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REVIEW OF THE INVESTIGATIONS ON THE BEAM-BEAM
INTERACTIONS AT THE ISR

by

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TABLE OF CONTENTS

1. INTRODUCTION
2. ELECTROMAGNETIC FIELDS DUE TO A PROTON BEAM
3. BEAM-BEAM TUNE SHIFT
4. NON-LINEAR RESONANCES
5. STRONG NON-LINEARITIES
6. OVERLAP KNOCK-OUT RESONANCES
7. CONCLUSIONS

1. INTRODUCTION

At the ISR, an extensive theoretical and experimental programme on the beam-beam interactions has been carried out. For coasting beams, analytical theories for beam-beam tune shifts and non-linear resonances have been developed and checked experimentally. Other theories, such as for Arnold diffusion, stochasticity and migration due to adiabatic change of the parameters have also been studied experimentally. The resonance theory has been generalized to the case of bunched beams, and the resonances due to the so-called beam-beam overlap knock-out have been observed in some detail. The present paper tries to summarize this work and to give the most interesting experimental results.

2. ELECTROMAGNETIC FIELDS DUE TO A PROTON BEAM

When two beams are crossing, a particle of beam 1 sees the electric field due to the charges in beam 2 and the magnetic field associated with the motion of these charges. The electric and magnetic forces are proportional and this may be shown from Maxwell's equations for a cylindrical beam of any section.

$$\begin{aligned} \oint \vec{E} \cdot d\vec{\ell} &= \iint \frac{\rho_e}{\epsilon_0} d\sigma = \frac{1}{\epsilon_0} c^2 \iint \rho_e d\sigma \\ \oint \vec{B} \cdot d\vec{\ell} &= \mu_0 \oint \vec{H} \cdot d\vec{\ell} = \mu_0 v \iint \rho_e d\sigma \end{aligned} \quad (1)$$

where ρ_e is the charge density of beam 2, ℓ is distance around perimeter of cylinder and $d\sigma$ an element of area of the cylinder cross-section.

Eqs (1) show that the fields are in a ratio v/c^2 . Since the magnetic force is proportional to the longitudinal speed v , the ratio between the forces is $v^2/c^2 = \beta^2$. Hence, if Φ is the electric potential, the effects of the electromagnetic fields can be described by

a total potential equal to

$$\Phi_{\text{tot}} = (1 \pm \beta^2) \Phi \quad (2)$$

The positive and negative signs are associated with antiparallel and parallel motion of the two beams, respectively. Using the Hamiltonian formalism, the Hamiltonian function for the perturbation can be calculated for an electric potential¹⁾

$$H = \frac{\Phi_{\text{tot}}}{\beta c |B\rho|} + \frac{1}{2} \left(\frac{\Phi_{\text{tot}}}{c |B\rho|} \right)^2 \cong \frac{\Phi_{\text{tot}}}{\beta c |B\rho|} = \frac{(1 \pm \beta^2) \Phi}{\beta c |B\rho|} \quad (3)$$

where $|B\rho|$ is the magnetic rigidity for beam 1 ($=p_1/e$).

Eqs (2) and (3) are basic expressions which enable the beam-beam interaction problems to be solved, provided that Φ can be calculated. For colliding beams, the positive sign is valid.

3. BEAM-BEAM TUNE SHIFT

The first effect to be considered is the focussing change associated with the electromagnetic fields of a beam. There is a linear effect due to a field gradient and commonly characterized by the linear tune shift, which can be calculated from²⁾

$$\Delta Q_y = \frac{1}{4\pi} \int_0^C \beta_y K_y ds \quad (4)$$

where y stands for x or z, s is the longitudinal coordinate and K_y is the focussing force. In the case of interest, this force has to be deduced from Eq. (3),

$$K_y = \frac{1 + \beta^2}{\beta c |B\rho|} \frac{\partial^2 \Phi}{\partial y^2} \quad (5)$$

Φ being given by $\text{div } \overrightarrow{\text{grad } \Phi} = \rho_e / \epsilon_0$.

