Ref.TH.2839-CERN

THE QUARK-NUCLEON PHASE DIAGRAM AND QUANTUM CHROMODYNAMICS

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ABSTRACT

The temperature-dependence of a conjectured first order phase transition between nuclear matter and quark-gluon matter is calculated for temperatures below $T = 200$ MeV. On the nuclear side a rather successful meson-nucleon mean field theory is applied while quark-gluon matter at large densities and finite temperatures is described perturbatively by Quantum Chromodynamics. Outside the finite volume of hot and dense quark-gluon matter the physical vacuum is characterized by the newly determined bag parameter $\Lambda_B = 235$ MeV. We observe a dramatic drop in the density of nuclear matter at the phase transition point as the temperature increases, if the scale parameter Λ of QCD is chosen as $\Lambda = 100$ MeV. For larger values of Λ the effect is less pronounced. Further work is required to settle this problem.

Ref.TH.2839-CERN
27 March 1980

INTRODUCTION

It is widely accepted that nuclear matter at normal densities is best described in terms of a nucleon medium. However, at large densities and zero temperature a first order phase transition is conjectured¹⁾ into quark matter. The second phase at large densities can be described perturbatively in the framework of Quantum Chromodynamics²⁾, since asymptotic freedom becomes operative in the medium when the quark Fermi momentum p_F is much larger than the characteristic hadronic scale parameter Λ .

The experimental value of Λ is a few hundred MeV from deep-inelastic lepton-nucleon scattering. The precise value of Λ depends on the theoretical definition of the quark-gluon coupling constant and the renormalization procedure in removing the unobservable divergences of the diagrams³⁾.

The density of quark matter at the conjectured first-order phase transition point is calculated on the basis of the Gibbs criteria,

$$\begin{aligned} p_1 &= p_2 \\ \mu_1 &= \mu_2 \end{aligned} \quad (1)$$

Eq. (1) requires equal pressures and equal chemical potentials for quark matter and nuclear matter in thermodynamical phase equilibrium. For self-consistency, the Fermi momentum p_F of perturbative quark matter as determined from Eq. (1) must be much larger than the scale parameter Λ .

In the present paper we wish to generalize the above calculational scheme to finite temperatures. The Gibbs criteria become then

$$\begin{aligned}T_1 &= T_2 \\p_1 &= p_2 \\ \mu_1(p_1, T_1) &= \mu_2(p_2, T_2).\end{aligned}\tag{2}$$

First, according to Eq. (2), we calculate the pressure $p(\mu, T)$ in both phases as a function of the chemical potential μ for some fixed value of T . The intercept is interpreted as a phase transition point of first order. Varying T we get a series of isotherms in the calculation.

The interest in the calculation of the temperature dependence of the conjectured first order phase transition is at least three-fold. First, the problem is relevant in heavy ion physics. The temperature dependence of the phase transition may be detected in the future in very high energy heavy ion collisions. Next, we may recall some cosmological problems from the early stage of evolution of the Universe. With regard to our calculation, perhaps the most interesting one is the estimate of relict quarks⁴⁾ from the Big Bang, if quark confinement is only partial, and limited to lower energies. Finally, the detailed study of how asymptotically free quark matter crosses over into nuclear matter may shed some light on certain aspects of the fundamental problem of quark confinement.

Quark-gluon matter at large densities and finite temperatures is described in our work⁵⁾ by Quantum Chromodynamics in the lowest non-trivial order of the quark-gluon coupling constant. The lowest order calculation is improved then by the renormalization group.

Since for practical applications (heavy ion collisions) we visualize the phase transition to occur in some finite volume, an additional parameter is required in the calculation of the

quark-gluon phase. Outside the finite volume of dense and hot quark-gluon matter there is the physical vacuum whose energy density is lower than that of the perturbative Fock vacuum inside. This energy difference Λ_B is often quoted as the bag parameter $B^{1/4}$ in the bag literature⁶⁾. The finite volume occupied by the perturbative, dense and hot quark-gluon matter is best described as a bag of macroscopic size immersed in the physical vacuum.

Our calculation of the quark-gluon phase is based on a very careful new determination⁷⁾ of the bag parameter Λ_B from the spectrum of heavy quark-antiquark systems. The isotherms strongly depend on the parameter Λ_B , since the energy difference between the physical vacuum and the inside Fock vacuum acts upon the finite volume of the quark-gluon phase as some inward pressure (vacuum pressure).

