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HIERARCHICAL FERMION MASSES FROM GRAND UNIFICATION

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ABSTRACT

A mechanism is presented for generating fermion masses as a reflection of the superheavy fermions existing in a class of grand unified theories. By means of it we propose a way of understanding how, giving a driving mass term for the top-generation, the charm-generation mass results as a relative first-order perturbation in the unified gauge coupling $g^2/4\pi \approx 10^{-2}$, and the up-generation mass as a second-order perturbation. An illustrative calculation of the charm-generation mass is described in a model based on E_6 .

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The wide variety of scales occurring in the mass spectrum of the elementary fermions in the $SU(3) \otimes SU(2) \otimes U(1)$ theory of strong and electroweak interactions represents the least appealing aspect of the theory itself. All of these mass scales are put in by hand and no explanation is given of their large ratios or of their relationship to the weak gauge boson masses.

It is in fact plausible to attribute this shortcoming to the lack of real unification between the different fundamental forces that is apparent in the $SU(3) \otimes SU(2) \otimes U(1)$ theory. However, not even in grand unified models has any concrete mechanism for generating such a variety of scales ever been proposed up to now, maybe indicating the need to go beyond grand unified theories. This is what we will do in this paper, by describing a class of models where a hierarchy of fermion masses can naturally occur. More specifically, the mechanism that we want to propose offers a way of understanding how, given a driving mass term for the top-generation (not too far from the W-boson mass), the charm-generation mass results as a relative first-order perturbation in the unified gauge coupling $g^2/4\pi \approx 10^{-2}$, and the up-generation mass arises as a second-order perturbation.

The basic scenario upon which our mechanism is built is the following. In standard grand unified models based on some group $G \supset SU(3) \times SU(2) \times U(1)$ the different generations of fermions appear as repetitive structures, having the same transformation properties under G . Let us suppose now that this description results from an underlying theory which is actually able to distinguish between the different families: this might, for example, arise from the fact that the observed generations of fermions are surviving pieces of inequivalent representations of a supergroup $G' \supset G$, or, perhaps, because quarks and leptons [maybe together with $SU(3) \times SU(2) \times U(1)$ gauge bosons and Higgses] are not fundamental particles but composites, with a very small radius, of more basic objects. In such a situation one can conceive the deeper fundamental theory as forbidding at the tree level all but the third-generation to get a "direct mass" from a standard Yukawa coupling to a Higgs field. On the other hand, the basic structure of the theory at super-high masses is likely to produce a number of superheavy fermions, because of their

occurrence in non-chiral representations under some conserved (or almost conserved) group. We want to show now that these superheavy fermions are able to give "radiative" masses, of natural order $(\alpha/\pi)M_W$, to the generations of conventional fermions which were unable to get a "direct" mass, not far from M_W itself, at the tree level. In its most general form the mechanism is the following.

Let us consider two different stages of symmetry breaking of the group G (or G') $\rightarrow G_1 \rightarrow G_2$ taking place at characteristic masses M_1 and M_2 . Some fermions will in general get G_1 -invariant direct masses m_F of order M_1 . As a reflection of these heavy fermions a radiative mass is in general induced through the diagram of Fig. 1 for the lower fermion sector (those fermions whose mass terms respect G_2 but not G_1). Its contribution is

$$\delta m = 3 \frac{\alpha}{\pi} m_F \sin \xi \cos \xi \left[\frac{M_a^2}{m_F^2 - M_a^2} \lg \frac{m_F^2}{M_a^2} - \frac{M_b^2}{m_F^2 - M_b^2} \lg \frac{m_F^2}{M_b^2} \right] \quad (1)$$

where m_F , M_a , M_b , all of order M_1 , are respectively the masses of the exchanged fermion and of the two gauge bosons with necessarily different transformation properties with respect to the group G_2 . Since the survival hypothesis¹⁾ demands that the internal fermion mass be a G_1 singlet, whereas the external fermion mass is not, the diagram goes through the mixing between these different gauge bosons with an angle

$$\sin \xi \simeq \frac{M_2}{M_a - M_b}$$

The order of magnitude of δm is therefore $(\alpha/\pi)M_2$, essentially independent of M_1 .