Starting from Eqs (4) and (5), different analytical expressions of ΔQ_y have been deduced, depending on the complexity of the beam model and on the geometry of the interaction region. For instance, when two beams are crossing at zero angle, the electric field can be calculated for an elliptic beam of Gaussian distribution, as in the space charge theory³⁾. It is even possible to integrate the contribution of elliptic pulses centred at different radial positions, in order to obtain the field of a wide stack³⁾. More relevant to the ISR is the case where the two beams cross horizontally at a small but non-zero angle. In this case, the geometry is more complicated but the beam model may be simplified. Analytical expressions of ΔQ_y have been worked out for a cylindrically symmetric Gaussian beam⁴⁾ and are given in an integral form,

$$\Delta Q_y = \frac{r_p E_0}{4 \pi e c^2} \frac{1 + \beta^2}{\beta^2 p_1} \frac{I_2 \beta_{y1}^* d}{\sigma_{y2}^{*2}} J_y(\xi, \omega) \quad (6)$$

where d is the interaction length, I_2 the beam 2 current, σ_{y2}^* the r.m.s. radius of beam 2, p_1 the momentum of beam 1, β_{y1}^* the betatron amplitude of beam 1 at the crossing point (taken as a local minimum) and

$$\xi = \frac{d}{2 \beta_{y1}^*}$$

$$\omega = \frac{\beta_{y1}^*}{\sigma_{y2}^*} \sin \psi$$

ψ being the crossing angle. In the ISR conditions where $\omega \gg 1$ and $\omega \gg \zeta$, the integral J_x vanishes while J_z is equal to $\sqrt{2\pi}/\zeta\omega$. Hence, the horizontal tune shift is zero but the vertical one is given by

$$\Delta Q_z = \frac{r_p E_o}{\sqrt{2\pi} e c^2} \frac{1 + \beta^2}{\beta^2 p_1} \frac{I_2 \beta_{z1}^*}{\sigma_{z2}^* \sin \psi} \quad (7)$$

where the physical constants have their standard meaning.

So far, only the linear tune shift was considered, but non-linear effects on off-centred particles do exist. If the two beams are vertically steered, the first non-linear field shifting the tune is the octupole component and this tune shift has been shown to be opposed to the linear one⁵⁾. Hence the total shift seen by the particles of beam 1 is distributed between zero and ΔQ_z (Eq. (7)) around an average value calculated to be⁵⁾

$$\Delta Q_{z,tot} \cong 0.7 \Delta Q_z \quad (8)$$

Combining (7) and (8) gives for the ISR

$$\Delta Q_{z,tot} = 2.48 \cdot 10^{-7} \frac{I_2 \beta_{z1}^*}{\gamma_1 h_{eff2}} \quad (9)$$

where γ_1 is the relativistic parameter of beam 1 and h_{eff2} is the effective height of beam 2 ($= 2\sqrt{\pi} \sigma_{z2}^*$).

Some measurements of the total tune shift have been done at 26 GeV/c for different current levels, as shown in Table 1.

Table 1: Total tune shifts measured in the ISR

Line	I ₁	I ₂	h _{eff2}	Constant	Comments
ELSA	6 A	23 A	5.0 mm	2.18 10 ⁻⁷	Not optimum steering
	Pulse	32 A	5.5 mm	2.78 10 ⁻⁷	Not optimum steering
FP	1.6 A	18.5 A	4.4 mm	2.22 10 ⁻⁷	Optimum steering
	5.8 A	11.2 A	7.0 mm	2.67 10 ⁻⁷	Optimum steering

The agreement between measurements and Eq. (9) is very good.

4. NON-LINEAR RESONANCES

Having the Hamiltonian (3) and following the perturbation treatment presented in Refs 1 and 6, it is possible to calculate the bandwidths Δe associated with isolated resonances. For an isolated resonance given by

$$e = n_x Q_x + n_z Q_z - p = 0, \quad N = n_x + n_z \quad (10)$$

the bandwidth can be written^{1, 6)}

$$\Delta e = \frac{(1+\beta^2) E_x^{n_x/2} E_z^{n_z/2}}{2^{(N-1)} \pi^{N/2} \beta c |B\rho| n_x! n_z!} \left(\frac{n_x^2}{E_x} + \frac{n_z^2}{E_z} \right) \quad (11)$$