To search for the transition point as determined by Eq. (2) we give here a brief description of the nuclear phase.

NUCLEAR MATTER AT FINITE TEMPERATURES

For convenience, we have chosen a successful nuclear mean field theory⁸⁾ to describe the nuclear phase at finite temperatures⁹⁾. For the sake of completeness we quote the most important results from Refs. 8, 9.

At $T=0$ a self-consistent set of equations is derived for the energy density $\mathcal{E}$, pressure p and baryon density n_B of the nuclear phase:

$$\mathcal{E} = \frac{1}{2} C_v^2 n_B^2 + \frac{1}{2C_s^2} (1-x)^2 + \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k (k^2 + x^2)^{\frac{1}{2}}, \quad (3)$$

$$\rho = \frac{1}{2} c_v^2 n_B^2 - \frac{1}{2c_s^2} (1-x)^2 + \frac{1}{3} \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k \frac{k^2}{\sqrt{k^2 + x^2}}, \quad (4)$$

$$n_B = \frac{\gamma}{(2\pi)^3} \int_0^{k_F} d^3k, \quad (5)$$

where the self-consistent nucleon mass $x=M^*/M$ is given by

$$x = \frac{1}{1 + c_s^2 \frac{\gamma}{(2\pi)^3} \int_0^{k_F} \frac{d^3k}{\sqrt{k^2 + x^2}}} \quad (6)$$

The spin-isospin degeneracy is $\gamma=4$ for nuclear matter with an equal number of protons and neutrons. The numerical values of the constants c_s^2 and c_v^2 are given in Ref. 8.

It is well known that Eqs. (3-6) reproduce the bulk properties of infinite nuclear matter⁸⁾ and the mean field theory model is also quite efficient when applied to finite large nuclei¹⁰⁾.

At finite temperatures we are interested in the thermodynamical potential

$$\Omega(\mu, V, T) = -T \ln \text{Tr} \exp(-\frac{1}{T}(H - \mu B)) \quad (7)$$

where H is the Hamiltonian and B is the baryon number operator.

The chemical potential μ of baryons is given by

$$\mu = \nu + n_B c_v^2 \quad (8)$$

with a parameter ν which appears in the calculations as follows. In Eqs. (3-6) one has to replace at finite temperatures

the integrals by the standard Fermi distributions

$$\int_0^{k_F} d^3k \longrightarrow \int d^3k [n + \bar{n}]$$

with

$$\begin{pmatrix} n \\ \bar{n} \end{pmatrix} = \frac{1}{1 + \exp\left\{\frac{1}{T} \sqrt{k^2 + x^2} \mp \frac{\nu}{T}\right\}}$$

Varying the parameter ν one calculates the pressure p for a fixed temperature T as a function of the chemical potential μ . The $p(\mu, T)$ curves from our numerical calculation are shown in Fig. 1. They are in agreement with the results of Ref. 9.

We proceed now to calculate the $p(\mu, T)$ curves for quark-gluon matter in order to search for the transition point.

THE QUARK-GLUON PHASE

For the sake of simplicity, and for applications in heavy ion physics, we shall be restricted to the case of two flavors (up and down quarks) to be compared with symmetric nuclear matter where the spin-isospin degeneracy is $\gamma = 4$.

The thermodynamical potential in lowest non-trivial order of the quark-gluon coupling constant g is calculated from the two-loop vacuum diagrams. In the Feynman rules of perturbative QCD the finite temperature T and the finite quark chemical potential μ_Q appear in a standard fashion. For massless quarks we find¹¹⁾

$$\begin{aligned} \frac{\Omega}{V} = -p = & -\frac{8\pi^2}{45} T^4 \left(1 - \frac{15}{16} \frac{g^2}{\pi^2}\right) - \frac{7\pi^2}{30} N_f T^4 \left(1 - \frac{25}{84} \frac{g^2}{\pi^2}\right) - \\ & - N_f \left(\frac{\mu_Q^2 T^2}{2} + \frac{\mu_Q^4}{4\pi^2} \right) \left(1 - \frac{g^2}{2\pi^2}\right) - \Lambda_B^4 \end{aligned} \quad (9)$$

where N_f is the number of quark flavors. The quark chemical potential μ_Q is related to the baryon chemical potential by $\mu_Q = \mu/3$.

