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DIRECT PHOTON PRODUCTION BEYOND LEADING ORDER IN QCD

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ABSTRACT

The significance of the quark-quark scattering process (quark + quark $\rightarrow$ quark + quark + photon) for the production of large q_T real photons is discussed in the framework of perturbative QCD. To extract the finite contribution of this process to the differential cross-section $d\sigma/dy d^2\vec{q}_T$ (hadron 1 + hadron 2 $\rightarrow$ photon + anything) we define the gluon distribution and the quark-to-photon fragmentation function beyond the leading approximation. The calculations are performed consistently in the dimensional regularization scheme. Our numerical estimates show the resulting finite $qq \rightarrow qq\gamma$ contribution to be a small (order $\alpha_s/2\pi$) correction in comparison with the basic QCD sub-processes.

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1. INTRODUCTION

The production of large q_T real photons in hadronic collisions has been recognized as an important test of quantum chromodynamics¹⁾⁻⁶⁾. A substantial γ/π^0 ratio rising with p_T and in agreement with the QCD predictions has been seen in experiments⁷⁾⁻⁹⁾. While the uncertainties are large, both in the experimental data and in the predictions of hard scattering models²⁾⁻⁵⁾, it is important, from the theoretical point of view, to determine also the higher order (non-leading) corrections from QCD. Especially interesting for pp collisions is the quark-quark scattering ($qq \rightarrow qq\gamma$) sub-process of Fig. 1 as the collision can take place between valence quarks. The leading contributions of this sub-process are already included in the basic QCD sub-processes, the Compton process ($qq \rightarrow q\gamma$, Fig. 2a) and the final quark bremsstrahlung process ($qq \rightarrow qq$ with $q \rightarrow q\gamma$, Fig. 2c), due to the factorization property of mass singularities. Thus one must necessarily go beyond the leading approximation to find the finite contribution of this order α_s^2 process. The analysis of the non-leading QCD terms has been recently completed for deep inelastic scattering¹⁰⁾⁻¹²⁾, and the problem of calculating beyond leading order has also been discussed for one-particle inclusive deep inelastic scattering¹³⁾ and massive lepton pair production¹⁴⁾⁻¹⁹⁾. An important lesson from the analysis of non-leading terms in deep inelastic scattering is that their functional form depends on the renormalization prescription and on the definition of parton distributions (and fragmentation functions) beyond leading order. However, the physical results are independent of any prescriptions when one calculates consistently in a chosen scheme.

In this work we use dimensional regularization to calculate the parton cross-sections, mainly for the reason that the anomalous dimensions at the two-loop level, which determine the Q^2 evolution of the parton distributions beyond leading order, have been calculated in this scheme¹⁰⁾⁻¹²⁾.

In Section 2 we illustrate our calculational scheme for deep inelastic scattering and define the parton distributions beyond leading order. In Section 3 we calculate the quark-quark scattering cross-section and after extracting the contributions to the basic QCD processes, we present our final result for the differential cross-section $d\sigma/dy d^2q_T(qq \rightarrow qq\gamma)$. Before this, we find it necessary to discuss also the fragmentation function $D_q^Y(z, Q^2)$ of a quark emitting a parallel real photon beyond the leading approximation.

In Section 4 some numerical estimates on the finite terms of the $qq \rightarrow qq\gamma$ process are given and compared with the basic QCD cross-sections.

Section 5 contains a discussion of the choice of the scaling violation parameter as well as a comparison with other works. The kinematical variables and other basic formulae, together with the parton distribution parametrizations used in the numerical calculations are given in the Appendix.

2. DEEP INELASTIC SCATTERING AND THE DEFINITION OF PARTON DISTRIBUTIONS

We first discuss deep inelastic scattering to illustrate our calculational procedure and to define the parton distributions. Especially, we reconsider the calculation of Ref. 14) for the contribution of gluons to the structure function F_2 . The definitions of the structure functions and kinematical variables can be found in Appendix A. The Compton process (Fig. 3b) contribution to $\hat{W}_{\mu\nu}$ is obtained by performing the following integral over the squared matrix element $\hat{H}_{\mu\nu}$:

$$\hat{W}_{\mu\nu} = \langle \frac{1}{2} \rangle \frac{\alpha_s}{2} \int \frac{d^4 p_1 d^4 p_2}{(2\pi)^2} \delta^+(p_1^2) \delta^+(p_2^2) \delta^4(k+q-p_1-p_2) (-\hat{H}_{\mu\nu}) \quad (1)$$

with normalization defined by Eq. (A4). The number in brackets is the colour factor of the process.

We perform our calculations for massless on-shell partons in n dimensional space ($\epsilon = 2 - n/2$). Contracting $\hat{H}_{\mu\nu}$ with $g^{\mu\nu}$ and $k^\mu k^\nu$ (for diracology, see Ref. 20)) one obtains

$$\begin{aligned} g^{\mu\nu} \hat{H}_{\mu\nu} &= 8(1-\epsilon) \left\{ (1-\epsilon) \left[\frac{u}{s} + \frac{t}{u} \right] + \frac{2q^2 s}{tu} - 2\epsilon \right\} \\ k^\mu k^\nu \hat{H}_{\mu\nu} &= -8s \end{aligned} \quad (2)$$

with the invariants $s = 2p_1 \cdot p_2$, $t = -2p_2 \cdot k$ and $u = -2p_1 \cdot k$. The phase-space integral needed to evaluate $\hat{W}_{\mu\nu}$ is given in the centre-of-mass system CMS($k+q$) and in n dimensions by

$$\begin{aligned} I(2-PS) &= \int \frac{d^n p_1}{(2\pi)^{n-1}} \frac{d^n p_2}{(2\pi)^{n-1}} \delta^+(p_1^2) \delta^+(p_2^2) (2\pi)^n \delta^n(k+q-p_1-p_2) \\ &= \frac{1}{8\pi} \left(\frac{4\pi e^{-\gamma_E} \mu^2}{s} \right)^\epsilon \int_0^1 dy y^{-\epsilon} (1-y)^{-\epsilon} \end{aligned} \quad (3)$$

where γ_E is Euler's constant and the integration variable is $y = (1 + \cos \theta)/2$. To make $\hat{W}_{\mu\nu}$ dimensionless we have introduced an arbitrary scale parameter μ (by substituting the strong coupling constant g by $g \cdot \mu^\epsilon$, for example). Expressing the invariants in terms of y , $z = Q^2/2k \cdot q$ and Q^2

$$s = Q^2 \frac{1-z}{z}, \quad t = -Q^2 \frac{1-y}{z}, \quad u = -Q^2 \frac{y}{z} \quad (4)$$

and performing the y integration one finally gets, combining Eq. (A2) with Eqs (1) and (2)

$$\begin{aligned} \frac{1}{2} \hat{F}_2(z, Q^2) = & \frac{\alpha_s}{2\pi} \left\langle \frac{1}{2} \right\rangle \cdot \left\{ 6z(1-z) \right. \\ & \left. + \left[-\frac{1}{\epsilon} - \ln 4\pi + \gamma_E + \ln \frac{Q^2}{\mu^2} + \ln \frac{1-z}{z} \right] \cdot \left[z^2 + (1-z)^2 \right] \right\} \end{aligned} \quad (5)$$

