Structure of hadron structure functions

A. De Rújula*

Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

François Martin

Centre Européen de Recherches Nucléaires, 1211 Geneva 23, Switzerland

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We investigate the comparison of the shapes, as functions of x , of nonsinglet structure functions of pions and nucleons, $F_{\pi,N}(x, Q^2)$. We argue that we can predict (to leading twist, all orders of quantum-chromodynamic perturbation theory) the shape of a pion structure function in terms of the observed shape of the nucleon structure function at the same Q^2 . The uncertainties due to our ignorance of the details of bound-state dynamics are at the 20% level. Our technology is to exploit in detail the relationship between two approaches to the description of structure functions: the operator-product expansion and the covariant parton model.

I. INTRODUCTION AND OUTLOOK

The inclusive scattering of leptons on hadrons and the production of lepton pairs in hadron-hadron collisions are described in terms of hadron structure functions, $F(x, Q_0^2)$, in the standard notation. It is generally agreed that quantum chromodynamics (QCD) predicts $F(x, Q^2)$ from $F(x, Q_0^2)$, provided $Q^2 > Q_0^2 \gg \Lambda^2$. Here Λ is the dynamical parameter setting the scale of the strong interactions in a given renormalization scheme. Whether the predictions are to be trusted in actual practice is still a subject of hot debate.

The Q^2 dependence of structure functions may in principle be understood but, what about their x dependence at fixed Q^2 ? In a naive language, the x dependence reflects the bound-state motion of the hadron's charged constituents. In a more formal language, the x moments of the structure function are hadron matrix elements of operators constructed with quarks and gluons.¹ In both languages the x dependence reflects the dynamics that bind quarks to make hadrons. The QCD bound-state problem has not been fully understood, much less solved. The conclusion that the x dependence of structure functions cannot yet be understood, much less predicted, seems to be inescapable. In this paper we present a challenge to this conclusion. We develop a qualitative understanding of the shape of structure functions by elaborating in detail the relation between the formal approach to structure functions¹ and the covariant parton model.² We attempt to be semiquantitative in understanding the comparison of the shapes of pion and nucleon structure functions.

Let $F_N(x, Q^2)$ be a nonsinglet nucleon structure function, for example, $x F_3(\nu N)$. The observed shape³ of this structure function at $Q^2 = -q^2 \sim 20$

GeV^2 is shown in Fig. 1. Let $F_\pi(x, Q^2)$ be a nonsinglet pion structure function, for example, $F_2(e\pi^-) - F_2(e\pi^+)$. This structure function has not been measured in electroproduction but it can be extracted from data on μ -pair production in pion-nucleon collisions, in a way that we shall review. The results⁴ for $F_\pi(x, Q^2)$ at $Q^2 = +q^2 \sim 20 \text{ GeV}^2$ are also shown in Fig. 1. The normalizations in the figure are arbitrary but the difference in shapes of $F_N(x, Q^2)$ and $F_\pi(x, Q^2)$ is striking. Within errors $F_N \sim x^{1/2}(1-x)^3$ while $F_\pi \sim x^{1/2}(1-x)$. We argue that we can successfully predict the shape of $F_\pi(x, Q^2)$ from the observed shape of $F_N(x, Q^2)$, to all orders of QCD perturbation theory, leading twist (terms of order $1/Q^2$ neglected). Let M and R be the mass and radius of a hadron. Our predictions for F_π are very insensitive to the unknown details of the bound-state dynamics provided, as is the case in nature, $(MR)_N^2 \gg 1 \gg (MR)_\pi^2$.

To relate F_π to F_N we must assume that there exists a momentum scale Q_0^2 (that need not be specified beyond its mere existence) at which it is a good approximation to think of a nucleon as a three-quark state⁵ (with no quark-antiquark "sea") and of a pion as a $q\bar{q}$ state. As we shall explain, we do not have to assume the absence of "binding glue". This "valence" approximation may be more doubtful for a pion than it is for a proton. Because the valence approximation is crucial to our analysis, we must pause in the Introduction to comment on its theoretical basis and experiment status, which we do in the next paragraph.

That the nucleon, as seen by a short-distance probe, has a small antiquark constituency we know from the success of a relation derived in the valence approximation,

$$F_2(\nu N) \simeq x F_3(\nu N), \quad (1.1)$$

FIG. 1. Data on "nonsinglet" nucleon (Ref. 3) and pion (Ref. 4) structure functions. The absolute normalizations are arbitrary.

where N stands for an isoscalar nuclear target. The above relation is satisfied by the data except for small deviations at $x < 0.3$. The absence of prominent multiquark states in the baryon and meson spectrum, as well as the relative suppression of Zweig-rule-forbidden decays are other indications of the relevance of a valence approximation. Perhaps the only theoretical justification for all of these results is the approximate validity of the $1/N_c$ expansion, with N_c the number of quark colors.⁶ The probability for a nonexotic nucleon to be in a $(qq\bar{q}\bar{q})$ state or a nonexotic meson to be in a $(q\bar{q}\bar{q})$ state is $\sim 1/N_c^2$, perhaps 10% for $N_c = 3$. The pion, because of its special status as the (quasi)Goldstone boson of (slightly) broken chiral symmetry, is often regarded as a system to which these naive considerations may not apply. The pion may be a very complicated collective excitation in which the valence quarks are submerged in an ocean of multiquark states. Yet, even in the original work of Nambu and Jona-Lasinio⁷ the pion occurs as a bound state in the two-body systems of quarks with nonzero dynamically generated mass, even if this system is infinitely complicated in the unbroken chiral symmetry basis. Also, recent work by Coleman and Witten⁸ indicates that chiral symmetry is broken in the large- N_c limit, assuming that confinement persists in this limit. These are indications that the pion may not be so different from other hadrons. Because the radius of the pion and nucleon are similar, the quark momenta in both states cannot be very different. In the language

of the $1/N_c$ expansion, this implies that the valence approximation must be similarly good for pions and protons. We do not expect these theoretical asides to convince the reader that the *real* pion is in any sense valence dominated. The ultimate proof is experiment. But we now have a strong experimental indication that the valence approximation for pions is very good. Consider the lepton-pair production processes

$$\sigma(\pi^- N): \pi^- N \rightarrow \mu^+ \mu^- + \text{hadrons}, \quad (1.2)$$

$$\sigma(\pi^+ N): \pi^+ N \rightarrow \mu^+ \mu^- + \text{hadrons}.$$

We know that the valence approximation for the isoscalar target is a good one: N consists of an equal number of u and d quarks with little antiquark contamination. Assume the valence approximation for pions $\pi^- \approx \bar{u}d$, $\pi^+ \approx u\bar{d}$. The above cross sections are then related by the square of quark charges:

$$d\sigma(\pi^- N) = 4d\sigma(\pi^+ N) \quad (1.3a)$$

or

$$d\sigma(\pi^- N) = \frac{4}{3} [d\sigma(\pi^- N) - d\sigma(\pi^+ N)]. \quad (1.3b)$$

In the opposite extreme, the "ocean" approximation where pions are assumed to consist of many pairs, one expects

$$d\sigma(\pi^- N) = d\sigma(\pi^+ N). \quad (1.4)$$

The structure functions $F(\pi^-)$ and $\frac{4}{3}[F(\pi^-) - F(\pi^+)]$ as extracted from the data⁴ on the reactions Eq. (1.1) are superimposed in Fig. 2. The π^- , π^+ difference does not vanish. On the contrary, within errors, Fig. 2 vindicates the valence approximation, Eq. (1.3b). Even at Q^2

FIG. 2. Data (Ref. 4) on π^- and $\pi^- - \pi^+$ structure functions, with the same normalization. The two data sets are compatible within errors, except, perhaps, below $x = 0.3$. This indicates a small "sea" component. Normalizations are arbitrary.