$$\int_0^C \beta_x^{n_x/2} \beta_z^{n_z/2} \frac{\partial^N \Phi}{\partial x^{n_x} \partial z^{n_z}} \exp [i(n_x \mu_x + n_z \mu_z - e \frac{s}{R})] ds$$

where E_x and E_z are the transverse emittances of a particle in beam 1, μ_x and μ_z the phase advances, and R the radius of the machine. Here again, the final expressions for Δe depend on the beam model and on the geometry of the interaction region. In the case mentioned in Section 3 where the two beams cross horizontally at a small angle, the bandwidths become⁷⁾

$$\Delta e = K_{n_x n_z} \Delta Q_z J_{n_x n_z}(\xi, \omega) \quad (12)$$

where $K_{n_x n_z}$ is a numerical factor, ΔQ_z is the linear tune shift given by Eq. (6), $J_{n_x n_z}$ is a correction factor of integral form and ξ and ω are the parameters defined in Section 3. In the ISR, where $\omega \gg 1$ and $\omega \gg \xi$, the correction factor $J_{n_x n_z}$ vanishes when $n_x \neq 0$ and is close to one and to z/σ_{z2}^* for the vertical resonances of even and odd order, respectively. Hence, vertical beam-beam resonances are dominant in the ISR and their bandwidths are proportional to the linear tune shift (Eq. (7)).

$$\begin{aligned} \Delta e &= \frac{N}{2^{(N/2-2)} (N-1)(N/2-1)!} \Delta Q_z \quad \text{for } N \text{ even} \\ \Delta e &= \frac{2N}{2^{(N-1)/2} \left(\frac{N-1}{2}\right)!} \frac{z}{\sigma_{z2}^*} \Delta Q_z \quad \text{for } N \text{ odd } \frac{z}{\sigma_{z2}^*} \ll 1 \end{aligned} \quad (13)$$

where z is the vertical distance between particle 1 and beam 2. From the bandwidth (13), it is possible to calculate the effects either of a rapid, non-adiabatic crossing of the resonance^{1, 6)} or of an adiabatic diffusion of particles into the resonance⁸⁾. The vertical size increase due to non-adiabatic crossing^{1, 6)} is approximated by

$$\frac{\Delta\sigma_z}{\sigma_z} = \sqrt{\frac{\pi c}{2R}} \frac{\Delta e}{N^{3/2} \sqrt{dQ_z/dt}} \quad (14)$$

while the loss rate associated with a tune diffusion can be estimated from⁸⁾

$$\left(\frac{dN_p}{N_p dt} \right)_{\max} = - \frac{P_T D_Q}{\Delta Q \delta Q} \quad (15)$$

where P_T is the trapping probability, D_Q the tune diffusion constant, ΔQ the total tune spread in the beam and δQ the tune change which causes the trapped particles to reach the walls.

It is interesting to calculate the orders of magnitude of the resonances and of their effects, for an intense beam (30 A) at 26 GeV/c with a size of $\sigma_{z2}^* = 1$ mm and a vertical distance of $z = 0.2$ mm. The results are summarized in Table 2, assuming either a rapid change of Q of 0.16 s^{-1} or a tune diffusion associated with an intrabeam momentum diffusion of $6.7 \cdot 10^{-11} \text{ s}^{-1}$.

Table 2: Calculated bandwidths, blow-ups and loss rates for beam-beam resonances.

Order N	4	5	6	7	8
Bandwidth Δe	$1.9 \cdot 10^{-3}$	$3.6 \cdot 10^{-4}$	$4.0 \cdot 10^{-4}$	$8.2 \cdot 10^{-5}$	$6.1 \cdot 10^{-5}$
Blow-up $\frac{\Delta\sigma_z}{\sigma_z}$	1.052	0.143	0.121	0.020	0.012
Loss rate $\frac{dN_p}{N_p dt}$ (ppm/min)	50	35	22	9	5
Working line	4C	5C	ELSA	ELSA	8C

In comparison with these beam-beam bandwidths, the measurement of the vertical 4th order resonance, made by octupole compensation, gives a bandwidth due to the machine hardware equal to $4.5 \cdot 10^{-6}$. In other words, the resonance excitation produced by machine hardware is not really harmful in the ISR, compared with the beam-beam resonance excitation. Another experiment shows this very well. The Schottky signal is monitored during a noise excitation of the beam and the transfer function is calculated using the Fast Fourier Transform⁹⁾. The amplitude of this function is locally modified by the presence of resonances in the stack. Fig. 1 shows that hardware resonances are not visible, while beam-beam resonances appear with a 15 A stack in the other ring. This technique should even give the possibility of measuring the resonance band width, provided that the signal is not too small, and this will be tried in a near future.