We effect the renormalization of the parton distribution in the minimal subtraction scheme by dropping the $1/\epsilon$ term and the factor $\ln 4\pi - \gamma_E$ can be reabsorbed into a redefinition of the scale μ^2 . Alternatively, in the $\overline{MS}$ scheme of Ref. 11) we can drop the $1/\epsilon + \ln 4\pi - \gamma_E$ term altogether. To relate the remaining part to the hadron level structure function $F_2(x, Q^2)$ we must define the parton distributions beyond the leading approximation. To achieve this, we will first discuss the formal approach to deep inelastic scattering based on operator product expansion (OPE) and renormalization group equations (RGE). OPE gives, for the moments of the (singlet part) structure function F_2 the form^{10), 11)}

$$M_n(Q^2) = A_n^q(\mu^2) C_n^q\left(\frac{Q^2}{\mu^2}, q^2\right) + A_n^g(\mu^2) C_n^g\left(\frac{Q^2}{\mu^2}, q^2\right) \quad (6)$$

where μ^2 is again an arbitrary renormalization scale, C_n^k 's are the coefficient functions and A_n^k 's the reduced matrix elements. The coefficient functions satisfy the RGE and can be written in the form

$$C_n^k\left(\frac{Q^2}{\mu^2}, q^2\right) = T \left[\exp - \int_0^{\ln Q^2/\mu^2} \gamma_n(\bar{g}(t')) dt' \right]_{kj} C_n^j(1, \bar{g}^2(Q^2)) \quad (7)$$

where $\bar{g}(Q^2)$ is the effective strong coupling and γ_n is the anomalous dimension matrix (T is the "time-ordering" operator). Writing Eq. (6) in the form

$$M_n(Q^2) = A_n^q(Q^2) C_n^q(1, \bar{g}^2) + A_n^g(Q^2) C_n^g(1, \bar{g}^2) \quad (8)$$

where

$$\begin{bmatrix} A_n^q(Q^2) \\ A_n^g(Q^2) \end{bmatrix} = T^n \left[\exp - \int_0^{\ln Q^2/\mu^2} \gamma_n^T(\bar{g}(t')) dt' \right] \begin{bmatrix} A_n^q(\mu^2) \\ A_n^g(\mu^2) \end{bmatrix} \quad (9)$$

we now define the universal (singlet part) quark distribution $q(x, Q^2)$ and gluon distribution $g(x, Q^2)$ to all orders in $\bar{g}^2$ by^{21), 22)}

$$\begin{aligned} A_n^q(Q^2) &= \int_0^1 dx x^{n-1} q(x, Q^2) \\ A_n^g(Q^2) &= \int_0^1 dx x^{n-1} g(x, Q^2) \end{aligned} \quad (10)$$

Equation (9) is then equivalent to the master equation

$$\frac{\partial}{\partial \ln Q^2} \begin{bmatrix} q(x, Q^2) \\ g(x, Q^2) \end{bmatrix} = \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}, \bar{g}^2\right) \begin{bmatrix} q(y, Q^2) \\ g(y, Q^2) \end{bmatrix} \quad (11)$$

where $P(z, \bar{g}^2)$ is the inverse Mellin transform of the anomalous dimension matrix γ_n and which equation in lowest order reduces to the Altarelli-Parisi-Lipatov evolution equation^{23), 24)}. We write the A-P-L equation down explicitly for the transitions $g \rightarrow q$ and $q \rightarrow g$:

$$\begin{aligned} \frac{\partial}{\partial \ln Q^2} q(x, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{qg}\left(\frac{x}{y}\right) g(y, Q^2) + \dots \\ \frac{\partial}{\partial \ln Q^2} g(x, Q^2) &= \frac{\alpha_s(Q^2)}{2\pi} \int_x^1 \frac{dy}{y} P_{gq}\left(\frac{x}{y}\right) q(y, Q^2) + \dots \end{aligned} \quad (12)$$

$$P_{qg}(z) = \left\langle \frac{1}{2} \right\rangle \cdot [z^2 + (1-z)^2]$$

$$P_{gq}(z) = \left\langle \frac{4}{3} \right\rangle \frac{1 + (1-z)^2}{z}$$

We can now identify the $\log Q^2$ term in Eq. (5) as the contribution of gluons to the quark distribution evolution (factor 1/2 on the left-hand side signifies equal contribution to antiquark evolution). With the definition of Eq. (10) of the quark distribution we are left with

$$\frac{1}{2} \hat{F}_2(z, Q^2) = \frac{\alpha_s(Q^2)}{2\pi} \langle \frac{1}{2} \rangle \left\{ 6z(1-z) + [z^2 + (1-z)^2] \ln \frac{1-z}{z} \right\} \quad (13)$$

as the finite contribution of the Compton sub-process to structure function F_2 . This result agrees with the result of the formal approach in the minimal subtraction scheme and with the same definition of parton distributions¹¹⁾. In the perturbative approach the turning of the coupling constant $\alpha_s(\mu^2)$ into the effective coupling $\alpha_s(Q^2)$ and the absorption of the $\log Q^2/\mu^2$ term into the parton distribution evolution in the basic process can be effected by choosing $\mu^2 = Q^2$ as the renormalization point.

We note the advantage of parton distributions defined by Eq. (10) over other definitions, as their Q^2 evolution is determined by the anomalous dimension γ_n alone. Also, γ_n is known in a dimensional minimal subtraction scheme at the two-loop level, which is needed for a consistent determination of non-leading effects.

Having indicated that the perturbative QCD approach reproduces the results of the formal approach in deep inelastic scattering, we can now apply the same procedure with the same definition of parton distributions to processes where one cannot use the powerful formal methods. Massive lepton pair production has been discussed in Refs 14)-19), and we will now generalize this approach to non-leading corrections due to three-particle final states in large q_T direct photon production.