$\sim 20 \text{ GeV}^2$ the valence approximation for pions seems to be reasonably good at $x > 0.3$.

There are two approaches to the analysis of structure functions. One of them is the "intuitive" parton model, in its infinite-momentum-frame or covariant realizations. The other is the formal field-theoretic approach, that deals with the expansion of two current at short distances into a sum of products of local operators, and with the target matrix elements of these operators. In Sec. II we give a derivation of the covariant parton model from the operator-product expansion. An aim is to attempt to make the connection between the formal and the intuitive approaches as precise as we can, in particular, with respect to $O(1/Q^2)$ "higher-twist" corrections. Target-mass effects⁹ and ξ scaling (as opposed to Bjorken's x scaling) are reviewed in Sec. III. Another aim of Secs. II and III is to express the results of the formal approach in terms of explicit functions describing the bound-state dynamics. Let $F_{\text{EXP}}(\xi, Q^2)$ be a nonsinglet structure function of a hadron of mass M , and let $F_{\text{LT}}(\xi, Q^2)$ be the leading-twist approximation to it. Here "leading twist" has two meanings. The first is that the ξ^n moments of F_{EXP} and F_{LT} coincide up to corrections of order n/Q^2 . The moments of F_{LT} at two momentum scales Q^2 and Q_0^2 are related by well known renormalization-group equations.¹ The second meaning of "leading twist" is that only certain operators bilinear in quark fields contribute to F_{LT} . The net result of Secs. II and III is a master formula relating $F_{\text{LT}}(\xi, Q^2)$ to functions describing the target as a bound state. We quote the result (ignoring spin complications to be discussed in the text) and proceed to explain it briefly:

$$F_{\text{LT}}(\xi, Q^2) = \int_0^\infty ds \int_{-\infty}^{M^2 - t - s/1-t} dt p(s, t), \quad (1.5a)$$

$$p(s, t) = [|\Gamma(s, t)|^2 |P(t)|^2 \text{Abs } P(s)]_{Q^2}. \quad (1.5b)$$

Here $p(s, t)$ is a "probability" for the constituent struck by the current to have an invariant mass t and the rest of the target ("spectator" quarks and gluons) to have an invariant mass s . The structure function is an integral over $p(s, t)$ with a $-t_{\text{min}}$ boundary imposed by the kinematics.¹⁰ The probability $p(s, t)$ has an explicit form, Eq. (1.5b), in terms of the amputated vertex function $\Gamma(s, t)$ describing the bound-state vertex $M \rightarrow t + s$, and the complete propagators $P(t)$ of the hit constituent and $P(s)$ of the spectators. All quantities in Eq. (1.5b) are renormalized at the momentum scale Q^2 . Equations (1.5) are the covariant parton model, as derived from the formal operator approach.

Sections IV and V deviate from the main line of the paper and are devoted to developing our intuition on the master formula Eq. (1.5). In Sec. IV we analyze a completely solvable problem: the deep-inelastic structure function of a nonrelativistic bound state.¹¹ We review how the structure function for a bound state of two particles with masses m_1 and m_2 has quasielastic peaks at $x_i = m_i/(m_1 + m_2)$. The half-width Δx around these peaks is of order $1/RM$, with R (M) the bound-state radius (mass). This suggests that a very naive "pion" would have a structure function much wider and peaking at larger x than a very naive "proton", a true statement for the actual pion and proton. Also, in Sec. IV we review results on the structure function of particles bound in a non-relativistic *confining* potential. This helps us clarify how the shape of the leading-twist structure function is sensitive to the target's bound-state dynamics but not to the long-distance forces presumed to be relevant to the confinement of the target's fragments.^{12, 11}

In Sec. V we analyze a "pointlike" model where the vertex function $\Gamma(s, t)$ of Eq. (1.5b) is a constant, and $P(s)$ and $P(t)$ are free-field-theory propagators. This section is meant to clarify the effect of relativistic kinematics on the shape of structure functions. Again, for another version of a naive pion and proton, we find that the structure function of the pion is wider and peaked at larger x than the proton structure function.

In Sec. VI we come back to the main line of the paper. We assume that there exists an unspecified value of Q_0^2 at which the pion is a $q\bar{q}$ state and the nucleon is a qqq state: The wave functions in a specific physical gauge have dominant projections on these Fock states. Although at Q_0^2 we assume that Fock states with gluons are negligible, this does not imply the absence of "glue", in the sense that not all of the target's momentum need be carried by the quarks. We construct a collection of *Ansätze* for the vertex function Γ and the propagators P occurring in Eqs. (1.5). In constructing these *Ansätze* we are guided by the intuition that the relevant quark momenta are $O(1/R)$, with R the bound-state radius. We expect the absorptive part of a quark propagator $\text{Abs } P(s)$ to have a width of order $1/R^2$, the momentum scale over which the strong interactions are strong. In practice we let the form of $\Gamma(s, t)$ and the parameters describing quark propagators vary over large ranges. This affects the form of $F_\pi(\xi, Q_0^2)$ and $F_N(\xi, Q_0^2)$ but has little effect on the comparison of their shapes [the ratios of their moments $F_\pi^n(Q_0^2)/F_N^n(Q_0^2)$]. The moment ratios depend strongly only on the parameters that are common to all our *Ansätze*: the masses and radii

of nucleons and pions, $(MR)_\pi$ and $(MR)_N$. The pion moments change only very slightly as $M_\pi \rightarrow 0$ at fixed R_π .

Section VII is an aside where we analyze the behavior of structure functions as $x \rightarrow 1$. For a nucleon target we find that

$$\lim_{x \rightarrow 1} F_N(x, Q_0^2) \sim (1-x)^3. \quad (1.6)$$

This coincides with the standard QCD "counting-rule" results,¹³ and is a consequence of the three-body nature of the nucleon. For a pion the result is more complicated:

$$\lim_{x \rightarrow 1} F_\pi(x, Q_0^2) \sim \frac{(1-x)^2}{s_0 + \bar{\Lambda}^2(1-x)}. \quad (1.7)$$

This coincides with the standard QCD result¹³ for $s_0 \neq 0$. Here $\bar{\Lambda}$ is a quantity of order $1/R$ and s_0 is the position of the pole of the quark propagator in perturbation theory, renormalized at a momentum scale Q_0^2 . We shall argue that for light $[u, d]$ quarks, $s_0/\bar{\Lambda}^2 < 0.1$. Thus we expect that $(1-x)^2$ behavior predicted by QCD to be relevant only at very large x values, $x > 0.9$. The s_0 dependence of Eq. (1.7) reflects the infrared sensitivity of the "standard" (though suspect) QCD counting rules.