The "radiative mass" is a natural candidate for giving the hierarchy in masses observed between the different fermionic generations. Let us see a possible concrete scenario. The group G is broken down to $SU(3) \times U(1)$ at M_W through at least the step involving $SU(3) \times SU(2) \times U(1)$ at M . Each generation of fermion fields, all taken to be left-handed, transforms as a representation ψ of G . The breaking of the group takes place through Higgs fields, which transform under G

as the Lorentz scalars that can be formed from bilinear products of two left-handed fields $\psi_\alpha \psi_\beta$, with α and β being group indices. These Higgs fields may either be fundamental degrees of freedom of the basic Lagrangian or the effective description of a fermion condensate. Two different Higgs representations under G are assumed to get vacuum expected values: one -- call it ϕ -- which is mainly responsible for the superlarge first breaking; and the other, H , which will also have non-vanishing vacuum expected values (v.e.v.) for the non-singlet components under $SU(3) \times SU(2) \times U(1)$, and therefore gives rise to the second step of breaking. In general the coupling $V(\phi, H)$ between the two different Higgs multiplets will also make the $SU(3) \times SU(2) \times U(1)$ singlets in H have a superlarge v.e.v., whereas the "miniaturization" of M_W relative to M will prefer the leaving of the $SU(3) \times SU(2) \times U(1)$ non-singlets in ϕ , with zero v.e.v., a positive superlarge mass squared and no couplings to the non-singlets in H .

As we said, these Higgs fields will also couple to fermions. If we start including explicitly the family degree of freedom, denoted by indices i, j , going from 1 to 3, the Lagrangian will contain the Yukawa couplings

$$\gamma_{ij} \psi_i H \psi_j + \Gamma_{ij} \psi_i \phi \psi_j \quad (2)$$

It is in constraining the form of the above Higgs couplings that the deeper underlying theory will play a crucial role, as we described. Here we assume that this post- G interaction will make H and ϕ each naturally couple only to one generation, in general different from each other. Making use of the arbitrariness of the basis chosen for ψ_i in generation space, this assumption is expressed by^{*)}

$$\gamma = a \mathbb{1}_3, \quad \Gamma = b U \mathbb{1}_3 U^T, \quad \mathbb{1}_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (3)$$

where a relative unitary transformation U is in general present.

*) Both H and ϕ must be contained in the symmetric product $\psi \times \psi$, so the coupling matrices γ and Γ need to be symmetric in generation space.

Denoting by f a generic "light" fermion field and by F a superheavy fermion field, the gauge currents are of three types: ff , fF , and FF . In the mass eigenstate basis, it is only the fF current which will make the different generations communicate as a reflection of the matrix U in Eq. (3). Clearly then, the diagram of Fig. 1 is the only effect present in the theory which is able to give masses to the generations other than the third one. Summing up all the contributions of this kind, we get the following mass matrix for any fermion of definite charge

$$\mathcal{M} = M_W \left(c \mathbb{1}_3 + \frac{\alpha}{\pi} d U \mathbb{1}_3 U^T \right) \quad (4)$$

with c and d being numerical factors of the order of 1, dependent only upon the charge sector we are considering. It is easily shown now that the above mass matrix has two eigenvalues of order M_W and $(\alpha/\pi)M_W$, respectively, and that the third eigenvalue is identically zero. This is because any 3×3 unitary matrix U can be parametrized as

$$U = \begin{pmatrix} R_1 & & \\ & 1 & \\ & & R_2 \end{pmatrix} \begin{pmatrix} 1 & & \\ & e^{i\delta} & \\ & & 1 \end{pmatrix} \begin{pmatrix} R_3 & & \\ & 1 & \\ & & R_4 \end{pmatrix} \quad (5)$$

with $R_{1,2,3}$ being two-dimensional rotations. Therefore, in a suitable basis, where γ is still proportional to $\mathbb{1}_3$, Γ can be written in a general way as

$$\Gamma \propto U \mathbb{1}_3 U^T = \begin{pmatrix} 1 & & \\ & 1 & \\ & & R_2 \end{pmatrix} \mathbb{1}_3 \begin{pmatrix} 1 & & \\ & 1 & \\ & & R_2^T \end{pmatrix} \quad (6)$$

This not only proves that the matrix $\mathcal{M}$ in Eq. (4) has one vanishing eigenvalue, but actually shows that one generation will stay massless also at higher orders.