3.1 THE QUARK-QUARK SCATTERING PROCESS ($qq \rightarrow qq\gamma$) IN DIRECT PHOTON PRODUCTION

In proton-proton collisions the Compton process (Fig. 2a) is the dominant QCD mechanism contributing to the direct photon production cross-section $d\sigma/dy d^2q_T$ ¹⁾⁻⁵⁾. The annihilation process (Fig. 2b) is suppressed by the fast decreasing antiquark distribution in the proton and the final quark bremsstrahlung process (Fig. 2c) is suppressed by the factor $\alpha_s/2\pi$. We shall now calculate the quark-quark scattering ($qq \rightarrow qq\gamma$) cross-section neglecting the interference between the two quark lines (Figs 1a and 1b) and neglecting also the interference due to identical particle symmetrization.

The one-particle inclusive cross-section due to $2 \rightarrow 3$ kinematics ($p_1 + p_2 = p_3 + p_4 + q$) is of the form

$$q_0 \frac{d\hat{\sigma}}{d^3q} = \frac{1}{2} \frac{1}{2s} \frac{1}{(2\pi)^5} \int d^4p_3 d^4p_4 \delta^+(p_3^2) \delta^+(p_4^2) \delta^4(p_1 + p_2 - p_3 - p_4 - q) |\mathcal{M}|^2 \quad (14)$$

at the parton level. We regularize our calculation again by evaluating Eq. (14) in n dimensions.

In CMS($p_3 + p_4$) the fixed momenta p_1 , p_2 and q are chosen to lie in the xz plane and thus $|\overline{\mathcal{M}}|^2$ in Eq. (14) can depend only on t , z and x components of p_3 and p_4 . The phase-space integral reduces after integration over irrelevant components into

$$\begin{aligned} I(3\text{-PS}) &= \int \frac{d^n p_3}{(2\pi)^{n-1}} \frac{d^n p_4}{(2\pi)^{n-1}} \delta^+(p_3^2) \delta^+(p_4^2) (2\pi)^n \delta^n(p_1 + p_2 - p_3 - p_4 - q) \\ &= \frac{1}{(2\pi)^2} \frac{1}{8} \left(\frac{4\pi e^{-\gamma_E} \mu^2}{s_{34}} \right)^\epsilon \int_{-1}^1 dz_1 (1-z_1^2)^{-\epsilon} \int_{-1}^1 dz_2 (1-z_2^2)^{-\frac{1}{2}-\epsilon} \end{aligned} \quad (15)$$

where we have introduced the arbitrary (renormalization) scale μ^2 and $s_{34} = (p_3 + p_4)^2 = (p_1 + p_2 - q)^2$. The integration is over the angles $z_i = \cos \theta_i$ ($i = 1, 2$) defining the direction of p_3 in CMS($p_3 + p_4$), i.e., in the notation $x = (t, z, x, \dots)$

$$\begin{aligned} p_3 &= p_{30} (1, \cos \theta_1, \sin \theta_1 \cos \theta_2, \dots) \\ p_4 &= p_{40} (1, -\cos \theta_1, -\sin \theta_1 \cos \theta_2, \dots) \end{aligned} \quad (16)$$

with $p_{30} = p_{40} = \sqrt{s_{34}}/2$.

In the Feynman gauge the separately gauge invariant squared terms $|\overline{\mathcal{M}}_a|^2$ and $|\overline{\mathcal{M}}_b|^2$ of Figs 1a and 1b are given by

$$\begin{aligned} |\overline{\mathcal{M}}_a|^2 &= -e_f^2 g^4 \frac{1}{(p_2 - p_4)^4} \frac{1}{4} \text{Tr} [p_4 \gamma^\sigma p_2 \gamma^\rho] \\ &\quad \otimes \text{Tr} \left[p_3 \left(\gamma^\sigma \frac{1}{p_1 - q} \gamma^\rho + \gamma^\rho \frac{1}{p_3 + q} \gamma^\sigma \right) p_1 \left(\gamma_\rho \frac{1}{p_3 + q} \gamma_\sigma + \gamma_\sigma \frac{1}{p_1 - q} \gamma_\rho \right) \right] \quad (17) \\ |\overline{\mathcal{M}}_b|^2 &= |\overline{\mathcal{M}}_a|^2 (p_1 \leftrightarrow p_2, p_3 \leftrightarrow p_4) \end{aligned}$$

When performing the integration over p_3 and p_4 the denominator in $|\overline{\mathcal{M}}_a|^2$ may vanish. Thus the configuration where $\vec{p}_4$ and $\vec{p}_2$ are parallel gives the mass singularity due to collinear emission of a gluon from quark (p_2). In the cross-section of Eq. (14) the corresponding $1/\epsilon$ term can be absorbed into the Compton cross-section giving rise to a scale violating gluon distribution. Similarly, when $\vec{p}_3$ and $\vec{q}$ are parallel, the corresponding $1/\epsilon$ divergence will be absorbed into the fragmentation function of a final state quark to a photon.

Evaluating the matrix elements in n dimensions and using the x variables defined in Appendix A, the integration over z_1 and z_2 gives

$$\begin{aligned}
 q_0 \frac{d\hat{\sigma}}{d^3q} &= e_f^2 2\alpha\alpha_s \frac{1}{q_T^4} \left\langle \frac{2}{g} \right\rangle \frac{\alpha_s}{2\pi} \cdot \left(\frac{4\pi e^{-Y_E} \mu^2}{s_{34}} \right)^\epsilon \cdot x_1 x_2 \\
 &\otimes \left\{ -\frac{1}{\epsilon} \frac{x_1}{1-x_1} \left[1 + (1-x_1)^2 \right] \frac{1 + (1-\tilde{z})^2}{\tilde{z}} \right. \\
 &\quad - \frac{1}{\epsilon} \frac{x_1 x_2}{x_1 + x_2} \frac{(x_1 + x_2)^2 + x_1^2}{x_2^2} \frac{1 + (1-\tilde{z})^2}{\tilde{z}} \\
 &\quad - 2 \frac{x_1}{x_2} \left[2 + (1-x_1)^2 + (1-x_1-x_2)^2 \right] \ln \frac{(1-x_1)(x_1+x_2)}{x_2} \\
 &\quad \left. + \frac{2x_1^2}{(x_1+x_2)^2} - \frac{(x_1+x_2)x_1}{1-x_1} + 4 \frac{x_1}{x_2} (x_1^2 + x_1 x_2 + x_2^2) \right\} \quad (18)
 \end{aligned}$$

where $z = x_2/(1-x_1)$ and $\tilde{z} = x_1 + x_2$. s_{34} can be written in terms of z or $\tilde{z}$ as

$$s_{34} = \frac{1-\tilde{z}}{\tilde{z}} \cdot \frac{q_T^2}{x_2} = (1-\tilde{z}) \cdot \frac{q_T^2}{x_1 x_2} \quad (19)$$

We will not give a detailed description of the integration as it is straightforward, although tedious. We only note that one naturally arrives at hypergeometric functions which must be expanded as power series in $\epsilon = 2 - n/2$.