It is also very interesting to compare the estimated loss rates of Table 2 with the loss rates observed in the ISR on different working lines, which cross resonances of different orders. The results, which do not depend on the actual beam current, but on the momentum p_1 and on $z/\sigma_{z_2}^*$, are given in Figs 2 to 4. The measured loss rates and their dependence with the order of the resonances present in the stack are in fairly good agreement with the calculated values.

5. STRONG NON-LINEARITIES

When the phase space energy or the nonlinearity is very large, the motion becomes unstable and the trajectories behave in a random fashion. In this case, stochastic regions start to appear in the phase space and the trajectories there have a strong dependence on the initial conditions. For the transverse motions in a synchrotron, stochastic regions appear first in the vicinity of the unstable fixed points and near the separatrices. The area covered by these regions increases with the nonlinearity and, at very high non-

linearity, the stochastic regions tend to occupy the whole available phase space. When looking for instance at the percentage of the phase space occupied by regular trajectories as a function of the nonlinearity, it seems that there is some sort of threshold, above which this percentage decreases strongly (Fig. 5). In terms of accelerator parameters, this threshold is reached when the sum of all the bandwidths within a unit square of the tune plane is equal to the area of the square. Since the bandwidths are proportional to the linear beam-beam tune shift (Eq. (13)), this condition corresponds to an upper limit for ΔQ_z . Because of intrabeam scattering and Arnold diffusion, it has been suggested that the practical limit is even lower^{10, 11)} and the limit adopted for proton machine design is

$$\Delta Q_z < 0.005 \quad (16)$$

In order to explore the beam behaviour around or above this limit, tentatives of increasing ΔQ_z (7) by varying one of the critical parameters I_2 , p_1 or β_{z1}^* , have been made. The first experiment was to collide a weak beam at 2 GeV/c with a strong beam at 26 GeV. This was however not really conclusive because of the strong effect of intrabeam scattering on the lifetime of the weak beam, but it showed that there is no noticeable effect on the lifetime for a tune shift of ~ 0.005 . The second experiment¹³⁾ simulated very strong beam-beam effects using a nonlinear lens, i.e. a pair of bars inside the vacuum chamber. This lens had the symmetry of the space charge corresponding to head-on collision and excites all resonances (10) for which n_x and n_z are even. When ΔQ_z changes between 0 and 0.09, the decay rate changes by about 3 orders of magnitude and depends strongly on the tunes. Fig. 6 shows that the decay rate saturates at tune shifts close to the stochasticity limit (reached at $\Delta Q_z = 0.05$ as shown by computer simulation). The third experiment¹⁴⁾ became possible with the installation of a low- β insertion in the ISR. The idea was to use

this installation to increase β_{z1}^* up to 200, in order to avoid the difficulties of the two earlier experiments, i.e. the intrabeam scattering problems at too low energies and the difficult question of whether the nonlinear forces of the lens bear any resemblance to those of a beam. The measurements of the decay rate or of the beam lifetime have shown that both vary exponentially with the tune shift (Fig. 7). At the limit of $\Delta Q_z = 0.005$ the lifetime is of the order of 100 hours and at the ISR values (< 0.001) the lifetime is around 1000 hours. Extrapolating these data to the approximate limit for electrons (0.06), gives a lifetime of the order of milliseconds, which is typically the order of the damping time.

These experiments do not show the existence of a well-defined threshold, but indicate that below the chosen limit (16) for non-radiating particles the beam lifetime is long enough (> 100 hours) for storage rings.

