E)$.
From (3.4),

$$e^{i\bar{\theta} \cdot \bar{w}} = 1 \quad \text{for } \bar{\theta} \in \Lambda_{\theta} .\tag{3.10}$$

This gives

$$\bar{\theta}^i \cdot \bar{w} = 2\pi k^i , \quad k^i \in \mathbb{Z} .\tag{3.11}$$

Therefore

$$\bar{w} = \sum_{i=1}^3 n^i \bar{w}^i , \quad n^i \in \mathbb{Z} ,\tag{3.12}$$

where

$$\bar{\theta}^i \cdot \bar{w}^j = 2\pi \delta^{ij} . \quad (3.13)$$

From (3.8) and (3.13) we see that

$$\bar{w}^j = \bar{e}^j . \quad (3.14)$$

Consequently, $\Lambda(H_E)$ is generated by the vectors $\bar{e}^i$ ($i=1,2,3$).

We now return to the quantization condition (3.1). It is equivalent to the relation

$$e^{4\pi i \bar{\beta} \cdot \bar{w}} = 1 \quad \text{for } \bar{w} \in \Lambda(H_E) . \quad (3.15)$$

One obtains

$$\bar{\beta} = \sum_{i=1}^3 n^i \bar{\beta}^i , \quad n^i \in \mathbb{Z} , \quad (3.16)$$

where

$$2 \bar{\beta}^i \cdot \bar{e}^j = \delta^{ij} . \quad (3.17)$$

Comparison of (3.8) and (3.17) gives

$$\bar{\beta}^i = \frac{\bar{\theta}^i}{4\pi} . \quad (3.18)$$

Thus, (3.16) together with (3.18) gives the lattice of the magnetic charges.

The lattice $\Lambda(H_M)$ of the magnetic group H_M can now be obtained from the lattice of the magnetic charges by the following rescaling¹¹⁾ :

$$\bar{m} = \bar{\beta} N^{-1} \in \Lambda(H_M) , \quad N = \begin{pmatrix} 3 & & \\ & 3 & \\ & & 1 \end{pmatrix} . \quad (3.19)$$

The lattice $\Lambda(H_M)$ is generated by the following three vectors :

$$\begin{aligned}\bar{m}^1 &= \left(\frac{1}{2\sqrt{3}} , \frac{1}{2} , 0 \right) \\ \bar{m}^2 &= \left(-\frac{1}{2\sqrt{3}} , \frac{1}{2} , 0 \right) \\ \bar{m}^3 &= \left(0 , \frac{1}{3} , \frac{1}{2} \right) .\end{aligned}\tag{3.20}$$

It is easily checked that $\Lambda(H_M)$ is identical to the lattice obtained from $\Lambda(H_E)$ by multiplying the third component of the weights by $\frac{3}{2}$. Thus, the magnetic group H_M is $SU(3)_M^c \otimes U(1)_M$, which is isomorphic to H_E . The generator Q_M of $U(1)_M$ is equal to $\frac{3}{2}Q_E$.

Instead of (3.20), we may choose the following, physically more transparent, basis in the lattice $\Lambda(H_M)$:

$$\begin{aligned}\bar{t}^1 &= \left(\frac{1}{2\sqrt{3}} , \frac{1}{6} , -\frac{1}{2} \right) \\ \bar{t}^2 &= \left(-\frac{1}{2\sqrt{3}} , \frac{1}{6} , -\frac{1}{2} \right) \\ \bar{t}^3 &= \left(0 , -\frac{1}{3} , -\frac{1}{2} \right) .\end{aligned}\tag{3.21}$$

This basis corresponds to a monopole which is a triplet under $SU(3)_M^c$ and carries $-\frac{1}{2}U(1)_M$ charge. It is a regular, dynamically stable¹⁴⁾ (under small perturbations), finite energy monopole. More generally, the dynamically stable monopoles are those that are singlets, triplets or antitriplets under $SU(3)_M^c$, and carry a $U(1)_M$ charge which is $\frac{3}{2}k$, $-\frac{1}{2}+\frac{3}{2}k$ or $\frac{1}{2}+\frac{3}{2}k$, respectively ($k \in \mathbb{Z}$). We note that the topological charge of the monopoles discussed in the previous section corresponds to their $U(1)_M$ charge.

The same analysis can be carried through for the case when $SU(5)$ breaks down to $K_E = SU(3)_c \otimes SU(2) \otimes U(1)_Y$. The corresponding magnetic group K_M turns out to be isomorphic to K_E . The generator Y_M of $U(1)_{YM}$ is found to be equal to $-3Y$. Under the magnetic group $K_M = SU(3)_M^c \otimes SU(2)_M \otimes U(1)_{YM}$, the dynamically stable monopoles transform as follows :

$$\begin{aligned}(\underline{1}, \underline{1})_{3k} , (\underline{3}, \underline{2})_{1+3k} , (\bar{\underline{3}}, \underline{1})_{-1-3k} , (\underline{1}, \underline{2})_{\frac{5}{2}+3k} , \\ (\underline{3}, \underline{2})_{\frac{5}{2}+3k} , (\bar{\underline{3}}, \underline{2})_{-\frac{5}{2}-3k} \quad (k \in \mathbb{Z}) .\end{aligned}\tag{3.22}$$

4. - MAGNETIC SYMMETRY AND CONFINEMENT

Very little is known about the quantum theory of monopoles. However, it is tempting to conjecture ¹¹⁾ that the monopole dynamics is effectively described by a gauge theory based on the magnetic group discussed in the previous section. Some regular monopoles would play the rôle of "elementary" matter fields in this theory. The magnetic theory may undergo a dynamical, partial Higgs breaking due to a formation of a condensate of bound states of the "elementary monopoles". This results in a formation of quantized electric vortices, which carry finite energy per unit length and confine some of the particles of the $SU(5)$ theory.