$q^0 d\hat{\sigma}^b/d^3q$ is then obtained by exchanging x_1 and x_2 in Eqs (18) and (19). We have separated the $1/\epsilon$ term into two parts with the appropriate z variables. In the first part we can identify the anomalous dimension function $P_{gq}(z)$ of Eq. (12) and the Compton cross-section given in Appendix A, Eq. (A9). In the second part we have the fragmentation function $D_q^Y(\tilde{z})$ of Eq. (A14) and the $qq \rightarrow qq$ cross-section of Eq. (A12). This demonstrates the well-known factorization property of mass singularities. Before extracting the finite $qq \rightarrow qq\gamma$ cross-section, we first discuss the definition of the fragmentation function $D_q^Y(z, Q^2)$ beyond the leading approximation.

3.2 FRAGMENTATION FUNCTION $D_q^Y(z, Q^2)$

The definition of parton distributions beyond the leading order was given in Section 2. In this section we will define the quark-to-photon fragmentation function $D_q^Y(z, Q^2)$ beyond the leading approximation.

$D_q^Y(z, Q^2)$ is defined from $e^+e^- \rightarrow \gamma X$ cross-section by

$$\frac{d\sigma}{dz} (e^+e^- \rightarrow \gamma X) = \sigma_{\mu\mu} \sum_f e_f^2 \left[D_{q_f}^Y(z, s) + D_{\bar{q}_f}^Y(z, s) \right] \quad (20)$$

where $z = 2E_\gamma/\sqrt{s}$ and $\sigma_{\mu\mu} = 4\pi\alpha^2/3s$. The Feynman diagrams for calculating D_q^Y are given in Fig. 4 and we can obtain the matrix element for the $e^+e^- \rightarrow q\bar{q}\gamma$ process by crossing (with obvious changes in couplings and colour factors) from the $q\bar{q} \rightarrow q\bar{q}\gamma$ matrix element $\mathcal{M}_a$. We again encounter the phase-space integral of Eq. (15) which we calculate in the centre-of-mass system of the final state quarks and in $n > 4$ dimensions. After integrating over the z_1 and z_2 variables, the angular distribution of the photon is given by

$$\begin{aligned} \frac{d\sigma}{dz d\Omega} &= \frac{s z}{4} \cdot q_0 \frac{d\sigma}{d^3q} \\ &= \frac{3\alpha}{16\pi^2} \sigma_{\mu\mu} \left(\frac{4\pi e^{-\gamma_E} \mu^2}{s_{34}} \right)^\epsilon \sum_f e_f^4 \\ &\quad \otimes \left\{ \left[-\frac{1}{\epsilon} \frac{1+(1-z)^2}{z} + z \right] (1+\cos^2\theta) + \frac{6-6z+z^2}{z} (1-\cos^2\theta) \right\} \end{aligned} \quad (21)$$

and integration over $\Omega = (\theta, \phi)$ results in

$$\begin{aligned} \frac{d\sigma}{dz} &= \frac{\alpha}{2\pi} \cdot 2 \sigma_{\mu\mu} \cdot \sum_f e_f^4 \\ &\quad \otimes \left\{ \frac{1+(1-z)^2}{z} \left[-\frac{1}{\epsilon} - \ln 4\pi + \gamma_E + \ln \frac{s(1-z)}{\mu^2} \right] + \frac{3}{2} \frac{1+(1-z)^2}{z} \right\} \end{aligned} \quad (22)$$

where we have used the relation $s_{34} = s(1-z)$. With the obvious assumption $D_q^Y = D_{\bar{q}}^Y$ the definition of Eq. (20) gives the result

$$D_{q_f}^Y(z, s) = \frac{\alpha}{2\pi} e_f^2 \frac{1+(1-z)^2}{z} \left[\ln \frac{s(1-z)}{\mu^2} + \frac{3}{2} \right] \quad (23)$$

which reproduces the leading approximation result given in Appendix A, Eq. (A14) and where we have treated the $1/\epsilon + \ln 4\pi - \gamma_E$ term as in Section 2.

3.3 THE FINAL $qq \rightarrow qqY$ CROSS-SECTION

We can now go back to our result of Eq. (18) and extract the final form for the contribution of the $qq \rightarrow qqY$ sub-process. We define the cross-section $\hat{\sigma}$ (CONSTANT) due to the "constant" (ϵ independent) terms of Eq. (18) as

$$\begin{aligned} q_0 \frac{d\hat{\sigma}^a}{d^3q} (\text{CONSTANT}) &= e_f^2 2\alpha_s \frac{1}{q_T^4} \left\langle \frac{2}{g} \right\rangle \frac{\alpha_s}{2\pi} X_1 X_2 \\ \otimes \left\{ \frac{2X_1^2}{(X_1+X_2)^2} - \frac{(X_1+X_2)X_1}{1-X_1} + 4 \frac{X_1}{X_2} (X_1^2 + X_1X_2 + X_2^2) \right. \\ &\quad \left. - 2 \frac{X_1}{X_2} [2 + (1-X_1)^2 + (1-X_1-X_2)^2] \ln \frac{(1-X_1)(X_1+X_2)}{X_2} \right\} \end{aligned} \quad (24)$$

