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THE U(1) PROBLEM AND THE HIGGS MECHANISM;
IN TWO AND FOUR DIMENSIONS

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ABSTRACT

The bosonized Schwinger model and the two-dimensional Stueckelberg model are shown to be equivalent, and both are generalized to four dimensions via the topological gauge field $A_{\mu\nu\rho}$. The resolution of the U(1) problem in the former and the Higgs mechanism in the latter are closely related; in covariant gauges they imply the decoupling of "dual" Kogut-Susskind dipoles.

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1. THE U(1) PROBLEM AND THE HIGGS MECHANISM

Goldstone's theorem requires the existence of massless bosons whenever a continuous symmetry is spontaneously broken. For gauge theories the assumption of manifest Lorentz invariance that goes into the proof of the theorem allows the well-known loop-hole by which the Goldstone boson can be avoided via the Higgs mechanism. For the case of U(1) chiral symmetry it has long been recognized that the existence of the chiral anomaly provides another possible loop-hole, but the exact mechanism by which this may be taken advantage of has only recently become clear through the work of several authors¹⁾, whose results may be summarized by effective Lagrangians²⁾.

These two mechanisms for the avoidance of Goldstone bosons appear to be quite distinct. The Higgs mechanism hinges on the spontaneous breakdown of a non-anomalous but local symmetry, while the resolution of the U(1) problem hinges on the spontaneous breakdown of a rigid but anomalous symmetry. Actually there is a close connection between them, which can be seen in the two-dimensional Schwinger model. There are two standard forms of this model, the first being massless QED₂:

$$\mathcal{L}_1 = -\bar{\psi}\phi\psi - \pi g i \bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F_{\mu\nu}^2 \quad (1.1)$$

This can be "bosonized" to give a second equivalent form:

$$\mathcal{L}_2 = -\frac{1}{2}(\partial_\mu\phi)^2 - ig\partial_\mu\phi\epsilon^{\mu\nu}A_\nu - \frac{1}{4}F_{\mu\nu}^2 \quad (1.2)$$

Both forms of the theory possess chiral symmetries, realized linearly on ψ in the case of $\mathcal{L}_1$, ($\delta\psi = \omega i\gamma_5\psi$), and non-linearly on ϕ in the case of $\mathcal{L}_2$ ($\delta\phi = \omega$). In both cases there is a conserved but gauge non-invariant current J_μ^5 and a gauge invariant current j_μ^5 with a divergence proportional to the topological charge density $i\epsilon^{\mu\nu}F_{\mu\nu}$. For $\mathcal{L}_2$ these currents are

$$J_\mu^5 = (\partial_\mu\phi) \quad , \quad j_\mu^5 = (\partial_\mu\phi + ig\epsilon^{\mu\nu}A_\nu) \quad (1.3)$$

J_μ^5 is conserved by virtue of the equations of motion. The analogous current for $\mathcal{L}_1$ is conserved by virtue of the equations of motion and the anomaly. From the point of view of $\mathcal{L}_1$, electric charges are screened and chiral symmetry is spontaneously broken^{3),4)}. This state of affairs is represented by $\mathcal{L}_2$ directly, with ϕ the pseudoscalar Goldstone field. Because of the anomaly there is a

$\phi - A_\mu$ coupling and $\mathcal{L}_2$ must be diagonalized before the physical spectrum is read off. It consists of one massive spin zero boson, thus evading the Goldstone theorem.

A more illuminating way to this conclusion is via the construction of a third form of the model. We write $\mathcal{L}_2$ in the equivalent form

$$\mathcal{L}'_2 = i\lambda \varepsilon^{\mu\nu} \partial_\mu L_\nu - \frac{1}{2} L_\mu^2 - ig L_\mu \varepsilon^{\mu\nu} A_\nu - \frac{1}{4} F_{\mu\nu}^2 \quad (1.4)$$

λ is a Lagrange multiplier field imposing the constraint $L_\mu = \partial_\mu \phi$, but we can instead eliminate L_μ in favour of λ to get a new Lagrangian

$$\mathcal{L}_3 = -\frac{1}{2} g^2 \left(A_\nu - \frac{1}{g} \partial_\nu \lambda \right)^2 - \frac{1}{4} F_{\mu\nu}^2 \quad (1.5)$$

We recognize $\mathcal{L}_3$ as the Stueckelberg model, a prototype for the Higgs mechanism in which only the gauge field A_μ and the Goldstone field λ appear. Choice of the "unitary gauge" $\lambda = 0$ leads immediately to the conclusion that the spectrum consists of one massive vector, equivalent in two dimensions to a massive pseudoscalar. The λ field is the "dual" of the original Goldstone field ϕ ⁵⁾. There is no chiral symmetry of $\mathcal{L}_3$ and hence no Noether current J_μ^5 , but there is the conserved but gauge variant topological current $J_\mu^5 = i\varepsilon^{\mu\nu} \partial_\nu \lambda$, and its anomalous but gauge invariant counterpart $j_\mu^5 = i\varepsilon^{\mu\nu} (\partial_\nu \lambda - gA_\nu)$.