Having computed the leading-twist moment ratios $F_\pi^n(Q_0^2)/F_N^n(Q_0^2)$ in Sec. VI, and found them to be rather model independent, we proceed in Sec. VIII to "evolve" these moments to another momentum scale Q^2 . In so doing we exploit a very useful trick, the fact that for a nonsinglet structure function, the expression

$$\frac{F_\pi^n(Q_0^2)}{F_N^n(Q_0^2)} = \frac{F_\pi^n(Q^2)}{F_N^n(Q^2)} \quad (1.8)$$

is correct to leading twist and to all orders of QCD perturbation theory, and independent of the actual strength of the strong interactions (the value of Λ). Moreover, since our covariant parton model is a model for the leading-twist structure function, the comparison of the leading-twist ratios $F_\pi^n(Q^2)/F_N^n(Q^2)$ with experiment is only in error by terms of order $n\bar{m}/Q^2$, with $\bar{m}$ a quantity of order 1 GeV or less, and Q^2 the large experimental value of order $\sim 20 \text{ GeV}^2$. We exploit these facts in Sec. X to predict $F_\pi(x, Q^2)$ (in the sense of its first few moments) from the observed $F_N(x, Q^2)$. The predicted $F_\pi(x, Q^2)$ is model dependent at the 20% level and coincides, at this level, with the observed structure function. After so much formalism and algebra we recover the naive expectations. First, because $(MR)_\pi^2 \ll (MR)_N^2$ the motion of quark constituents implied by the uncertainty principle is more violent in a pion than in a nucleon, this makes the pion x dis-

tribution wider than the nucleon one. Second, the pion has two valence quarks and the nucleon three; this makes the pion structure function peak at larger x than the proton structure function.

Section IX, dealing with the relation between lepton-pair production and lepton scattering, has to precede the comparison of theory and experiment, because the pion structure function is extracted from lepton-pair-production (LPP) data, while our input nucleon structure function is taken from deep-inelastic neutrino scattering (DIS). The comparison of structure functions, as extracted by experimentalists, from these two types of experiment is known in QCD¹⁴ to receive corrections of order α_s . In terms of moments the largest corrections to LPP data are factorizable into their separate effects on each of the colliding hadrons. The result of the comparison of the two sets of data is predicted in QCD perturbation theory to be of the form

$$F_\pi^n(\text{LPP}) = F_\pi^n(\text{DIS}) \left(1 + \frac{\alpha_s}{4\pi} f_n \right). \quad (1.9)$$

The numbers f_n happen to be very large¹⁴ and the use of perturbation theory is doubtful. Fortunately, we are interested in the shape, not the normalization, of structure functions. The shape corrections involve only ratios of Eq. (1.9) for two different moments. The QCD corrections to these ratios are small and consistently calculable in perturbation theory.

In Sec. XI we comment on the light-quark (u or d) and strange-quark contributions to kaon structure functions. For a pion $(MR)^2 \ll 1$, while for a kaon this is not the case. A consequence of this is that the pion structure function is rather insensitive to quark masses, but the kaon structure functions depend sensitively on them. Thus, in our approach, a measurement of kaon structure functions is a sensitive test of quark masses. The detailed analysis of kaon structure functions will be given in a subsequent paper.

We offer brief conclusions in Sec. XII.

II. FORMAL DERIVATION OF THE COVARIANT PARTON MODEL

In a field theory like QCD, the formal approach to the analysis of inclusive structure functions is formulated in terms of their Cornwall-Norton (or Nachtmann) moments and on the operator expansion of the product of two currents at short distances. Let F denote a nonsinglet structure function, for example, $xF_3(\nu N)$ or $F_2(ep - en)$. Let x be the usual Bjorken variable. We postpone the discussion of target-mass effects till the next section. Let A_n be the leading-twist contribution

to the moments, defined by an inverse-power-series expansion in q^2 :

$$\int_0^1 x^{n-2} F(x, q^2) dx = A_n (\ln q^2) + O(1/q^2). \quad (2.1)$$

The quantities A_n are target matrix elements of operators of twist = dimension - spin = 2. For a nonsinglet structure function there is only one such tower of operators, and

$$A_n p_{\mu_1} \cdots p_{\mu_n} = \langle p | \bar{\psi}(0) \gamma_{\mu_1} D_{\mu_2} \cdots D_{\mu_n} \psi(0) | p \rangle \left[\frac{-(i)^n}{n!} \right]$$

+ permutations,

$$D_\mu \equiv \partial_\mu - ig_s (\lambda^a/2) A_\mu^a, \quad (2.2)$$

where g_s is the color coupling constant and A_μ^a are gluon fields.

To proceed, we introduce an auxiliary vector $\tilde{q}$ orthogonal to q :

$$\tilde{q} \equiv q - \frac{q^2}{pq} p, \quad \tilde{q} \cdot q = 0, \quad (2.3a)$$

$$v \equiv p \cdot q, \quad p^2 \equiv M^2 \quad (2.3b)$$

and choose to work in an axial gauge

$$\tilde{q} \cdot A^a = 0. \quad (2.4)$$

Multiply both sides of Eq. (2.2) by $\tilde{q}_{\mu_1}, \dots, \tilde{q}_{\mu_n}$, to obtain

$$A_n v^n \left[1 - \frac{q^2}{v^2} M^2 \right]^n = -i^n \langle p | \bar{\psi} \tilde{q} (\tilde{q} \cdot \partial)^{n-1} \psi | p \rangle. \quad (2.5)$$

The right-hand side of this equation has the momentum-space diagrammatic interpretation given in Fig. 3(a). In the figure, $P(k)$ is the full propagator of the "struck" quark, $P'(p-k)$ is the full propagator of the "spectators", the "rest" of the target, and $V(k, p-k)$ is the amputated vertex function (a matrix in spin space) describing the target as a bound state. In terms of these functions, the matrix element in Eq. (2.5) is

$$\begin{aligned} & \langle p | \bar{\psi} \tilde{q} (\tilde{q} \cdot \partial)^{n-1} \psi | p \rangle \\ &= i^n \int \frac{d^4 k}{(2\pi)^4} \sum_{\text{spins}} \langle p | VP(k) \tilde{q} (\tilde{q} \cdot k)^{n-1} \\ & \quad \times P^\dagger(k) V^\dagger P'(p-k) | p \rangle. \end{aligned} \quad (2.6)$$

It is convenient to parametrize k in the manner of Sudakov:

$$\begin{aligned} k &= zp + y \frac{M^2}{v} q + k_T, \\ qk_T &= pk_T = 0, \end{aligned} \quad (2.7)$$

$$\tilde{q} \cdot k = z\tilde{q} \cdot p = zv[1 - (q^2/v^2)M^2],$$

FIG. 3. Connection between the operator matrix elements of the formal approach to structure functions and the covariant-parton-model diagrams. For details and notation, see text.