Let us restrict ourselves to the case where the unbroken subgroup of $SU(5)$ is $H_E = SU(3)_C \otimes U(1)_{EM}$. The corresponding magnetic group is $H_M = SU(3)_M^C \otimes U(1)_M$. One expects that the stable and regular $(\bar{3})_{-\frac{1}{2}}$ and $(\bar{3})_{\frac{1}{2}}$ monopoles are included among the "elementary" matter fields of the magnetic theory. Clearly, the $U(1)_M$ coupling is strong, since it is inversely proportional to the $U(1)_{EM}$ coupling. Consequently, the $(\bar{3})_{-\frac{1}{2}}$ and $(\bar{3})_{\frac{1}{2}}$ monopoles will tend to form strongly coupled bound states which carry zero net $U(1)_M$ charge. Because of the strong $U(1)_M$ coupling, the absolute value of the binding energy is likely to exceed the sum of the monopole and antimonopole mass. This may lead to condensation of these bound states in the vacuum and, consequently, to a breaking of the magnetic symmetry H_M down to $U(1)_M$. These considerations are meant to provide only a qualitative picture for the requirement that composite operators made out of the "elementary" monopole and antimonopole fields develop a non-zero vacuum expectation value.

Due to the breaking of the magnetic symmetry, quantized electric vortices would be present in the theory. The quantized electric flux carried by the vortices is characterized by the elements of $\pi_1(H_M/U(1)_M)$ ¹⁵⁾. In order to compute $\pi_1(H_M/U(1)_M)$, let us recall [cf. (2.5)] that

$$\pi_1(H_M) = \{x^n, n \in \mathbb{Z}\}.$$

Any curve in H_M which belongs to the homotopy class $z = x^3$ can be continuously deformed to lie entirely in $U(1)_M$. It is, therefore, homotopically trivial in $H_M/U(1)_M$. Hence

$$\pi_1(H_M/U(1)_M) = \{1, x, x^2\}, \quad (4.1)$$

where x can now be thought of as a set of closed curves $g(t)$ ($0 \leq t \leq 1$), entirely lying in $SU(3)_M^c$, with the property $g(1) = a^2 g(0)$, where a is defined in (2.4).

Thus, the electric vortices carry one or two units of chromoelectric $(SU(3)_c)$ flux, and, therefore, confine particles that are not invariant under the centre of $SU(3)_c$. These include the quarks and the X bosons of the $SU(5)$ theory.

The colour magnetic fields of the various monopoles, on the other hand, are screened by the colour magnetic condensate. The $U(1)_M$ magnetic fields of the monopoles are, however, long range.

At this stage it is worth while to stress once again the difference between an $SU(3)_c$ gauge theory considered on its own and an $SU(3)_c$ gauge theory embedded in a unified gauge group, such as $SU(5)$. In the former case, the theory does not contain any regular monopoles, whereas in the latter case, as we discussed above, the theory admits regular stable monopoles which form magnetic colour triplets, and simultaneously carry a $U(1)_M$ charge. Bound states of these monopoles, via the $U(1)_M$ coupling, may provide a colour magnetic condensate which then squeezes the chromoelectric flux.

Finally, we comment on the question of production of $SU(5)$ monopoles in the early Universe. Recently, it has been argued ¹⁶⁾ that an unacceptably large number of these monopoles might have been produced (in the early Universe), indicating a possible discrepancy between unified theories and standard big bang cosmology. It seems to us, however, that such claims are premature for the following reason. For $SU(5)$ broken down to $K_E = SU(3)_c \otimes SU(2) \otimes U(1)_Y$, the monopoles are classified under the magnetic group $K_M = SU(3)_M^c \otimes SU(2)_M \otimes U(1)_{YM}$. The monopoles considered in Ref. 16) belong to the $(\underline{1}, \underline{1})_{3k}$ representation of K_M [see Eq. (3.22)]. The "elementary monopoles", however, correspond to the $(\underline{\bar{3}}, \underline{2})_{\frac{1}{2}}$ representation. The $(\underline{1}, \underline{1})_3$ monopole, for instance, can be regarded to be made out of six $(\underline{\bar{3}}, \underline{2})_{\frac{1}{2}}$ monopoles. Thus, it is likely that the $(\underline{1}, \underline{1})_3$ monopole is a composite object ^{*}). Moreover, it has been shown by Polyakov ¹⁸⁾ that at high enough temperatures quarks are liberated due to condensation

^{*}) Cf., for example, some recent work of Weinberg ¹⁷⁾ on monopoles in the Prasad-Sommerfield limit.

of the chromo-electric vortices. This would lead to a squeezing of chromo-magnetic flux, and therefore to confinement of monopoles that carry $SU(3)_M^c$ quantum numbers. The $(1,1)_3$ monopole can then be regarded as a magnetic dibaryon with respect to $SU(3)_M^c$. Similarly, all the other monopoles can be considered as composite objects made out of the $(\bar{3},2)_{\frac{1}{2}}$ monopole and its antimonopole. With $SU(3)_M^c$ in the confining phase, the simplest unconfined monopole is the $(1,2)_{\frac{3}{2}}$ monopole, which is a magnetic baryon under $SU(3)_M^c$. Without much elaboration, one can see that the production estimates for such magnetic baryons are expected to be lower than the estimates given in Ref. 16). To see this, let us note that formation of a magnetic baryon-antibaryon pair requires simultaneous production of three magnetic mesons [confined pairs of a $(\bar{3},2)_{\frac{1}{2}}$ monopole and its antimonopole] on top of each other. The resulting lowering of the production estimates, however, may not be sufficient to remove the apparent conflict between big bang cosmology and grand unified models.

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