The second $1/\epsilon$ term in Eq. (18) is according to the definition of D_q^Y in Section 3.2

$$\begin{aligned} q_0 \frac{d\hat{\sigma}^a}{d^3q} (\text{FRAGMENTATION}) &= 2\alpha_s^2 \frac{1}{q_T^4} \left\langle \frac{2}{g} \right\rangle \frac{(X_1X_2)^2}{X_1+X_2} \frac{(X_1+X_2)^2 + X_1^2}{X_2^2} D_{q_f}^Y(X_1+X_2, q_T^2) \\ &\quad + e_f^2 2\alpha_s \frac{1}{q_T^4} \left\langle \frac{2}{g} \right\rangle \frac{\alpha_s}{2\pi} \frac{(X_1X_2)^2}{X_1+X_2} \frac{(X_1+X_2)^2 + X_1^2}{X_2^2} \\ \otimes \left(\ln \frac{1}{X_1X_2} - \frac{3}{2} \right) \cdot \frac{1 + (1-X_1-X_2)^2}{X_1+X_2} \end{aligned} \quad (25)$$

where the first part is the final state quark bremsstrahlung cross-section of Eq. (A12) with scale $Q^2 = q_T^2$ and the second part is the term arising from the correct substitution of the fragmentation function from definition (23). In numerical calculations we will treat the bremsstrahlung part separately and only the second part [denoted by $\hat{\sigma}$ (CONST. FRAG.)] contributes to the $qq \rightarrow qqY$ cross-section. The $\ln 1/X_1X_2$ term appears because of our choice of q_T^2 as the scaling variable $q_T^2 = \hat{s}X_1X_2$.

Finally, the first $1/\epsilon$ term in Eq. (18) is seen to contribute to the gluon distribution evolution. We first effect renormalization by dropping the $1/\epsilon + \ln 4\pi - \gamma_E$ term as in the case of deep inelastic scattering. At the hadron level, the contribution of the rest is given by [see Eq. (A7)]

$$q_0 \frac{d\hat{\sigma}^a}{d^3q} = 2\alpha\alpha_s \frac{1}{q_T^4} \langle \frac{1}{6} \rangle e_f^2 \int_{x_1}^{1-x_2} \frac{dx_1}{x_1} \left[\frac{x_1}{x_1} q_f \left(\frac{x_1}{x_1} \right) \right] \int_{x_2}^{1-x_1} \frac{dx_2}{x_2} \left[\frac{x_2}{x_2} q_{f'} \left(\frac{x_2}{x_2} \right) \right] \quad (26)$$

$$\otimes \frac{x_1^2 x_2}{1-x_1} [1 + (1-x_1)^2] \frac{\alpha_s}{2\pi} \frac{4}{3} \frac{1 + (1-z)^2}{z} \cdot \left[\ln \frac{q_T^2}{\mu^2} + \ln \frac{1-z}{zx_1} \right]$$

with $z = x_2/(1 - x_1)$. With our definition of the gluon distribution, only the $\ln q_T^2/\mu^2$ term contributes to the gluon evolution in the Compton cross-section of Eq. (A9), namely with $x = X_2/(1 - x_1)$, $y = X_2/x_2$ and $z = x/y$, we write

$$\Delta g(x, \mu^2) = \frac{\alpha_s}{2\pi} \ln \frac{q_T^2}{\mu^2} \int_x^1 \frac{dy}{y} P_{gq} \left(\frac{x}{y} \right) q(y) \quad (27)$$

Choosing now $\mu^2 = q_T^2$ gives the renormalization group improved Compton cross-section with $g(x, q_T^2)$ and $\alpha_s(q_T^2)$. The $\ln [(1-z)/zx_1]$ term contributes with our choice of the parton distributions as a finite term (denoted by $\hat{\sigma}$ (CONST. COMPTON)) to the $qq \rightarrow qq\gamma$ cross-section

$$q_0 \frac{d\hat{\sigma}^a}{d^3q} (\text{CONST. COMPTON}) = e_f^2 2\alpha\alpha_s \frac{1}{q_T^4} \langle \frac{2}{9} \rangle \frac{\alpha_s}{2\pi} \quad (28)$$

$$\otimes x_1^2 [1 + (1-x_1)^2] \cdot \left[1 + \left(1 - \frac{x_2}{1-x_1} \right)^2 \right] \ln \frac{1-x_1-x_2}{x_1 x_2}$$

We can now summarize our results for the calculation of the $qq \rightarrow qq\gamma$ Feynman diagrams of Fig. 1. We have separated the contribution to the final state quark bremsstrahlung process, the cross-section of which is given at the hadron level by (symmetrizing in x_1 and x_2)

$$q_0 \frac{d\hat{\sigma}}{d^3q} (\text{BREHSS.}) = 2\alpha_s^2 \frac{1}{q_T^4} \langle \frac{2}{9} \rangle \sum_{f,f'} \quad (29)$$

$$\otimes \int_{x_1}^{1-x_2} \frac{dx_1}{x_1} \left[\frac{x_1}{x_1} q_f \left(\frac{x_1}{x_1} \right) \right] \int_{x_2}^{1-x_1} \frac{dx_2}{x_2} \left[\frac{x_2}{x_2} q_{f'} \left(\frac{x_2}{x_2} \right) \right] \frac{x_1^2 x_2^2}{x_1 + x_2}$$

$$\otimes \left[D_{q_f}^{\gamma} (x_1 + x_2, q_T^2) \frac{(x_1 + x_2)^2 + x_1^2}{x_2^2} + D_{q_{f'}}^{\gamma} (x_1 + x_2, q_T^2) \frac{(x_1 + x_2)^2 + x_2^2}{x_1^2} \right]$$

The diagrams also contribute to the basic Compton cross-section of Eqs (A7)-(A10) which we should thus calculate with scale-breaking parton distributions $f(x, q_T^2)$ and with effective coupling $\alpha_s(q_T^2)$. Summing the contributions $\hat{\sigma}$ (CONSTANT), $\hat{\sigma}$ (CONST. FRAGM.) and $\hat{\sigma}$ (CONST. COMPTON) given in Eqs (24), (25) and (28) we obtain the final result