The two equivalent theories $\mathcal{L}_2$ and $\mathcal{L}_3$ both manage to avoid their respective Goldstone bosons ϕ and λ appearing as massless particles in the physical spectrum. Their respective mechanisms for achieving this are therefore obviously related. For the remainder of this paper we are concerned with making this connection more precise in the context of a generalization of the Schwinger model to four dimensions.

2. GENERALIZING THE SCHWINGER MODEL

The unusual properties of the Schwinger model are usually thought to be a consequence of the two-dimensional kinematics and therefore not generalizable to four dimensions. A closer look at the mechanisms operating in the Schwinger model, however, suggests that this conclusion is too hasty. A crucial feature is the presence of a "topological" gauge field, that is a gauge field propagating no physical degrees of freedom when massless, and whose dual curl is a topological charge density. In two dimensions the ordinary Maxwell field fits this description, but in four dimensions we need a three index gauge field $A_{\mu\nu\rho}$, with a

generalized gauge invariance $\delta A_{\mu\nu\rho} = (\partial_\mu \Lambda_{\nu\rho} + 2 \text{ terms})^{6),7)}$. The next point to appreciate is which of the equivalent forms of the Schwinger model we should generalize. $\mathcal{L}_2$ and $\mathcal{L}_3$ are the forms that are closer to the actual physics, and indeed $\mathcal{L}_2$ has an immediate generalization to four dimensions:

$$\mathcal{L}_2(4 \text{ dim.}) = -\frac{1}{2}(\partial_\mu \phi)^2 - ig \partial_\mu \phi \varepsilon^{\mu\nu\rho\sigma} A_{\nu\rho\sigma} - \frac{1}{48} F_{\mu\nu\rho\sigma}^2 \quad (2.1)$$

where $F_{\mu\nu\rho\sigma} = (\partial_\mu A_{\nu\rho\sigma} + 3 \text{ terms})$ is the gauge invariant field strength for $A_{\mu\nu\rho}$. Again we have a non-linearly realized chiral symmetry $\delta\phi = \omega$, with a conserved but gauge variant current $J_\mu^5 = \partial_\mu \phi + ig \varepsilon^{\mu\nu\rho\sigma} A_{\nu\rho\sigma}$. The corresponding gauge invariant current $j_\mu^5 = \partial_\mu \phi$ has the anomalous divergence

$$\partial_\mu j_\mu^5 = -\frac{ig}{4} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \quad (2.2)$$

in precise analogy with the two-dimensional model. To obtain the four dimensional generalization of $\mathcal{L}_3$ we can again repeat the steps of (1.4) and (1.5) to obtain

$$\mathcal{L}_3(4 \text{ dim.}) = -\frac{1}{12} g^2 \left(A_{\mu\nu\rho} - \frac{1}{g} (\partial_\mu \Theta_{\nu\rho} + 2 \text{ terms}) \right)^2 - \frac{1}{48} F_{\mu\nu\rho\sigma}^2 \quad (2.3)$$

where $\Theta_{\mu\nu}$ is an antisymmetric tensor field transforming as $\delta\Theta_{\mu\nu} = g\Lambda_{\mu\nu}$. It also has its own gauge transformation $\delta\Theta_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$ and so can be considered as the "dual" of ϕ in (2.1). This Lagrangian is the natural generalization, in this context, of the Stueckelberg model. In the unitary gauge $\Theta_{\mu\nu} = 0$ there remains only the massive field $A_{\mu\nu\rho}$, which in four dimensions is equivalent to a massive pseudoscalar⁸⁾.

Our four dimensional version of $\mathcal{L}_2$ in (2.1) is in fact the simplest prototype of the effective Lagrangians proposed²⁾ for QCD, in the large N limit. The code of correspondence is

$$\frac{1}{4!} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu\rho\sigma} \propto Q(x) \equiv \frac{g^2}{64\pi^2} \varepsilon^{\mu\nu\rho\sigma} F_{\mu\nu}^a F_{\rho\sigma}^a \equiv \partial_\mu K^\mu \quad (2.4)$$

$$\phi(x) \propto \left\{ \ln \det(\bar{q}_L q_R) - \ln \det(\bar{q}_R q_L) \right\}$$