6. OVERLAP KNOCK-OUT RESONANCES

In a synchrotron, the Hamiltonian (3) is always periodic since the machine is circular, and Fourier's development of this function gives rise to isolated resonances of the type (10). In the case where the perturbation, e.g. the electric potential Φ , varies with time, this variation can also be analysed in Fourier's series and a new frequency spectrum is thus superposed upon the spectrum due to the machine periodicity. This superposition corresponds to the appearance of a fine structure in the resonance lines and the exact treatment¹⁾ shows that the new resonance condition is

$$e = n_x Q_x + n_z Q_z - p - b \frac{f_{per}}{f_{rev}} = 0 \quad (17)$$

where f_{per} is the basic perturbation frequency and f_{rev} is the revolution frequency of the particle of beam 1.

The bandwidth of such a resonance is given by an expression similar to Eq. (11), but multiplied by the harmonic coefficient c_b of the time varying part of Φ .

In the case of a bunched beam interacting with a coasting beam, the associated electric potential Φ varies with time at a given position of the circumference, since Φ is proportional to the bunched beam current I_2 . The revolution frequency of the bunched beam is then f_{per} and the coefficients c_b are the harmonic amplitudes of the bunch spectrum. According to whether the two beams are in the same ring or in two different rings, the corresponding perturbation is called single-beam or two beam overlap knock-out respectively. For single-beam overlap knock-out, the electric potential Φ_{tot} is essentially due to the coupling forces induced by the bunches via the surrounding material and analytical predictions are difficult. For two-beam overlap knock-out, on the contrary, there is a finite number of rather short interaction regions and Φ_{tot} is essentially due to direct space charge forces of equation (2). Algebraic treatment becomes possible, in particular when the crossing angle is not vanishing. This has been done in the ISR and the expressions obtained for the bandwidths are¹⁾

$$\begin{aligned} \Delta e(Q_z = p + b \frac{f_{\text{per}}}{f_{\text{rev}}}) &= \sum_{\text{c.p.}} K \operatorname{erf}\left(\frac{z}{\sqrt{2} \sigma_{z2}^*}\right) \exp\left[i\left(\mu_z - e \frac{s}{R}\right)\right] \\ \Delta e(NQ_z = p + b \frac{f_{\text{per}}}{f_{\text{rev}}}) &= \sum_{\text{c.p.}} \frac{K N^2}{\sqrt{\pi} \sqrt{2}^{(N-3)} N!} \exp\left(-\frac{z^2}{2 \sigma_{z2}^{*2}}\right) \\ &\quad H_{N-2}\left(\frac{z}{\sqrt{2} \sigma_{z2}^*}\right) \exp\left[i\left(N\mu_z - e \frac{s}{R}\right)\right] \end{aligned} \quad (18)$$

where

$$K = \frac{M_0 (1 + \beta^2)}{4 \pi \beta^2} \frac{I_2 \beta_{z1}^* c_b}{|B_p| \sigma_{z2}^* \sin \psi} = \frac{e^2 c^2 M_0 c_b}{2 \sqrt{2} \pi E_0 r_p} \Delta Q_z$$

and H_n are the Hermite polynomials, which satisfy the recursion formula

$$H_{n+1}(x) = 2x H_n(x) - 2n H_{n-1}(x)$$

The positions in the tune diagram of the overlap knock-out resonances (18) can be calculated from Eq. (17), using the fact that $n_x = 0$ and that b is a multiple of the RF harmonic number h , i.e. $b = nh$ (the side bands due to the fact that only 20 bunches occupy the 30 buckets can be neglected in a first approximation). If the two beams are at the same energy, the resonance positions are given in Fig. 8 for the ISR working line ELSA.

Other conditions of overlap knock-out can exist in the ISR, when the two beams have different energies and when the exciting beam consists of empty buckets inside an accelerated stack (the sole harmonics $b = nh$ are then present).

This is for instance the case when one beam is stacked at 26 GeV/c or accelerated between 26 and 31 GeV/c, while the other one is stable at 31 GeV/c. The resonance lines then appear at different positions in the stack and Table 3 summarizes very briefly which resonances may be harmful.

Table 3: Overlap knock-out resonances in the ISR during stacking and acceleration against a 31 GeV/c beam.

