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PHASE TRANSITION IN FOUR-DIMENSIONAL COMPACT QED

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A B S T R A C T

The energy and the specific heat of the four-dimensional $U(1)$ lattice gauge model is evaluated by Monte Carlo simulations on lattices of size L^4 , where $L = 4, 5$ and 6 , and evidence is presented for the occurrence of a second-order phase transition. A finite size scaling analysis of our results gives the critical value of the coupling constant $e_c^2 = 0.995$ and a correlation length exponent $\nu \simeq \frac{1}{3}$.

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It is known that compact quantum electrodynamics ¹⁾ [discrete U(1) gauge theory] on a four-dimensional lattice confines static electric charges in the strong coupling limit ²⁾, while for weak coupling the interaction obeys the familiar Coulomb law ³⁾. These properties imply the occurrence of a phase transition at some intermediate value of the coupling constant ¹⁾, but in spite of considerable theoretical work on this model ⁴⁾ there exist at present only conjectures on the order of this phase transition. Recently Creutz, Rebbi and their collaborators ⁵⁾⁻⁸⁾ have carried out a series of Monte Carlo studies for Abelian and non-Abelian lattice gauge models. For the U(1) model in four dimensions, Creutz found that the plaquette energy exhibits a hysteresis loop indicative of a phase transition ⁹⁾. He suggested that this is a higher order transition because of the occurrence of large fluctuations and slow convergence of the energy as a function of the number of Monte Carlo iterations. However, to determine the properties of this phase transition it is essential to evaluate also the specific heat or possibly higher order derivatives of the energy as a function of the coupling constant e^2 , and the size of the lattice. These quantities depend on the correlations between plaquettes which are expected to increase at large distances in the neighbourhood of a higher order phase transition. The correlation length ζ diverges at the phase transition point e_c^2 as $\zeta \sim |e^2 - e_c^2|^{-\nu}$, where ν is a critical exponent. For a finite system of linear dimension L , $\zeta \lesssim L$, and therefore it is important to apply finite size scaling theory ¹⁰⁾ to obtain the critical properties of the infinite system. We present in this letter Monte Carlo results obtained by computer calculations for the energy and the specific heat of the discrete U(1) gauge model on four-dimensional lattices of size L^4 , where $L = 4, 5$ and 6 . The analysis of our data with the finite size scaling theory ¹⁰⁾ described below indicates clearly the occurrence of a second-order phase transition in this model at $e_c^2 = 0.995$ with a correlation exponent $\nu \approx \frac{1}{3}$.

The Monte Carlo procedure used in our work is the heat bath method of Creutz, Jacobs and Rebbi ⁵⁾. We show in Fig. 1 the mean plaquette energy E_p as a function of $\beta = 1/e^2$ for a lattice of size $L = 5$. To achieve convergence to around 1% starting from cold and hot initial lattice configurations, we have found it necessary to sample up to 7000 iterations near the critical region, $0.90 < \beta < 1.0$. For comparison we have also plotted on Fig. 1 our calculations for E_p obtained from low and high temperature series expansions ¹¹⁾. These agree very well with our Monte Carlo results except in the critical region.

We have evaluated also the specific heat of the $U(1)$ lattice gauge model as a function of β and the lattice size L . We present in Fig. 2 our results for the specific heat per plaquette divided by the single plaquette energy fluctuation¹²⁾, for values of β near the critical region and $L = 4, 5$ and 6 . This normalized specific heat ρ is given by

$$\rho = 1 + \sum_{p' \neq p} [\langle E_p E_{p'} \rangle - \langle E_p \rangle^2] / [\langle E_p^2 \rangle - \langle E_p \rangle^2] \quad (1)$$

where E_p is the energy (action) of the p -th plaquette, and $\langle \rangle$ denotes the thermal average computed by Monte Carlo simulations. It is evident from Fig. 2 that ρ has a maximum as a function of β which increases rapidly as a function of L indicating the occurrence of a second-order phase transition in this model.