where K_μ is the well-known⁹⁾ combination of glue fields entering the anomaly equation in QCD. This correspondence preserves the transformation laws for ϕ and $A_{\mu\nu\rho}$. The unusual kinetic term for K_μ is re-interpreted¹⁰⁾ as the standard kinetic terms of $A_{\mu\nu\rho}$. Therefore, as the bosonized Lagrangian $\mathcal{L}_2$ of Section 1

can be seen as an effective Lagrangian for QED_2 , so we can plausibly argue that QCD, in the large N limit, is the appropriate generalization to four dimensions of QED_2 . From this point of view we see that the lessons to be had from the Schwinger model and its generalizations may be applied to QCD. For instance, one might wonder how to reconcile the two statements that "the $U(1)$ Goldstone boson receives a mass because of the anomaly" as suggested by the $1/N$ expansion¹¹⁾, and "the massless Goldstone boson decouples", as suggested by the Kogut-Susskind³⁾ mechanism. The answer is exactly as for the Higgs mechanism. It is not really the Goldstone boson that becomes massive but the gauge field to which it couples. A massive topological gauge field happens also to describe a massive pseudoscalar boson. This is all manifest in a unitary gauge, but in a covariant gauge there really is a Goldstone boson which decouples as a consequence of gauge invariance. The general framework for exhibiting the decoupling of a Goldstone boson is that of the Goldstone quartet¹²⁾, which is a generalization of the Kogut-Susskind mechanism. This is what we now discuss.

3. DECOUPLING OF THE GOLDSTONE BOSON

In covariant gauges we expect a chiral $U(1)$ Goldstone boson to be present, but to decouple. First then, let us consider the covariant quantization of $A_{\mu\nu\rho}$. From the Lagrangian (A, ϕ) of (2.1) we construct the functional integral

$$Z = \int [d\phi] [dA_{\mu\nu\rho}] \Delta'_A \exp i \left\{ \int \mathcal{L}(A, \phi) \right\} d^4x \quad (3.1)$$

Δ'_A is the Faddeev-Popov, (FP), determinant, but as the FP matrix has zero eigenvalues these should first be removed. Equivalently, the ghost Lagrangian will have its own gauge invariance which must be fixed and which will require further ghosts¹³⁾. In general a whole tower of ghosts is needed⁷⁾ and this fact is indicated by the prime on Δ'_A . Now the δ function in (3.1) can be replaced by a Lagrange multiplier as follows,

$$Z = \int [d\phi] [dA_{\mu\nu\rho}] [dB_{\nu\rho}] \delta(\partial_\mu B_{\nu\rho}) \Delta'_A \Delta'_B \exp i \left\{ \int \mathcal{L}(A, \phi) + B_{\nu\rho} (\partial_\mu A_{\mu\nu\rho}) \right\} d^4x \quad (3.2)$$

As $B_{\nu\rho} (\partial_\mu A_{\mu\nu\rho})$ is invariant under $\delta B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu$ integration over $B_{\mu\nu}$ requires that we fix a gauge for it¹⁴⁾. Hence the new δ function and its corresponding ghost determinant. Repeating the whole above procedure twice leads finally to

$$Z = \int [d\phi] [dA_{\mu\nu\rho}] [dB_{\mu\nu}] [d\lambda_\mu] [dg] \Delta'_A \Delta'_B \Delta_\lambda \exp i \left\{ \mathcal{L}(A, \phi) + \right. \\ \left. + B_{\nu\rho} (\partial_\mu A_{\mu\nu\rho}) + \lambda_\nu (\partial_\mu B_{\mu\nu}) + g (\partial_\mu \lambda_\mu) \right\} d^4x \quad (3.3)$$

Had we used 't Hooft's gauge weighting trick to provide quadratic terms for $B_{\mu\nu}$, λ_μ and g in the exponent we would have needed also¹⁴⁾ other Nielsen-Kallosh type ghosts. In unweighted gauges they do not appear. Exponentiating the ghost determinants we find the following effective Lagrangian

$$\mathcal{L}_{\text{eff.}} = \mathcal{L}(\phi, A_{\mu\nu\rho}) + B_{\mu\nu} (\partial_\lambda A_{\lambda\mu\nu}) + \lambda_\nu (\partial_\mu B_{\mu\nu}) + g (\partial_\mu \lambda_\mu) \\ + \frac{1}{12} \bar{C}_{\mu\nu} (\Box \delta_{\mu\rho} \delta_{\nu\sigma} + 2\partial_\nu \partial_\rho \delta_{\mu\sigma} + \partial_\sigma \partial_\mu \delta_{\nu\rho}) C_{\rho\sigma} \\ + \frac{1}{4} \bar{C}_\mu (\Box \delta_{\mu\nu} - \partial_\mu \partial_\nu) C_\nu + \bar{C} \Box C + (\text{ghost gauge fixing terms} + \text{ghosts of ghosts}) \quad (3.4)$$