Order N of the resonance	Beam 2 conditions (Beam 1 at 31 GeV on ELSA)	Values of n ($b=nh$) present in beam 1	Beam 1 regions where resonances appear
1	Stacking at 26 GeV/c	> 11	Top
	Acceleration to 31 GeV/c	> 7	Top and bottom
2 to 5	Stacking and acceleration	> 10	
6	Stacking and acceleration	> 8	Bottom
	Stacking and acceleration	> 3	Bottom, if beam 1 line is down to $Q_z = 8.840$
7 to 9	Stacking and acceleration	> 1	Bottom

The strength of these resonances depends on the values of the coefficients c_b (18). In the case of bunches, the bunch frequency spectra are given in Fig. 9 for two different RF voltages, i.e. two different bunch lengths. The coefficients c_b , with $b = nh$, become negligible for $n \geq 7$ and $n \geq 5$, depending on the voltage. Hence, a reduction of the cavity voltage from 16 kV to 4 kV as soon as the bunches are trapped and a slight reduction of the tunes by 0.015 (Fig. 8) during stacking practically eliminates the effects of overlap knock-out for two beams at the same energy. In the case of acceleration with empty buckets, the harmonic c_h is predominant

and the coefficients c_{nh} become very small for $n > 3$. Hence, it seems that only resonances of order 6 to 9 (Table 3) may be harmful during acceleration, at the bottom of the stable beam at 31 GeV/c.

Vertical blow-up and beam losses have effectively been observed at the top of ELSA, for two beams at 26 GeV/c and a cavity voltage of 16 kV, and at the bottom of a 31 GeV/c beam, when stacking and accelerating in the other ring, in agreement with these deductions. Specific measurements of current losses have also been made with an aperture limited stack on ELSA and a bunched pulse at injection, both at 26 GeV/c. The loss dependence on the RF voltage and on the tune value has been measured. Table 4 gives the results obtained for single beam overlap knock-out resonances.

Table 4: Single beam overlap knock-out resonance effects

Tune dependance				RF voltage dependence	
Q_x	Q_z	Plane	dI/dt^*	V_{RF} (kV)	dI/dt^*
8.955	8.935	H	17	0.7	0.2
		V	12	4.0	8.2
8.975	8.955	H	25	8.0	17.5
		V	32	16.0	25.0

* in arbitrary units

Similar measurements have been done for two-beam (26 GeV/c) overlap knock-out resonances, Since the two beams interact at the crossing points only, vertical resonances are excited and vertical beam separations become very important. Since Q_z is close to 9, large bumps in the 8 intersections cancel the effects (good bumps)

while large bumps of opposite sign in diametrically opposed intersections are additive (bad bumps). Table 5 summarizes the effects on beam losses of the tune, of the RF voltage and of the bumps.

Table 5: Two-beam overlap knock-out resonance effects.

Tune dependence		RF voltage dependence		Bump dependence	
Q_z	dI/dt^*	V_{RF} (kV)	dI/dt^*	Bumps	dI/dt^*
8.900	30	0.7	6	zero	7.0
8.920	95	4.0	13	2 mm "good"	3.0
8.940	200	8.0	110	4 mm "good"	0.7
8.950	> 250	16.0	150	4 mm "bad"	145

* in arbitrary units

In another experiment, a small and vertically limited beam of ~ 80 mA was left circulating at the top of ELSA while moving bunches in the other ring. Enhancements of current loss when overlap knock-out resonances are crossed, have been observed (Fig. 10).

It is finally interesting to mention the measurement made in the ISR of two bunched beam overlap knock-out resonances¹⁵⁾. The measured effects are indeed very strong under the conditions of the experiment, since the first harmonic c_h excited the resonances and since the resonances were crossed many times because of the synchrotron oscillations. Fig. 11 gives an example of the observation of a dipole resonance, the two beams having different energies, i.e. 11 and 26 GeV/C. This figure shows that a bunched beam of only ~ 150 mA induced by overlap knock-out a loss of ~ 30 % in the other ring.

7. CONCLUSIONS

The theories, briefly presented here, can satisfactorily account for all the beam-beam effects observed in the ISR. In the case of strong non-linearities, experiments have shown that there is no noticeable effect on the beam lifetime below the threshold deduced from the stochasticity theory and adopted for machine design. It is perhaps interesting to note that most of the presented theories can be applied to electron beams and that some experiments, such as overlap knock-out measurements, may give useful informations for electron storage rings.