and to introduce names for the invariant masses of the struck quark and the spectators:

$$\begin{aligned} t \equiv k^2 &= zM^2 + \frac{zs + k_T^2}{z-1}, \\ s &= (p-k)^2. \end{aligned} \quad (2.8)$$

To present the gist of the rest of the argument let us, for the time being, concentrate on the quark-antiquark contribution to V in a pseudo-scalar-meson bound state and write the quark propagators in leading order of perturbation theory:

$$P(k) = \frac{\not{k} + \mu}{t - \mu^2 + i\epsilon}, \quad (2.9a)$$

$$P'(p-k) = \frac{\not{p} - \not{k} + m}{s - m^2 + i\epsilon}, \quad (2.9b)$$

$$V = \gamma_5 \Gamma(s, t), \quad (2.9c)$$

where Γ is a scalar function.¹⁵ We will come back to the general case below. Equation (2.6) can now be written

$$\nu A_n = \int_{-\infty}^{\infty} \frac{dz}{(2\pi)^3} \frac{z^n}{2|1-z|z} \int_0^{\infty} dk_T^2 \int_{-\infty}^{\infty} ds \frac{|\Gamma|^2 \text{Tr}[\gamma_5 (\not{k} + \mu) \not{q} (\not{k} + \mu) \gamma_5 (\not{q} - \not{k} + m)]}{(t - \mu^2 + i\epsilon)^2 (s - m^2 + i\epsilon)}. \quad (2.10)$$

The expression for A_n is given in the form of a z^n moment integral. It is tempting to identify $x = z$ and trivially undo the original Mellin transform Eq. (2.1). This is acceptable if we first prove that the support of the z integral is the correct one: $0 < z < 1$. Consider the s integration in Eq. (2.10) in the complex s plane. The pole in the s propagator is below the real axis at $s = m^2 - i\epsilon$. The double pole in t is at

$$s = -\frac{k_T^2}{z} + \frac{(1-z)}{z} [zM^2 - \mu^2] + i\epsilon \frac{1-z}{z}. \quad (2.11)$$

For $(1-z)/z < 0$ ($z > 1$ or $z < 0$) these poles are also below the real axis. One may close the s -integration contour on the upper half plane and, assuming $\Gamma^2(s, t)/s t^2$ is sufficiently well behaved, obtain a null result. Thus, the integrand in (2.10) vanishes unless $0 \leq z \leq 1$. Q.E.D. We may now identify $z = x$, undo the Mellin transform, perform the traces and obtain

$$F(x) = x \int 2 \frac{ds dt}{(2\pi)^2} |\Gamma(s, t)|^2 \times \theta(M^2 x(1-x) - xs - (1-x)t) \times \frac{g(x, s, t) \delta(s - m^2)}{(t - \mu^2 + i\epsilon)^2}, \quad (2.12a)$$

$$g(x, s, t) = x(M^2 + 2m\mu - s) - t + (1-x)\mu^2, \quad (2.12b)$$

where θ is the usual step function.

Several comments and generalizations are now in order.

We have considered only an extreme case where the meson bound state is dominated by a purely quark-antiquark contribution. The quark spins are explicitly correlated in a manner that is reflected in the trace spin sum of Eq. (2.10). The opposite extreme case is the one where the spectators are a quark plus many gluons and quark pairs. In this case the spin correlation is lost, the sum over the struck-quark spin can be performed independently, the trace in Eq. (2.10) becomes $\text{Tr}[\not{q}\not{k}]$ and $g = x$ in Eq. (2.12b). We will refer to this situation as "uncorrelated" spin. Formally, the uncorrelated-spin assumption is $V^\dagger P'(p-k)V \approx 1$, a unit matrix in spin space.

In a meson the recoiling quanta may not be just a single quark. In a baryon they definitely are not. In either case, the propagator $P'(p-k)$ of the spectators is not just the simple pole of Eq. (2.9b). Similarly, if we work to all orders of

QCD perturbation theory, the propagators $P(k)$ of the struck quark will deviate from the simple-pole expression Eq. (2.9b). Let

$$P(k) = \Pi(k^2) \not{k} + M(k^2). \quad (2.13)$$

QCD's asymptotic freedom implies that Π and M satisfy unsubtracted dispersion relations,¹⁶ even though the propagator cannot be reliably computed in perturbation theory at low momenta. Let the spectral representation of Π be

$$\Pi(s) = \int_{-\infty}^{\infty} \frac{\text{Abs}\Pi(s')}{s - s' + i\epsilon} ds' \quad (2.14)$$

and similarly for M . The analytic properties of $P(k)$ and $P'(p-k)$ are enough to paraphrase the preceding derivation of the support of the z integral in Eq. (2.10). The general uncorrelated-spin result for arbitrary propagators is simply

$$F(x, q^2) = x \int ds dt \theta(s) \theta(x(1-x)M^2 - xs - (1-x)t) \times g(x, s, t) p(s, t), \quad (2.15a)$$

$$p(s, t) = [\text{Abs}\Pi'(s) |\Pi(t)|^2 |\Gamma(s, t)|^2]_{q^2}, \quad (2.15b)$$

$$g(x, s, t) = x, \quad (2.15c)$$

where we have absorbed numerical factors into the vertex function Γ . To be precise we have written in the above expression an implicit q^2 dependence: The vertex function and propagators are to be understood as quantities renormalized at a Euclidean momentum scale of order q^2 . Notice that $\text{Abs}\Pi(s)$ in Eq. (2.15) has replaced the simple pole $\delta(s - m^2)$ of Eq. (2.12). Equation (2.15) has been derived in an axial gauge, for which $\text{Abs}\Pi(s) > 0$. Thus the integrand is positive, and the expression for the structure function has the probabilistic interpretation of Fig. 3(b). All that has happened is that we have "opened up" the operator-product-expansion diagram of Fig. 3(a) into the covariant-parton-model absorptive "handbag" diagram of Fig. 3(b). The structure function is an integral over the positive probabilities $p(s, t)$ of Eq. (2.15b) for the struck quark (recoiling particles) to have an invariant squared four-momentum $t(s)$. The causal step function $\theta(s)$ reflects the support of the cut in the recoiling particles collective propagator. The step function $\theta(x(1-x)M^2 - xs - (1-x)t)$ is the $-t_{\min}$ boundary of the allowed phase space for the reaction in Fig. 3(b). In the covariant approach the non-

trivial x dependence fully originates in this θ function. In terms of the vector k_T introduced in Eq. (2.7), the second θ function simply states $|\vec{k}_T|^2 \geq 0$. In a covariant language this is just the requirement that a vector orthogonal to p and q be spacelike.