The result in Eq. (9) is independent of the renormalization scheme, since it is of lowest order in g^2 . The first term in Eq. (9) describes the thermodynamics of the gluon gas whereas the second and third terms correspond to the contribution of quarks. The vacuum pressure Λ_B^4 was defined before, numerically $\Lambda_B = 235$ MeV is taken from Ref. 7.

In order to decide about the most suitable renormalization scheme one should calculate the next order to Eq. (9) and choose the scheme in which the correction is the smallest one. This procedure would fix the numerical value of g at some normalization point, if the corresponding scale parameter Λ of the theory is taken from deep-inelastic data.

The thermodynamical potential beyond the lowest non-trivial order in g^2 becomes a function of a dimensional parameter s when the subtraction procedure is specified in the diagrams:

$$\Omega = \Omega(g, T, \mu_Q, s)$$

The leading $\ln(T/s)$ and $\ln(\mu_Q/s)$ terms can be summed by the machinery of the renormalization group. The summation is equivalent to replacing g^2 in Eq. (9) by the running coupling constant g^2 . For $T \ll \mu_Q$ the dangerous logarithms are powers of

$\log(\mu_Q/s)$. These are taken into account by

$$g^2 \rightarrow \bar{g}^2(\mu_Q) = \frac{8\pi^2}{\frac{29}{3} \ln \frac{\mu_Q}{\Lambda}} \quad (10)$$

where the dependence on the scheme is hidden in the numerical value of the scale parameter Λ . The running coupling constant $\bar{g}(\mu_Q)$ of Eq. (10) is used in our numerical analysis below $T=200$ MeV.

THERMODYNAMICAL BEHAVIOR

The pressure $p(\mu, T)$ for different temperatures is shown in Fig. 1. The transition points are determined for different temperatures from the intercepts of the nuclear and quark-gluon curves. The Gibbs criteria of Eq. (2) are equivalent to the well-known Maxwell construction when the isotherms are plotted in Fig. 2. The densities of quark-gluon matter and nuclear matter at the transition point are plotted against the temperature T in Fig. 3. As an interesting observation, we note that for $\Lambda_B=0$ a first order phase transition is not supported by our QCD calculation in lowest non-trivial order of g^2 .

A comparison of Figs. 2 and 3 with the corresponding textbook diagrams for a van der Waals medium shows essential differences. In a van der Waals gas the isotherms on a pressure-density (p - n) plane have no crossing points. At any arbitrary fixed value of n , p is an increasing function of T . The two values of the densities for the gaseous and liquid phase at the transition point move into opposite directions as T varies. With increasing T their difference decreases and at some temperature T_{crit} they merge into the same critical density n_{crit} , so that

the phase transition becomes of second order. This is the expected usual behavior.

However, it is easy to show that the van der Waals behavior is not the most general one. Indeed, our calculation of quark-gluon matter in perturbative QCD yields an example for very different thermodynamical behavior. The pressure at the transition point drops as a function of increasing temperature in Fig. 2. The densities of quark-gluon matter and nuclear matter at the transition point move in the same direction in Fig 3 as a function of temperature. Physically, the phenomenon of asymptotic freedom is responsible for this peculiar behavior. Namely, when quark-gluon matter is more and more compressed at fixed temperature with increasing density it approaches the behavior of an ideal gas, against naive intuition.

Let us study briefly the most general conditions which support the van der Waals behavior for a general thermodynamical system which is fully characterized by a thermodynamical potential expressed in terms of its proper thermodynamical variables. For example, the free energy density f as a function of the density n and temperature T yields a complete thermodynamical description. In principle (but not in practice), f as a function of n and T is calculable in QCD for arbitrary values of the independent thermodynamical variables even in the non-perturbative nuclear regime.