$$\begin{aligned}
 q_0 \frac{d\sigma}{d^3q} (qq \rightarrow qq\gamma) &= 2\alpha\alpha_s \frac{1}{q_T^4} \left\langle \frac{2}{3} \right\rangle \frac{\alpha_s}{2\pi} \cdot \sum_{f,f'} e_f^2 \\
 &\cdot \int_{x_1}^{1-x_2} \frac{dx_1}{x_1} \left[\frac{x_1}{x_1} q_f \left(\frac{x_1}{x_1} \right) \right] \int_{x_2}^{1-x_1} \frac{dx_2}{x_2} \left[\frac{x_2}{x_2} q_{f'} \left(\frac{x_2}{x_2} \right) \right] \cdot x_1 x_2 \\
 &\cdot \left\{ \frac{2x_1^2}{(x_1+x_2)^2} - \frac{(x_1+x_2)x_1}{1-x_1} + 4 \frac{x_1}{x_2} (x_1^2 + x_1 x_2 + x_2^2) \right. \\
 &- 2 \frac{x_1}{x_2} \left[2 + (1-x_1)^2 + (1-x_1-x_2)^2 \right] \ln \frac{(1-x_1)(x_1+x_2)}{x_2} + \frac{x_1 x_2}{x_1+x_2} \frac{(x_1+x_2)^2 + x_1^2}{x_2^2} \\
 &\cdot \frac{1 + (1-x_1-x_2)^2}{x_1+x_2} \left(\ln \frac{1}{x_1 x_2} - \frac{3}{2} \right) + \frac{x_1}{x_2} \left[1 + (1-x_1)^2 \right] \left[1 + \left(1 - \frac{x_2}{1-x_1} \right)^2 \right] \\
 &\cdot \ln \frac{1-x_1-x_2}{x_1 x_2} + \left[x_1 \leftrightarrow x_2 \right] \left. \right\} \quad (30)
 \end{aligned}$$

4. NUMERICAL CALCULATIONS

We shall now give a numerical comparison of the final $qq \rightarrow qq\gamma$ cross-section of Eq. (30) with the leading QCD contributions (Compton, annihilation and final quark bremsstrahlung) given by Eqs (A9)-(A13). First, some cautionary remarks must be made: even with our well defined prescription for obtaining the non-leading terms, our numerical estimates are, at best, approximate. For a consistent analysis, the parton distributions should be determined from deep inelastic scattering data using the next-to-leading order formulae^{11),12)}. However, even for valence quarks, the parton distributions are presently only leading logarithmic estimates.

Much more important is the uncertainty for the Compton cross-section due to the poor determination of the gluon distribution. However, we find that over most of the kinematical region and with rather different gluon parametrizations, the Compton scattering process dominates the real photon production and our qualitative conclusions concerning the smallness of the "constant" term will be quite general.

We shall use the parton distribution parametrization of Baier et al.²⁵⁾ (Case I) based on a Buras-Gaemers type fit to the CDHS data²⁶⁾ as well as that of Owens and Reya²⁷⁾ (Case II) based on counting rules and which also includes

leading scaling violations. The details of these parametrizations can be found in Appendix B. The resulting direct photon production cross-sections are shown in Figs 5 and 6 for $\sqrt{s} = 53$ GeV and $y = 0$. Predictions at other energies and rapidities turn out to be qualitatively the same. In Fig. 5 the predictions for the leading QCD processes are shown together with some of the experimental points of Refs 8) and 9). The uncertainty due to the gluon distribution is clearly seen; for the non-scaling parametrization II (curve C) the Compton cross-section is below the scaling prediction of Ref. 2) (curve A) by a factor of 20 at $q_T = 10$ GeV/c. The annihilation term is less than 15% of the Compton term at small q_T decreasing rapidly for all parametrizations. The bremsstrahlung cross-section (curve D) is essentially the same for the two parametrizations; it amounts to 15-30% of the Compton cross-section calculated with parametrization I, but is almost equal to it, at large p_T , for parametrization II. We have also calculated the bremsstrahlung term with the QCD leading approximation fragmentation function $D_q^Y(z, Q^2)_{\text{QCD}}^{(28)-(30)}$ using the parametrization (see Ref. 31))

$$D_{q_f}^Y(z, Q^2)_{\text{QCD}} = e_f^2 \frac{\alpha}{2\pi} \frac{0.08 + 1.12z - 0.20z^2}{z^2 (0.89 - 0.64 \ln(1-z))} \ln \frac{Q^2}{\Lambda^2} \quad (31)$$

The resulting cross-section is at all q_T about half of the result with the lowest order fragmentation function.

We come, finally, to the main concern of this work, namely the magnitude of the $qq \rightarrow qq\gamma$ cross-section. We have presented, in Fig. 6 separately, the contribution of the "constant" terms of Eq. (24) (curve C), the sum of the "constant" and "constant Compton" terms (curve D) and finally, the total $qq \rightarrow qq\gamma$ cross-section (curve E) together with the leading contributions. For the two parametrizations these curves are indistinguishable, reflecting the good determination of valence quark distributions from deep inelastic scattering.

The "constant" term is positive for $q_T > 4$ GeV/c decreasing slowly, but it is partly cancelled at large q_T by the "Compton constant" term. The "fragmentation constant" term is positive and of the same order of magnitude as the sum of the two previous terms (see curve E). The ratio of the final $qq \rightarrow qq\gamma$ cross-section to the leading QCD contributions is shown in Fig. 7 for $\sqrt{s} = 25$ GeV and $\sqrt{s} = 53$ GeV. For parametrization I it is less than 10% for the whole q_T range. For parametrization II we note an anomalous behaviour at the kinematical boundary ($q_T = 12.5$ GeV/c for $\sqrt{s} = 25$ GeV). This effect signals the breakdown of parametrization II at large x values as the limit $x_T \rightarrow 1$ is dominated by the behaviour of the parton distributions near $x = 1$.

5. DISCUSSION

We have chosen q_T^2 as the scale both in the parton distributions and the fragmentation functions but any other parameter could have been used provided our formulae are adequately modified. Choosing $\hat{s}$ (the sub-process centre-of-mass energy squared) amounts to a redistribution of terms between the leading and next-to-leading contributions, namely the logarithmic term of Eq. (28) becomes $\ln(1 - x_1 - x_2)$ and the term $\ln 1/x_1 x_2$ in Eq. (25) disappears. The parton distributions, however, are obtained from an analysis valid typically in the range $5 \leq Q^2 \leq 200 \text{ GeV}^2$. At ISR energies this corresponds roughly to the domain of variation of q_T^2 , whereas one has, at $\sqrt{s} = 53 \text{ GeV}$, $4q_T^2 \leq \hat{s} \leq 2800 \text{ GeV}^2$. Clearly, q_T^2 should be preferred. Also, another definition of the parton distributions would be possible, which would include the factorizable $\ln(1 - z)/z$ terms (see Eq. (5)). However, this would give a more complicated evolution of the parton distributions than the one given in Section 2.

In a recent paper Contogouris et al.³²⁾ arrived at conclusions quite different from ours concerning the magnitude of the $qq \rightarrow qq\gamma$ term. However, the results are not directly comparable because of their different regularization method and obviously different definition of parton distributions, which seems to introduce a large non-leading term to the gluon evolution. Furthermore, in taking the zero-mass limit of the corresponding virtual photon production cross-section they do not explicitly factorize the final quark fragmentation (i.e., bremsstrahlung) term. We have seen that for parametrization II this term can be rather important.