The structure function as computed from the handbag diagram (HD) of Fig. 3(b) coincides with the leading-twist result of the operator-product expansion (OPE), Fig. 3(a), only up to terms that vanish as inverse powers of Q^2 . Indeed, the OPE result is

$$F(x, q^2) = \int_0^\infty ds \int_{-\infty}^{M^2 x - xs / (1-x)} dt p(s, t), \quad (2.16)$$

while the HD result is

$$F(x, q^2) = \int_0^{-q^2(1-x)/x} ds \int_{q^2/x+s}^{M^2 x - xs / (1-x)} dt p(s, t). \quad (2.17)$$

At any fixed $|q^2| < \infty$ the kinematics of the handbag diagram imply an upper limit on s and a lower limit on t that are not present in the OPE analysis. In QCD we shall see that $p(s, t)$ vanishes for large s or $|t|$ as an inverse power of these quantities. Thus Eqs. (2.16) and (2.17) coincide up to corrections that are inverse powers of q^2 . Strictly speaking, it is not correct to say that the OPE result coincides with the cut diagram of Fig. 3(b). Its correct interpretation at finite q^2 is the half-cut diagram of Fig. 3(c). In practice we will be led to use the QCD renormalization group equations for the q^2 evolution of the leading-twist moments of structure functions. The normalization of these moments at fixed q^2 is to be read off the leading-twist OPE expression Eq. (2.16), not the handbag-model expression Eq. (2.17).

A typical quick reaction to the cut diagram of Fig. 3(b) is to state that it implies that quarks "get out" and is thus in contradiction with confinement. Therefore, the argument goes, we must have made somewhere an assumption that is not justified in QCD. This is not correct. The $F(x)$ of Eq. (2.15) is not the full structure function, but the leading-twist approximation to it. Whether in a confining theory or not, bound states do not occur to any finite order of perturbation theory. We have neglected, among others, the higher-twist contributions of Fig. 4(b). The final-state interactions of Fig. 4(c) are no doubt responsible for the resonance peaks in the actual structure function. The leading-twist $F(x)$ is to be regarded as a smooth interpolation of the actual structure function, in the sense of Ref. 12, and is insensitive to the long-distance forces responsible for the confinement of the fragments of the

FIG. 4. (a) A leading-twist operator matrix element. (b) An example of a next-to-leading-twist operator matrix element. (c) Parton-model counterpart of (b).

target. We offer more insight into this question in a solvable confining theory in Sec. IV.

The spin-correlated general results with full propagators, analog to the spin-uncorrelated expression Eq. (2.15), are further discussed in Sec. VI.

III. TARGET-MASS EFFECTS AND ξ SCALING

The kinematical effects of a nonzero target mass M can be correctly treated to all orders⁹ in M^2/q^2 . So far, in the derivation of Eqs. (2.12) and (2.15), we have been cavalier in this endeavor. To deal with tensors of well defined-spin (and twist) both sides of Eq. (2.2) must be made traceless. This does not affect the right-hand side, since $\not{p}$ and D^2 can be exchanged, via the equations of motion, for operators of lower twist.^{1,17} The derivation of the covariant parton model proceeds as usual. The only new ingredient is a redefinition of $\tilde{q}$ in Eq. (2.3a) as a lightlike vector

$$\tilde{q} \equiv q + \xi p, \quad \tilde{q}^2 = 0. \quad (3.1)$$

The quantity ξ such that $\tilde{q}^2 = 0$ is

$$\xi = \frac{-q^2}{2\nu} \frac{2}{1 + (1 - q^2 M^2 / \nu^2)^{1/2}} \\ \equiv \frac{2x}{1 + (1 - 4M^2 x^2 / q^2)^{1/2}}, \quad (3.2)$$

that is, the usual ξ -scaling variable.⁹ Notice that, in analogy to Eq. (2.4), we are now naturally led to work in light-like gauge. In the contraction of the left-hand side of Eq. (2.2) with a string of $\tilde{q}$'s, the trace terms drop out ($\tilde{q}^2 = 0$). The rest of the derivation proceeds with no change. In the

spin-uncorrelated case, for instance, the target structure is described in terms of a function of ξ :

$$F(\xi) = \int ds dt \theta(s) \theta(\xi(1 - \xi)M^2 - \xi s - (1 - \xi)t) \\ \times \text{Abs} \Pi(s) |\Pi(t)|^2 |\Gamma(s, t)|^2, \quad (3.3)$$

where the q^2 dependence is implicit in the re-normalization scale, as in Eq. (2.15b). The only extra complication is that for vector currents the conventionally defined structure functions are not the objects whose moments are the A_n of Eq. (2.1). But the relevant relations are well known. For νW_2 , for instance,^{9, 12}

$$\nu W_2(q^2, \nu) = x^2 \left(\frac{\xi}{2x - \xi} \right)^3 \left[F(\xi) + 6 \frac{M^2}{(-q^2)} x \left(\frac{\xi}{2x - \xi} \right) \int_0^1 d\xi' F(\xi') + 12 \left(\frac{M^2}{q^2} \right)^2 x^2 \left(\frac{\xi}{2x - \xi} \right)^2 \int_0^1 d\xi' (\xi' - \xi) F(\xi') \right]. \quad (3.4)$$

It is the above relationship that ultimately gives to q^2 and ξ their meaning in terms of quantities measured with the lepton probe. In the derivation leading to Eq. (3.3), q^2 and ξ are "dummy" external variables.

From the point of view of the covariant parton model depicted in Fig. 3(b), the occurrence of ξ in Eq. (3.3) comes as no surprise. Indeed, when terms of $O(M^2/q^2)$ are included, the argument of the second θ function,

$$\xi(1 - \xi)M^2 - \xi s - (1 - \xi)t = \tilde{k}_T^2 > 0, \quad (3.5)$$

still is the correct expression for the squared modules of a spacelike vector orthogonal to p and q . Alternatively, the minimum $-t$ allowed by Eq. (3.5) still is the correct $-t_{\min}$. The meaning of ξ scaling is now clear: ξ is the kinematical combination of the initial-state variables $p^2 = M^2$, q^2 , and $p \cdot q$ that enters into the correct expression for $t_{\min}$ or $|\tilde{k}_T|^2$, in Eq. (3.5).

At any finite q^2 the observed structure function has a support in ξ :

$$0 < \xi < 2/[1 + (1 - 4M^2/q^2)^{1/2}], \quad (3.6)$$

while Eqs. (3.3) and (3.4) have the support $0 < \xi < 1$. This is not a contradiction,¹⁸ $F(\xi)$ is the inverse Mellin transform of the leading-twist moments, not the complete structure function with its physical support. The moments of $F(\xi)$ in the interval $0 < \xi < 1$ are the objects whose q^2 dependence satisfies well known renormalization-group equations.

Why should one include target-mass effects when higher-twist effects of order $\bar{m}^2/q^2$ are neglected? Here $\bar{m}^2$ is an unknown mass scale defining the relative size of twist-2 matrix elements [Fig. 4(a)] and twist-4 matrix elements [Fig. 4(b)].