In order to obtain the critical parameters for the infinite lattice we apply a finite size scaling analysis to our data. Assume that the correlation length $\zeta_L(\beta)$ for a system of linear dimensions L takes the form

$$\zeta_L(\beta) = L \beta(x) \quad (2)$$

where

$$x = |\beta - \beta_L| L^{1/\nu} \quad (3)$$

and $s(x)$ is a scaling function with the properties that : a) $s(x)$ is analytic for small x and $s(0) = 1$; b) as $x \rightarrow \infty$ $s(x) \sim x^{-\nu}$, where ν is the correlation length critical exponent. Then $\zeta_L \sim |\beta - \beta_L|^{-\nu}$ for $\beta \sim \beta_L$, and ζ_L has a maximum value $\zeta_L = L$ at $\beta = \beta_L$. It can be readily shown that Eqs. (2), (3) imply that for large L

$$\beta_L - \beta_c \sim L^{-1/\nu} \quad (4)$$

where β_c is the critical value of β for the infinite system, and the normalized specific heat ρ has the properties

$$\rho_L = c L^{2/\nu - d} \quad \text{at} \quad \beta = \beta_L \quad (5)$$

$$\rho_L = |\beta - \beta_L|^{-\alpha} \quad \text{for} \quad \beta \sim \beta_L \quad (6)$$

where α satisfies the hyperscaling relation $\alpha = 2 - \nu d$, d = dimensionability and c is a constant.

We have applied Eqs. (3)-(5) to our Monte Carlo data for the normalized specific heat ρ_L to obtain ν and β_c . The data show that the maximum value of $\rho_L \sim L^2$ which, according to Eq. (5) implies $\nu \sim \frac{1}{3}$ for $d = 4$ while a fit to Eq. (4) gives $\beta_c = 1.005$. The curves shown in Fig. 2 correspond to a simple parametrization of the scaling function $s(x)$, Eq. (2), incorporating its asymptotic properties,

$$s(x) = (1 + \lambda x^2)^{-\nu/2} \quad (7)$$

We obtain a very good fit to our data with $\nu = \frac{1}{3}$, $\lambda = 0.6$, and $c = 0.92$, and then apply these parameters to evaluate ρ for the limit $L \rightarrow \infty$.

We should point out that the convergence of ρ and E as a function of the number of Monte Carlo iterations starting from cold and hot lattices becomes increasingly slow as the critical region is approached. This is illustrated for ρ in Fig. 3 in the case $L = 5$ and $\beta = 0.990$. While this behaviour is expected because of the increasing lattice relaxation time in this region, we have found that the approach to equilibrium is not always monotonic, but is marked by occasional rapid changes indicating the possible occurrence of long-lived metastable states in the $U(1)$ lattice.

A direct consequence of the scaling behaviour of compact QED discussed in this paper is that the coefficient a of the area dependence of the Wilson loop should be proportional to ζ^2 near the critical region in the strong coupling regime¹³⁾. This implies that $a \sim |\beta - \beta_c|^{2\nu}$ where we have found $\nu \sim \frac{1}{3}$. A Monte Carlo study of the Wilson loops for this system should verify this behaviour and provide another determination for β_L and ν .

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FIGURE CAPTIONS

Figure 1 The U(1) plaquette energy E_p as a function of $\beta = 1/e^2$ for a lattice of size 5^4 . The curves labelled a) and b) are obtained from low and high temperature series expansions respectively, where ¹¹⁾

a)
$$E_p = 1 - \frac{1}{4\beta} - \frac{1}{32\beta^2}$$

b)
$$E_p = \frac{\beta/2 + \frac{1}{2}(\beta/2)^3 + \frac{49}{12}(\beta/2)^5}{1 + (\beta/2)^2 + \frac{1}{4}(\beta/2)^4 + \frac{1}{36}(\beta/2)^6}$$

Figure 2 The normalized specific heat ρ as a function β near the critical region for lattices of size L^4 where $L = 4, 5$ and 6 . The curves are obtained from the scaling function $s(x)$, Eq. (7), with $\nu = \frac{1}{3}$, $\lambda = 0.6$, and $c = 0.92$.

Figure 3 The normalized specific heat ρ as a function of the number of Monte Carlo iterations for $\beta = 0.990$ and $L = 5$ starting from a cold and a hot lattice.

FIG. 1

FIG. 2

FIG. 3