6. CONCLUSIONS

Our functional form for the non-leading contribution of the $qq \rightarrow qq\gamma$ process gives a small correction of order $\alpha_s(q_T^2)/2\pi$ to the direct photon production cross-section as can be expected. The correction term of Eq. (28) due to consistent treatment of the gluon distribution beyond leading order plays a significant role in changing the form of the final $qq \rightarrow qq\gamma$ cross-section at large q_T . However, all terms in the $qq \rightarrow qq\gamma$ cross-section lie clearly below the Compton cross-section, and our conclusions are not essentially sensitive to the choice of the gluon distribution parametrization except at the kinematical boundary.

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APPENDIX A

For deep inelastic scattering $P + q \rightarrow X$ ($p + q \rightarrow x$ at the parton level) we define the usual scaling variables

$$x = \frac{Q^2}{2P \cdot q}, \quad z = \frac{Q^2}{2p \cdot q} \quad (A1)$$

where P (p) is the four momentum of the incoming hadron (parton). From the parton model assumption $p = \xi P$ we get the relation $x = \xi z$.

The parton level structure functions $\hat{F}_i(z, Q^2)$ ($i = 1, 2$) are defined by the parton tensor $\hat{W}_{\mu\nu}$ decomposition

$$\hat{W}_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) \hat{F}_1 + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \frac{z}{p \cdot q} \hat{F}_2 \quad (A2)$$

The hadron level structure functions $F_i(x, Q^2)$ ($i = 1, 2$) are defined by the

$$F_i(x, Q^2) = \int_0^1 \frac{d\xi}{\xi} f(\xi) \hat{F}_i\left(\frac{x}{\xi}, Q^2\right) \quad (A3)$$

where $f(\xi)$ is the longitudinal momentum distribution of the parton in the incoming hadron. The normalization is determined by the basic $q\gamma^* \rightarrow q$ process (Fig. 3a) which gives the naive parton model result

$$\hat{F}_1(z, Q^2) = \hat{F}_2(z, Q^2) = \delta(1-z) \quad (A4)$$

and at the hadron level

$$F_i(x, Q^2) = q(x) \quad (A5)$$

For a large q_T process $P_1 + P_2 \rightarrow q + X$ ($p_1 + p_2 \rightarrow q + x$ at the parton level) we define the x variables by

$$X_i = \frac{q_T^2}{2P_i \cdot q}, \quad x_i = \frac{q_T^2}{2p_i \cdot q} \quad (A6)$$

where P_i (p_i) are the four momenta of the incoming hadrons (partons) related in the parton model by $p_i = \xi_i P_i$ resulting into $X_i = \xi_i x_i$ ($i = 1, 2$).

The relation between the cross-sections at the hadron and parton levels is

$$\begin{aligned}
 q^0 \frac{d\hat{\sigma}}{d^3q} (X_1, X_2, q_T^2) \\
 &= \sum_{j,k} \int \frac{d\xi_1}{\xi_1} \left[\xi_1 f_j'(\xi_1) \right] \int \frac{d\xi_2}{\xi_2} \left[\xi_2 f_k^2(\xi_2) \right] q^0 \frac{d\hat{\sigma}}{d^3q} (x_1, x_2, q_T^2) \\
 &= \sum_{j,k} \int_{X_1}^{1-X_2} \frac{dx_1}{x_1} \left[\frac{X_1}{x_1} f_j' \left(\frac{X_1}{x_1} \right) \right] \int_{X_2}^{1-X_1} \frac{dx_2}{x_2} \left[\frac{X_2}{x_2} f_k^2 \left(\frac{X_2}{x_2} \right) \right] q^0 \frac{d\hat{\sigma}}{d^3q} (x_1, x_2, q_T^2)
 \end{aligned} \tag{A7}$$

where $f(\xi)$'s are the parton distributions and where the integration limits are obtained from the parton level constraint

$$p_x^2 = (p_1 + p_2 - q)^2 \geq 0 \iff 1 - x_1 - x_2 \geq 0 \tag{A8}$$

In CMS($P_1 + P_2$) X_1 and X_2 are given by $X_1 = x_T e^y/2$, $X_2 = x_T e^{-y}/2$, where the transverse momentum variable x_T and rapidity are defined by $x_T = 2q_T/\sqrt{s}$ and $y = 1/2 \ln [(q_0 + q)/(q_0 - q)]$.

Next we will give a list of the basic QCD cross-sections contributing to direct photon production in terms of the x variables. The Compton cross-section of Fig. 2a is at the parton level

1° quark in hadron 1

$$q^0 \frac{d\hat{\sigma}}{d^3q} = e_f^2 2\alpha_s \left\langle \frac{1}{6} \right\rangle \frac{1}{q_T^4} (x_1 x_2)^2 \delta(1 - x_1 - x_2) \left[\frac{1}{x_2} + x_2 \right] \tag{A9}$$

2° gluon in hadron 1

$$q^0 \frac{d\hat{\sigma}}{d^3q} = e_f^2 2\alpha_s \left\langle \frac{1}{6} \right\rangle \frac{1}{q_T^4} (x_1 x_2)^2 \delta(1 - x_1 - x_2) \left[\frac{1}{x_1} + x_1 \right] \tag{A10}$$

and the annihilation cross-section of Fig. 2b is at the parton level

$$q^0 \frac{d\hat{\sigma}}{d^3q} = e_f^2 2\alpha_s \left\langle \frac{4}{9} \right\rangle \frac{1}{q_T^4} (x_1 x_2)^2 \delta(1 - x_1 - x_2) \left[\frac{x_1}{x_2} + \frac{x_2}{x_1} \right] \tag{A11}$$

The final quark bremsstrahlung cross-section (Fig. 2c) is for different quark flavours given by

1° bremsstrahlung from quark (flavour f) in hadron 1

$$q^0 \frac{d\hat{\sigma}}{d^3q} = 2 \alpha_s^2 \frac{1}{q_T^4} D_{q_f}^{\gamma}(x_1+x_2) \left\langle \frac{2}{g} \right\rangle \frac{(x_1 x_2)^2}{x_1+x_2} \frac{(x_1+x_2)^2 + x_1^2}{x_2^2} \quad (A12)$$

2° bremsstrahlung from quark (flavour f') in hadron 2

$$q^0 \frac{d\hat{\sigma}}{d^3q} = 2 \alpha_s^2 \frac{1}{q_T^4} D_{q_{f'}}^{\gamma}(x_1+x_2) \left\langle \frac{2}{g} \right\rangle \frac{(x_1 x_2)^2}{x_1+x_2} \frac{(x_1+x_2)^2 + x_2^2}{x_1^2} \quad (A13)$$

where the fragmentation function D_q^{γ} is given in the leading approximation by the formula

$$D_{q_f}^{\gamma}(z, Q^2) = \frac{\alpha}{2\pi} e_f^2 \frac{1 + (1-z)^2}{z} \ln \frac{Q^2}{\Lambda^2} \quad (A14)$$

APPENDIX B

We quote here the details of the parton distribution parametrizations used in Section 4.