A safe answer is that the ξ -scaling analysis is correct, and removes from higher twists the kinematical effects of the target mass M . For ξ scaling to be also numerically useful it must be that $\bar{m}^2 \ll M^2$. The argument that this is the case for baryons¹² can be sharpened in the language of the $1/N_c$ expansion.⁶ Let $R \sim \Lambda^{-1}$ be the distance at which the strong interactions become strong, $\alpha_s \sim 1$. Consider the large- N_c , fixed- Λ limit. The mass of a baryon, a bound state of N_c quarks, increases like $N_c \Lambda$. The matrix element of a twist-2 operator [Fig. 4(a)] increases like N_c : the number of quarks contributing to the structure function. The matrix element of a twist-4 operator increases like $\Lambda^2 N_c (N_c - 1)/2$; Λ^2 is its dimension, the rest is the number of ways a gluon can be exchanged in Fig. 4(b). Thus $\bar{m}^2/M^2 \sim (N_c - 1)/2N_c^2 = \frac{1}{9}$ for $N_c = 3$, a somewhat marginal case. For a pion, a $q\bar{q}$ state independently of N_c , $\bar{m}^2/M^2$ does not decrease with N_c . But nobody would defend the numerical usefulness of ξ scaling for a particle as light as a pion.

IV. THE DEEP-INELASTIC STRUCTURE FUNCTION OF A NONRELATIVISTIC BOUND STATE

It is useful, in trying to develop an understanding of the physics contained in the covariant-parton-model structure function of Eq. (2.15), to investigate it in simple situations. We devote this and the next section to such an endeavor.

Consider a quantum-mechanical nonrelativistic bound state of two spinless particles of mass μ and m , $M = m + \mu - B$, with the binding energy $B \ll m, \mu$. Let the current also be scalar and couple to the constituent of mass μ , as in Fig. 5. The expression for the structure function in this

FIG. 5. Diagram underlying the calculation of the structure function of a nonrelativistic bound state.

case is

$$F(x) = \int \frac{ds dt}{(2\pi)^3} |\Gamma(s, t)|^2 \theta(M^2 x(1-x) - xs - (1-x)t) \times \frac{\delta(s - m^2)}{(t - \mu^2 + i\epsilon)^2}. \quad (4.1)$$

In the nonrelativistic limit

$$\theta(xM^2(1-x) - xs - (1-x)\mu^2) \rightarrow \theta(|\vec{k}| - M|x - \mu/(m+\mu)|), \quad (4.2)$$

where $|\vec{k}|$ is the center-of-mass-system (c.m.s.) momentum of the recoiling constituents. Let $\psi(|\vec{k}|)$ be the momentum wave function, with normalization

$$\int \frac{d^3k}{(2\pi)^3} |\psi(\vec{k})|^2 = 1. \quad (4.3)$$

In the nonrelativistic limit, Eq. (4.1) reduces to

$$F(x) = \frac{M}{8\pi^2} \int_{k_{\min}^2}^{\infty} dk^2 |\psi(\vec{k})|^2 + O(1/Q^2), \quad (4.4a)$$

$$k_{\min} \equiv M \left| x - \frac{\mu}{m+\mu} \right| \equiv M |x - x_0|. \quad (4.4b)$$

The quark probability distribution $F(x)$ peaks at $k_{\min} = 0$ or $x = x_0 = \mu/(m+\mu)$. This is the quasi-elastic peak of the structure function. The non-relativistic approach breaks down away from the quasielastic peak as the constituents' momenta sampled in Eq. (4.4a) become large $k > k_{\min} \sim O(M)$. But one may still compute the second derivative of $F(x)$ at the quasielastic peak, and relate it to the mean square radius R^2 of the bound state. Just to give an example, let the target ground-state wave function have a Lorentzian form

$$|\psi(k^2)|^2 = \frac{8\pi}{\sqrt{2R(k^2 + 1/2R^2)^2}}. \quad (4.5)$$

The corresponding structure function is

$$F(x) = \frac{1}{\sqrt{2\pi MR} [(x - x_0)^2 + 1/2R^2 M^2]}. \quad (4.6)$$

This simple expression displays many of the qualitative features that we will find in a relativistic treatment of bound states. Let us, for the sake of discussion, call a "pion" a bound state with $m = \mu$ ($x_0 = \frac{1}{2}$) and a "proton" a bound state with $m \approx 2\mu$ (corresponding to two slowly recoiling quarks, and $x_0 = \frac{1}{3}$). The width of the x distribution around the $x = x_0$ peak is $\Delta(x - x_0) \sim 1/RM$. For two particles with similar radii (like the real pion and proton), the x width of the structure function about the quasielastic peaks becomes larger as the mass of the target diminishes. This is a reflection of the uncertainty principle: The motion of the constituents becomes more violent and the structure function flatter as the mass decreases at fixed radius. The pion and proton structure functions of Eq. (4.5) are shown in Fig. 6, for the actual values of $1/RM$ (2.3 for a pion, 0.26 for a proton). Let us mention in passing that Eq. (4.5) does not satisfy the constituent-counting rule

$$\int_0^1 F(x) dx = 1, \quad (4.7)$$

the integral being strictly unity only over the interval $(-\infty, \infty)$. The correct normalization and

FIG. 6. "Proton" and "pion" nonrelativistic structure functions described in text.

support properties have been lost in the nonrelativistic limit Eq. (4.2). The deviations from the correct sum rule Eq. (4.7) are small for a consistent nonrelativistic bound state ($RM \gg 1$), which the pion is not.

The completely solvable problem of a nonrelativistic-bound-state structure function is also useful in the analysis of higher-twist effects. First of all, mass effects to all orders in $(M^2, \mu^2, m^2)/Q^2$ just change Eqs. (4.4) to the corresponding Nachtmann-scaling result

$$\left(1 - \frac{4M^2 x^2}{q^2}\right)^{1/2} F(\xi) = \frac{M}{8\pi^2} \int_{k_{\min}^2}^{\infty} dk^2 |\psi(\vec{k}^2)|^2, \quad (4.8)$$

$$k_{\min} = M|\xi - \mu/(m + \mu)|.$$

More illuminating is the study of final-state interactions between the scattered constituents, depicted in Fig. 7(a). This problem has been thoroughly analyzed by Bell¹¹ and much of what follows paraphrases his work. To take an extreme view, let the binding potential be a confining one: $V(x) \sim x$, say. In computing the leading-twist diagram of Fig. 5 we have approximated the final states $|n\rangle$ in the sum

$$\langle p | J^\dagger(z) J(0) | p \rangle = \sum_n \langle p | J^\dagger(z) | n \rangle \langle n | J(0) | p \rangle \quad (4.9)$$

by free constituent states. One may include the final-state interactions to all orders by summing over the actual levels of the potential. For a confining potential the exact structure function is a sum of δ functions $\delta(\xi - \xi_n)$ at the positions of the elastic peak and the excited levels

$$\xi_n = \frac{2[1 - (M_n^2 - M_0^2)/2\nu]}{1 + [1 + (2M_0^2/\nu)[1 - (M_n^2 - M_0^2)/2\nu]]^{1/2}}, \quad (4.10)$$

$$M = M_0.$$

FIG. 7. (a) Final-state interactions in a potential model. (b) The wave function in ladder approximation.

FIG. 8. Comparison of the leading-twist structure function with the complete results (a sum of an elastic and a series of quasielastic peaks) in a solvable nonrelativistic confining-potential model.