Because of the Abelian nature of the theory we have many independent sets of BRST transformations. Let us concentrate on the particular invariance under the transformations

$$\delta A_{\mu\nu\rho} = (\partial_\mu C_{\nu\rho} + 2 \text{ terms}) \Lambda \\ \delta \bar{C}_{\mu\nu} = 12 B_{\mu\nu} \Lambda, \quad \delta C_{\mu\nu} = \delta B_{\mu\nu} = 0 \quad (3.5)$$

with Λ the anticommuting BRST parameter. What is relevant¹⁵⁾ to the U(1) problem is the transformation of J_μ^5 , which rotates into a "ghost current" C^μ . The corresponding antighost current $\bar{C}^\mu$ rotates into a fourth current B^μ . Because of the nilpotency of BRST transformations these four currents are all we can get. They are

$$J_\mu^5 = \partial_\mu \phi + ig \varepsilon^{\mu\nu\rho\sigma} A_{\nu\rho\sigma} \\ C_\mu = i \varepsilon^{\mu\nu\rho\sigma} \partial_\nu C_{\rho\sigma} \\ \bar{C}_\mu = i \varepsilon^{\mu\nu\rho\sigma} \partial_\nu \bar{C}_{\rho\sigma} \\ B_\mu = \frac{i}{g} \varepsilon^{\mu\nu\rho\sigma} \partial_\nu B_{\rho\sigma} \quad (3.6)$$

Now the breaking of chiral symmetry implies a massless asymptotic field for J_μ^5 . For this to decouple via the quartet mechanism has been shown by Kugo¹⁵⁾ to require only the existence of massless asymptotic fields for the other three currents of (3.6). That they do exist is obvious from (3.6) so that the decoupling of the U(1) Goldstone boson is automatic. In four dimensions the existence of an asymptotic field for C^μ generally depends on the formalism of a bound state in the C^μ channel because of the non-linearity of the anomaly. But the higher rank Abelian gauge theory of $A_{\mu\nu\rho}$ has a linear anomaly, as in the two-dimensional models, so that this objection is overcome.

Let us say that quartet of currents (3.6) has the following set of asymptotic fields

$$\begin{aligned} J_\mu^5 &\rightarrow \partial_\mu \chi & B_\mu &\rightarrow \partial_\mu \beta \\ C_\mu &\rightarrow \partial_\mu \gamma & \bar{C}_\mu &\rightarrow \partial_\mu \bar{\gamma} \end{aligned} \quad (3.7)$$

The fields $(\chi, \beta, \gamma, \bar{\gamma})$ constitute the Goldstone quartet whose decoupling from the physical spectrum resolves the U(1) problem. β, γ and $\bar{\gamma}$ are the duals of $B_{\mu\nu}, C_{\mu\nu}$ and $\bar{C}_{\mu\nu}$. Let us call $\theta_{\mu\nu}$ the dual of χ . Then $(\theta_{\mu\nu}, B_{\mu\nu}, C_{\mu\nu}, \bar{C}_{\mu\nu})$ constitute another Goldstone quartet. $\theta_{\mu\nu}$ can be identified with the Goldstone field appearing in the generalized Stueckelberg model (2.3), so that the decoupling of this quartet ensures the absence of a Goldstone boson via the Higgs mechanism. Now we see very clearly the relationship between the mechanism of the U(1) problem and the customary Higgs mechanism. They correspond to the decoupling of dual Goldstone quartets.

4. COMMENTS

Our purpose in this paper has been twofold. Firstly, to elucidate the connection between the U(1) problem and the Higgs mechanism by exploiting the equivalence of the Stueckelberg to the Schwinger model, both in two dimensions. Secondly to generalize these insights to four dimensions by means of the Abelian gauge theory for $A_{\mu\nu\rho}$. Our conclusion is that the mechanism by which the U(1) Goldstone boson decouples is dual to the Higgs mechanism, in the sense that they correspond to the decoupling of dual Goldstone quartets. Our analysis is in no way restricted to two or four dimensions, but is equally applicable to any number of dimensions.

In the two dimensional Schwinger model the topological gauge field A_μ not only resolves the $U(1)$ problem but also is responsible for the screening of "electric charge" (confinement in massive QED_2). By analogy we expect the topological field $A_{\mu\nu\rho}$ in four dimensions also to be relevant to colour confinement in QCD⁶⁾, in which it would have to appear as a bound state of colour gauge fields. Indeed, Kugo has argued¹⁶⁾ on general grounds that the $U(1)$ problem and the problem of colour confinement in QCD are quite the same.

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