Case I [Ref. 25)]

The distributions are parametrized as $x p_i(x, Q^2) = C_i(Q^2) x^{\eta_i^i(Q^2)} (1-x)^{\eta_2^i(Q^2)}$, where for valence quarks ($u_v = 2d_v$)

$$\begin{aligned} C_v(Q^2) &= 1/B(\eta_1^v, \eta_2^v + 1) \\ \eta_1^v(Q^2) &= 0.56 - 0.92 \bar{s} \cdot 4/25 \\ \eta_2^v(Q^2) &= 2.71 + 5.08 \bar{s} \cdot 4/25 \end{aligned} \tag{B1}$$

with $\bar{s} = \ln[(\ln Q^2/\Lambda^2)/(\ln Q_0^2/\Lambda^2)]$, $Q_0^2 = 5 \text{ GeV}^2$ and $\Lambda = 0.47 \text{ GeV}$. For gluons (now with $\Lambda = 0.6 \text{ GeV}$)

$$\begin{aligned} C_g(Q^2) &= 2.01 - 3.56 \bar{s} + 1.98 \bar{s}^2 \\ \eta_1^g(Q^2) &= -1.13 \bar{s} + 0.48 \bar{s}^2 \\ \eta_2^g(Q^2) &= 2.9 + 5.08 \bar{s} \cdot 4/25 \end{aligned} \tag{B2}$$

For sea quarks ($\bar{q} = \bar{u} = \bar{d} = \bar{s} = s$)

$$\begin{aligned} C_{\bar{q}}(Q^2) &= 0.99 + 0.72 \bar{s} + 0.96 \bar{s}^2 \\ \eta_1^{\bar{q}}(Q^2) &= 0. \\ \eta_2^{\bar{q}}(Q^2) &= 8.10 - 1.49 \bar{s} + 5.10 \bar{s}^2 \end{aligned} \tag{B3}$$

Case II [Ref. 27)]

For valence quarks the same parametrization is used as above with

$$\begin{aligned} \eta_1^v(Q^2) &= 0.7 - 0.176 \bar{s} \\ \eta_2^v(Q^2) &= 2.6 + 0.80 \bar{s} \end{aligned} \tag{B4}$$

and now $\bar{s}$ is defined at $Q_0^2 = 1.8 \text{ GeV}^2$ and $\Lambda = 0.5 \text{ GeV}$.

For gluon and sea quarks, the following functional form is used

$$x p_i(x, Q^2) = A_i (1-x)^{\eta_i} + A'_i (1-x)^{\eta'_i} + B_i e^{-C_i x} \quad (B5)$$

with the parameters A_i , etc. (second order polynomials in $\bar{s}$) given in the original publication.

In both parametrizations we use the value $\alpha_s(Q^2) = 12\pi/25 \ln Q^2/\Lambda^2$ for the effective strong coupling.

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FIGURE CAPTIONS

- Fig. 1 : The quark-quark scattering Feynman diagrams: $qq \rightarrow qq\gamma$
a) photon from quark flavour f
b) photon from quark flavour f' .
- Fig. 2 : The basic QCD sub-processes for direct photon production
a) the Compton diagrams: $qg \rightarrow q\gamma$
b) the annihilation diagrams: $q\bar{q} \rightarrow g\gamma$
c) the quark-quark scattering diagrams $qq \rightarrow qq$ with final state quark fragmentation into a real photon.
- Fig. 3 : Feynman diagrams for deep inelastic scattering
a) basic $q\gamma^* \rightarrow q$ process
b) gluon contribution: $g\gamma^* \rightarrow q\bar{q}$
- Fig. 4 : Feynman diagrams for $e^+e^- \rightarrow q\bar{q}\gamma$
- Fig. 5 : The direct photon production cross-section $q_T^4 d\sigma/dy d^2q_T$ for pp collisions at $\sqrt{s} = 53$ GeV and $y = 0$. The Compton (+ annihilation) cross-section is shown for three different parton distribution parametrizations: the scaling parametrization of Ref. 2) with $\alpha_s = 0.3$ (curve A) and the non-scaling parametrizations given in Appendix B; Case I (curve B) and Case II (curve C). The bremsstrahlung cross-section (curve D) is calculated with parametrization I and with $D_q^\gamma(z, q_T^2/\Lambda^2)$ of Eq. (23) ($\Lambda = 0.47$ GeV). Also shown are experimental points from Refs 8) and 9) obtained with the fit:
 $E d\sigma/d^3p(pp \rightarrow \pi^0 X) = 8.78 \cdot p_T^{-7.8} \cdot e^{-0.51 p_T}$ mb/GeV².
- Fig. 6 : The direct photon production cross-section $q_T^4 d\sigma/dy d^2q_T$ for pp collisions at $\sqrt{s} = 53$ GeV and $y = 0$. The Compton + annihilation + bremsstrahlung cross-section with parametrizations I (curve A) and II (curve B). The "constant" part of the $qq \rightarrow qq\gamma$ cross-section [Eq. (24), (curve C)] sum of the "constant" and "Compton constant" parts [Eqs (24) and (28) (curve D)] and the final $qq \rightarrow qq\gamma$ cross-section [Eq. (30) (curve E)] all with parametrization II.
- Fig. 7 : The ratio of the final $qq \rightarrow qq\gamma$ cross-section to the sum of the basic QCD cross-sections, $R = \sigma(qq \rightarrow qq\gamma)/\sigma(\text{Compton} + \text{annih.} + \text{bremsstr.})$, at $\sqrt{s} = 53$ GeV and $\sqrt{s} = 25$ GeV. The full lines are for parametrization I and the dashed lines are for parametrization II.

FIG. 1

FIG. 2

FIG. 3

FIG. 4

FIG. 5

FIG. 6

FIG. 7