The exact structure function and the smooth leading-twist approximation to it are symbolically depicted in Fig. 8. They do not look very much the same, yet their ξ moments are equal up to corrections of order $1 + m/R^2Q^2$, where m is the order of the moment. There is an equivalent way of stating this result: smear the exact structure function with an uncertainty $\Delta\xi \sim (R^2Q^2)^{-1}$ and it will match the shape of the leading-twist structure function. The physics underlying this statement is simple. To relate the product of currents in Eq. (4.9) to the structure function one must Fourier transform it with a measure $\exp(iqz)$. Let the desired smearing in ξ (or $x = -q^2/2\nu$) be introduced by allowing for an uncertainty Δq^2 . The relevant intervals dominating the Fourier transform of Eq. (4.9) are then such that $\Delta z^2 \Delta q^2 \sim 1$. Bad resolution and not high energy (as it is often stated) is responsible for light-cone dominance. An uncertainty in ξ thus corresponds to the enhancement of short-distance dynamics. The leading-twist results interpolate (in the sense of a few moments) the exact structure function.¹² The leading-twist structure function does not reflect whether the potential confines or not, though it certainly reflects the bound-state "motion" of the target's constituents.

V. THE POINTLIKE LIMIT OF STRUCTURE FUNCTIONS

In the previous section we investigated the way a structure function reflects the bound-state dynamics in a solvable nonrelativistic case. This section is complementary, its aim is to analyze the role of relativistic kinematics on the shape of structure functions. To isolate this role, we work

FIG. 9. Diagram underlying the calculation of the structure function for a pointlike coupling of a neutral target to charged "constituents".

out the structure function of a zero charge particle with a pointlike trilinear coupling to charged particles, to which we may incorrectly refer as constituents. To leading order of perturbation theory, the structure function to be computed corresponds to the diagram of Fig. 9. Let us, for maximum simplicity, refer to a scalar current, and scalar target and constituents. The contribution of the mass μ constituent (whose charge is taken to be unity) to the leading-twist structure function is then Eq. (4.1). For a pointlike coupling, $|\Gamma(s, t)|^2$ is a constant and

$$F(x) \propto \int dt \theta(M^2 x(1-x) - xm^2 - (1-x)t) \frac{1}{(t - \mu^2 + i\epsilon)^2} \propto \frac{(1-x)}{(x-x_0)^2 + \bar{\gamma}^2}, \quad (5.1a)$$

$$x_0 \equiv \frac{M^2 - m^2 + \mu^2}{2M^2} \begin{cases} \frac{1}{2} \quad (m = \mu, \text{ "pion"}), \\ \frac{1}{3} \left[1 - \frac{B}{M} \left(1 + \frac{B}{2M} \right) \right] \quad (m = 2\mu, \text{ "nucleon"}), \end{cases} \quad (5.1b)$$

$$\bar{\gamma}^2 \equiv \frac{B(M+m+\mu)[M^2 - (m-\mu)^2]}{4M^4}, \quad (5.1c)$$

$$B \equiv m + \mu - M. \quad (5.1d)$$

In the above expressions B is the binding "gap". We take it to be positive, so that the target is stable. The structure function Eq. (5.1a), being proportional to the square of the $M \rightarrow m + \mu$ pointlike coupling, does not satisfy the sum rules of constituent counting. The pointlike example is not a bound-state model and the constituents are not always there.

The expression Eq. (5.1a) vanishes as $x \rightarrow 1$. This reflects the behavior of the struck constituents' propagator that in this limit is driven far off shell. The denominator in Eq. (5.1a) is qualitatively similar to the corresponding one in the

FIG. 10. The structure function $F(x)/x$ for pointlike couplings, described in text.

nonrelativistic result Eq. (4.6). Again we encounter a quasielastic enhancement at $x = x_0$, with $x_0 = \frac{1}{2}$ for equal-mass constituents, and $x_0 \approx \frac{1}{3}$ for $m = 2\mu$ and a small binding gap $B \ll M$. The width γ^2 of the peak is now proportional to the binding fraction B/M , rather than the uncertainty $1/2R^2M^2$ of Eq. (4.6). The pion and nucleon pointlike structure functions $F(x)/x$ for a vector current and uncorrelated-spin- $\frac{1}{2}$ constituents of mass $\mu = 340$ MeV are shown in Fig. 10. As in the nonrelativistic case the pion structure function is much wider than the nucleon structure function. This feature will be reproduced by more realistic calculations to be discussed below and can essentially be traced back, as in this and the previous section, to the fact that in a quark model the pion is in some sense more bound than the nucleon.

In the language of the infinite-momentum-frame parton model, x (or, more precisely, ξ) is the fraction $(k_0 + k_3)/(p_0 + p_3)$ of quark versus target momenta in the "plus" direction. If all the momentum is carried by quarks, one naively expects $\langle x \rangle = 1/n$, with n the number of constituents. The observation that $\langle x \rangle < \frac{1}{3}$ for nucleons (or $\langle x \rangle < \frac{1}{2}$ for pions) is attributed to missing momentum carried by gluons. These considerations are not strictly correct, as is clear from Eqs. (5.1). Consider the pion case, since in the nucleon case the treatment of the spectators ($m = 2\mu$) is un-

realistic and the $x \rightarrow 1$ behavior of Eq. (5.1a) is incorrect (see Sec. VII). For $B/M \ll 1$, $\langle x \rangle \sim \frac{1}{2}$ as expected. But as the binding gap increases $B/M \gg 1$, $\langle x \rangle \rightarrow \frac{1}{3}$. The missing momentum is not carried by gluons, or any other particle, it resides in the binding gap and is a consequence of the relativistic treatment. In constructing valence-dominated structure functions at a certain Q^2 in the next section, we will not feel compelled to impose the "momentum sum rule" $\langle x \rangle \sim 1/n$, that does not follow from general principles.

VI. MESON AND BARYON STRUCTURE FUNCTIONS AT SMALL q^2

Consider the nonsinglet, leading-twist, covariant-parton-model uncorrelated-spin structure function derived from first principles in Secs. II and III [and related to measured functions as in Eq. (3.4)]:

$$F(\xi, q_0^2) = \int ds dt \theta(s) \theta(M^2 \xi(1 - \xi) - \xi s - (1 - \xi)t) \times p(s, t, q_0^2), \quad (6.1a)$$

$$p(s, t, q_0^2) \equiv [|\Gamma(s, t)|^2 |\Pi(t)|^2 \text{Abs} \Pi'(s)]_{q_0^2}. \quad (6.1b)$$

We postpone to the end of the section our comments on the complications arising from spin correlations among the target's constituents.

To proceed to investigate Eq. (6.1) we make an assumption that we defended in the Introduction: there exists a value $q^2 = q_0^2$, for which it is a good approximation to describe a meson as a quark-antiquark state and a nucleon as a three-quark state. In our context, the precise meaning of this assumption is the following. Consider the wave function of a meson (or nucleon) in the $\bar{q}A^a = 0$ lightlike gauge, with $\bar{q} = q + \xi p$, as in Eq. (3.1). The QCD wave function has projections on the Fock states $q\bar{q}$, $q\bar{q} + \text{gluon}$, ... for a meson; and on the states qqq , $qqq + \text{gluons}$, ... for a nucleon. We assume that at a certain renormalization scale q_0^2 , the projection on the $q\bar{q}$ and qqq Fock states is dominant. This is depicted in Fig. 11, and means that there are no "valence" gluons at $q^2 = q_0^2$. But this does not mean that all reference to "glue", as opposed to valence gluons, is lost. Indeed we will find in all our *Ansätze*, to be discussed below, that $\langle x \rangle < \frac{1}{2}$ for pions and $\langle x \rangle < \frac{1}{3}$ for nucleons. Much as in the explicit relativistic discussion of the previous section, some "plus" momentum resides in the binding, or the glue.