REFERENCES

1. G. Guignard, A general treatment of resonances in accelerators, CERN 78-11 (1978).
2. E.D. Courant and H.S. Snyder, Theory of alternating-gradient synchrotron, Ann. Phys. (USA) 3, 1 (1958).
3. B. Zotter, Q-shift due to direct space-charge field of stacked particle beams, CERN ISR-TH/75-5 (1975).
4. E. Keil, C. Pellegrini, A.M. Sessler, Tune shifts for particle beams crossing at small angles in the low- β section of a storage ring, CERN ISR-TH/73-44 (1973).
5. E. Keil, Non-linear space charge effects, CERN ISR-TH/72-7 (1972).
6. G. Guignard, The general theory of all sum and difference resonances in a three-dimensional magnetic field in a synchrotron, CERN 76-06 (1976).
7. P.M. Hanney and E. Keil, The width of non-linear resonances excited by beams crossing at small angles in low- β sections, CERN ISR-TH/73-55 (1973).
8. M. Month, Beam loss in a coasting beam from a high-order isolated resonance, Proc. 9th Internat. Conf. on High Energy Accelerators, Stanford, 1974, (CONF 740522, USAEC, Washington, 1974), p. 402.
9. J. Borer, G. Guignard, A. Hofmann, E. Peschardt, F. Sacherer, B. Zotter, Information from beam response to longitudinal and transverse excitation, presented at the 1979 Part. Acc. Conf., San Francisco.
10. B.V. Chirikov, Research concerning the theory of non-linear resonance and stochasticity, Nuclear Physics Institute, Novosibirsk, Report 267, 1969 (Translation, CERN 71-40, 1971).
11. E. Keil, Incoherent space charge phenomena in the ISR, Proc. 8th Internat. Conf. on High Energy Accelerators, CERN, 1971, p. 372.
12. K. Hübner, A search for beam-beam effect in the ISR, Proc. 9th Internat. Conf. on High Energy Accelerators, Stanford, 1974, (CONF 740522, USAEC, Washington, 1974), p. 63.
13. E. Keil and G. Leroy, Effects of a non-linear lens on the stored proton beam in the ISR, IEEE Trans. Nuclear Sci. NS-22, 1370 (1975).
14. B. Zotter, Experimental investigation of the beam-beam limit of proton beams, Proc. 10th Internat. Conf. on High Energy Accelerators, 1977, Vol. 2, p. 23.
15. S. Myers, Overlap knock-out resonances with colliding bunched beams in the CERN ISR, presented at the 1979 Part. Acc. Conf., San Francisco.

Fig 1: Amplitude of the vertical transfer function of beam 1 as a function of the beam 2 current, showing beam-beam resonances of order 7 to 9.

Fig 2: Loss rates observed on the 5C line in the ISR run 353 with 8 A and 9 A stacks at 26 GeV/c. The initial effective height was 3.2 mm and the vacuum pressure was between 2.5 and 3×10^{-11} Torr.

Fig 3: Loss rates observed on the ELSA line in the ISR run 643 with 25 A and 24 A stacks at 26 GeV/c. The bottom of the stack crossed 7th order resonances, the initial effective height was 4 mm and the vacuum pressure was between 1.5 and 2×10^{-11} Torr.

Fig 4: Loss rates observed on the 8C line in the ISR run 418 with 10 A stacks at 26 GeV/c. The initial effective height was 3.2 mm and the vacuum pressure was between 2 and 3.5×10^{-11} Torr.

Fig 5: Percentage of the phase space occupied by regular trajectories, versus the non-linearity, for coupled oscillators.

Fig 6: Beam decay rate as a function of the excitation of a non-linear lens.

Fig 7: Decay rate and beam lifetime, as functions of the beam-beam tune shift achieved with a high-β insertion.

Fig 8: Positions of the overlap knock-out resonances in a stack on the ELSA line, with a pulse at the injection in the other ring.

Fig 9: Bunch frequency spectra for two different RF voltages.

Fig 10: Overlap knock-out resonances of order 1 and 2 for two beams at the same energy.

Fig 11: Dipole overlap knock-out resonance with two bunched beams.