It may be that in a concrete realization this situation turns out to be realistic. This would indicate the need for a different mechanism to generate the tiny mass for the lightest generation. The observed fermion mass spectrum

($u < d \neq e$; $c > s \approx \mu$; $t > b = \tau$) with its characteristic inversion between $Q = 2/3$ and $Q = -1/3$ quark masses, plus the badly broken equality of $Q = -1/3$ quark and $Q = -1$ lepton masses in the lightest generation, may suggest that something funny goes on concerning the u , d , and e masses. Notice that in any case the situation we have discussed is an approximate one, since a "tiny" mass (relative to M) has also to be given to one generation of superheavy fermions which does not get a direct mass through its direct coupling to H or ϕ . Here the post-G interactions, gravity included, could become effective²⁾ in a way that escapes our present comprehension.

It is, however, striking that the mass scale relevant to the lightest generation fits as an order of magnitude with $(\alpha/\pi)^2 M_W$, which is of course suggestive of a two-loop radiative correction effect. Can one speculate on a way to get such an effect? An iterative mechanism suggests itself, with one more step of intermediate symmetry breaking and a third Higgs system mainly coupled to the fermions (one generation only) that get a direct mass at this level of breaking. Here too, communication will occur between different generations only through currents that connect fermions of different levels. The wanted cascade of masses, with the lightest family getting a mass of order $(\alpha/\pi)^2 M_W$, will result if only adjacent levels of fermions are allowed to communicate through the one-loop diagram.

A very special realization of the scenario described above is offered by the group E_6 with the fermions of each family in the fundamental 27-representation, as discussed in an earlier paper¹⁾. The nice property of the 27 is that under $SU(3) \times SU(2) \times U(1)$ it splits into the observed 15-plet of "light" fermions plus a self-conjugate 12-plet of fermions which will become superheavy, and is just enough to describe the superlarge breaking of E_6 in terms of the fermion-fermion operator. Mainly for illustrative purposes we have looked into this model at the generation of a radiative mass for the second family, with the lightest one kept massless at this level. The 12-plet of superheavy fermions transforms under the subgroup $SU(5)$ as $\underline{5} + \bar{\underline{5}} + \underline{1} + \underline{1}$; we call them (D_i, E^C, N^C) , (D_i^C, E, N) , ν^C , and L . Of the two last neutral superheavy leptons, L is also an $SO(10)$ singlet, whereas

ν^c is the member of the 16-plet under $SO(10)$ which is apparently missing from the observed spectrum. As to the Higgs representations, 351 and 27, the first is capable of describing the breaking $E_6 \rightarrow SU(3) \times SU(2) \times U(1)$, whereas the smaller one is assumed to give the last stage of breaking and, at the same time, a direct mass to the top-generation. The masses of the superheavy fermions can be connected with the relevant vacuum expected values. A relatively simple pattern of radiative masses emerges for the charm generation if one assumes a hierarchy in these superheavy fermion masses according to the suggestive cascade $E_6 \rightarrow SO(10) \rightarrow SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$. In this case a careful inspection^{*)} shows³⁾ that the dominant contribution to the μ - and the s-quark masses comes from the L exchange (Fig. 2), whereas the charm quark receives both a contribution from the L exchange and from the ν^c exchange (Fig. 3). These contributions are in general comparable, because the higher mass of L relative to ν^c is compensated by the higher masses of the gauge bosons exchanged in Fig. 2 relative to those of Fig. 3. This comes from the postulated hierarchical pattern, which in turn suppresses all the diagrams with D_i or E exchanges. A similar suppression actually takes place for the contributions to the $Q = -1/3$ and $Q = -1$ masses coming from ν^c exchange, basically because they have superheavy partners unlike the $Q = 2/3$ quark. From the diagram of Fig. 2, $m_s = m_\mu$ arises at superhigh energies, whereas the additional contribution to the c-quark mass from Fig. 3 -- although not possible to estimate numerically because of the extra parameters involved -- might explain its higher value relative to m_μ . As we said, all of these masses are of the order of $(\alpha/\pi)M_W$, which is phenomenologically correct.

In conclusion we think that the mechanism we have described offers an interesting insight into the fermion mass hierarchy problem. Needless to say it leaves us with a number of unanswered questions, which we do not even try to list but certainly deserve further investigation.

*) We are indebted to A. Masiero for his collaboration on this point. Full details will be published elsewhere³⁾.

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Figure captions

- Fig. 1 : The basic diagram for the radiative mass.
- Fig. 2 : Radiative mass contribution to s , μ , and c from L exchange.
- Fig. 3 : Radiative mass contribution to c from ν^c exchange.

FIG. 1

FIG. 2

FIG. 3