The thermodynamical potential f exists everywhere including the unstable region between the characteristic densities of quark-gluon matter and nuclear matter where the medium separates into two different phases. The free energy density f is a continuous function of n and T in all known thermodynamical examples. Let us express now the chemical potential μ and pres-

sure p in terms of the derivatives of the free energy density as

$$\begin{aligned}\mu &= f, n \\ p &= nf, n - f\end{aligned}$$

Consider now the Gibbs criteria for phase equilibrium (Eq. (2)) at two neighbouring T values. We obtain

$$\frac{d\eta_i}{dT} (f, nn)_i = \frac{(f, \tau)_2 - (f, \tau)_1}{n_2 - n_1} - (f, n\tau)_i \quad (11)$$

where $i=1$ or 2 for the two phases, respectively, and n_i is the density at the transition point in the i^{th} phase. We assume that f, τ is a smooth function of n between n_1 and n_2 so that

$$(f, nn)_{(2)} \frac{d\eta_{(2)}}{dT} = \pm f, Tnn \Delta - \frac{1}{3} f, Tnnn \Delta^2 \quad (12)$$

$$\Delta = \frac{n_2 - n_1}{2}, \quad n_2 > n_1,$$

where the derivatives are taken at the density $(n_1 + n_2)/2$. In Eq. (12) the terms with fourth or higher order derivatives with respect of n are neglected because of the assumed smoothness properties of f .

One notes that f, nn must be positive in both phases because the equilibrium states are stable. Eq. (12) shows that n_1 and n_2 move in opposite directions if f, Tnn dominates $f, Tnnn \Delta$, and otherwise they move together. Even if f, Tnn dominates, its sign may be negative in the case of non-van der Waals behavior. Namely, the density difference $n_2 - n_1$ increases with increasing T .

For illustration we can guess the derivatives f, Tnn and $f, Tnnn$ from the equation of state for the perturbative quark-

-gluon phase (see Eq. (9)). Introducing n instead of μ , we obtain the derivatives of the free energy density from the function $p(\mu, T)$. We find that f_{Tnn} dominates at high temperatures and there it is negative. Thus, indeed, one cannot expect van der Waals behaviour in the quark-gluon system.

CONCLUSION

We have calculated the first order phase transition between perturbative quark-gluon matter and nuclear matter at finite temperatures. The most serious ambiguity in the calculation of the perturbative quark-gluon phase is identified as the choice of the hadronic scale parameter Λ . With $\Lambda = 100$ MeV one can see in Fig. 3 a dramatic drop in the density of the nuclear phase at the transition point as a function of temperature. The average Fermi momentum of quarks is much larger than Λ at the transition point, so that the perturbation theory is applicable in a self-consistent manner. With an increasing value of Λ the dramatic drop in the nuclear density as a function of temperature gradually disappears and perturbation theory on the quark-gluon side becomes less and less applicable at the transition point. Only the next to leading order correction to the equation of state in Eq. (9) can decide about the right value of Λ provided it is available in some renormalization scheme from deep inelastic data.

We estimated the non-perturbative instanton effects in the quark-gluon phase but the conclusion is not very informative. The size of the instanton corrections to the quark-gluon equation of state is very sensitive to the choice precisely where around the Debye screening length the cut-off in the in-

stanton size is put. Only a more serious calculation in which the deformation of instantons in the quark-gluon medium is taken into account can answer this ambiguity in a precise manner. The effects of instantons can be also interpreted as some unknown density and temperature dependence of the parameter Λ_B . We hope to return to this problem in a later publication.

We have obtained very similar results when we replaced the nuclear mean field theory in the description of the nuclear phase by a recent hard sphere model¹²⁾.

We are aware of a paper¹³⁾ in the literature where the phase transition of nuclear matter into quark-gluon matter at finite temperatures is estimated along the line of a similar strategy. A very different approach to the problem is also being pursued¹⁴⁾.

ACKNOWLEDGEMENTS

The authors are indebted for very useful discussions to Drs. P. Hasenfratz, G. Marx and J. Zimányi. One of us (J.K.) acknowledges a helpful conversation with prof. R. Hagedorn.

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FIGURE CAPTIONS

Fig.1a The pressure of the quark-gluon phase and also of the nuclear phase is plotted for various temperatures as a function of the baryon chemical potential μ . The marked intercepts are the phase transition points.

Fig.1b The Λ -dependence of the perturbative quark-gluon pressure is represented here for a fixed value of the temperature.

Fig.2 The isotherms for the choice $\Lambda = 100$ MeV indicate a strong non-van der Waals behaviour in the system.

Fig.3 The densities at the transition point are plotted against the temperature for various values of the scale parameter Λ .

Fig. 2.

Fig. 3.