Wave functions are not renormalization-scale-invariant concepts. As q^2 increases, a gluon and a sea constituency are developed, in the sense of the renormalization group: The hadron is still composed only of valence quarks, but their per-

FIG. 11. The "valence" approximation for pions and nucleons at $q^2 = q_0^2$.

turbative substructure is revealed by the shorter-distance probe. The renormalization-group equations relating $p(s, t, q_0^2)$ to $p(s, t, q^2)$ in Eqs. (6.1) are equivalent to the renormalization-group equations obeyed by the moments of the leading-twist structure functions. It is a matter of convention to renormalize the matrix elements at q_0^2 and let the operators "evolve" in q^2 , or to renormalize directly the matrix elements at q^2 . Figure 12 is a pictorial representation of this statement.

Let us first concentrate on a meson structure function, dominated by its quark-antiquark constituency at $q^2 = q_0^2$. Equation (6.1) is telling us that we have to specify the complete quark propagator and the bound-state vertex function at this momentum scale. Since we have not solved the QCD bound-state problem, we are forced to make a variety of reasonable guesses. We inform the impatient reader that the predicted pion structure function will turn out to be rather guess independent, while the kaon structure function will be very sensitive to quark "masses".

As we discussed in Sec. II, in QCD perturbation theory the quark propagator satisfies a dispersion relation [Eqs. (2.13) and (2.14)]. In perturbation theory the propagator $\Pi(s)$ has a pole at the square of the renormalized quark mass $s = s_0$ (Q_0^2) and a cut $s > s_0$. But the violent infrared behavior of QCD prevents us from computing

FIG. 12. The evolution of structure function moments in the "operator" (left) or the "wave functions" (right).

$\text{Abs}\Pi(s)$ for small s , a calculation to $O(\alpha_s)$ would diverge as $s \rightarrow s_0 + \Lambda^2$ from above. No doubt this singular behavior of QCD perturbation theory is meaningless. All we know for sure is that $\Pi(s) \sim s^{-1}$ at large s . To proceed, we make a family of nonsingular guesses for $\text{Abs}\Pi(s)$:

$$\text{Abs}\Pi(s) = \frac{\pi^{-1}\gamma^2}{(s - m^2)^2 + \gamma^4}. \quad (6.2)$$

In our approach, the quantity m in Eq. (6.2), or more precisely, $(\langle s \rangle)^{1/2}$, defines a constituent quark mass. The width γ reflects the range at moderate s over which the strong interactions have a large coupling strength ~ 1 . Presumably

both m and γ (for u, d quarks) are $\sim \Lambda$. Yet, in the numerical analysis, we will vary both m and γ in a range from zero to ~ 600 MeV. A propagator with an absorptive part like Eq. (6.2) does not have a free-particle pole, except for $\gamma \rightarrow 0$. Presumably this is not an embarrassment for confined quarks.

Next, we have to guess the vertex function $|\Gamma[s = (p - k)^2, t = k^2]|^2$. Here we are guided by the intuition that the uncertainty in relative momentum of the constituents is of order R^{-1} , with R the radius of the bound state. Let κ^2 denote either the c.m.s. relative three-momentum or the relative four-momentum

$$\kappa^2 = \begin{cases} (\vec{k}_{\text{REL}})^2_{\text{c.m.s.}} = \frac{\lambda(M^2, s, t)}{4M^2} \\ \equiv \frac{M^4 + s^2 + t^2 - 2M^2s - 2M^2t - 2st}{4M^2}, \\ |2k - p|^2. \end{cases} \quad (6.3a)$$

$$(6.3b)$$

We expect the *ground-state* wave function or its closely related relativistic counterpart Γ , to be a simple bell-shaped function with no wiggles. Accordingly, we make a family of *Ansätze*:

$$\text{Gaussian: } |\Gamma|^2 = \exp(-\kappa^2 R^2 \alpha), \quad (6.4a)$$

$$\text{Exponential: } |\Gamma|^2 = \exp(-|\kappa| R \beta), \quad (6.4b)$$

$$\text{Power: } |\Gamma|^2 = (\kappa^2 + R^{-2} \gamma)^{-\rho}, \quad (6.4c)$$

where the numbers α, β, γ are in each case adjusted such that $\langle (\vec{x} - \vec{y})^2 \rangle = R^2$. We find in our numerical analysis that, as long as the radius of the meson is kept at its observed value, the different choices in Eq. (6.4) lead to very similar

results for the structure functions. The large- t behavior of $\Gamma(s, t)$, relevant to the $x \rightarrow 1$ limit of the structure function, is actually computable in QCD. We devote the next section to a discussion of this point.

Let us now turn our attention to a nucleon structure function. As for the pion, we momentarily dismiss spin complications, or, equivalently, discuss the uncorrelated-spin case. The notation for the valence-dominated, three-body kinematics is given in Fig. 13. The explicit expression for $p(s, t)$ in Eq. (6.1b) now involves an integration over the (k_1, k_2) relative phase space of the spectators:

$$p(s, t) = |\Pi(t)|^2 F(s, t), \quad (6.5)$$

$$F(s, t) = \int d^4k_1 d^4k_2 \text{Abs}\Pi(s_1) \text{Abs}\Pi(s_2) \delta^4(k' - k_1 - k_2) \theta(s_1) \theta(s_2) |\Gamma(s, t, s_1, s_2, k_1 \cdot k)|^2 \\ \propto \int ds_1 ds_2 \text{Abs}\Pi(s_1) \text{Abs}\Pi(s_2) \theta(s_1) \theta(s_2) \theta(s - (\sqrt{s_1} + \sqrt{s_2})^2) \frac{\lambda^{1/2}(s, s_1, s_2)}{s} \int_{-1}^1 d\cos\alpha |\Gamma(s, t, s_1, s_2, \cos\alpha)|^2. \quad (6.6)$$

In the above expression α is the angle between $\vec{k}_1$ and $\vec{k}$ in the $(\vec{k}_1, \vec{k}_2)$ center-of-mass system. There is a feature of Eq. (6.6) that is not shared by its quark-antiquark counterpart Eq. (6.1): $F(s, t)$ vanishes as s^2 for small s , a property of two-body phase space. This feature will be most

relevant to the $x \rightarrow 1$ limits discussed in the next section.

To compute $F(s, t)$ in Eq. (6.6) we use again a variety of *Ansätze*. For the propagators we take Eq. (6.2), as before. For the vertex function Γ we first define, in analogy to Eqs. (6.3), the quan-

titles

$$\kappa^2 = \begin{cases} |\vec{k}^2|_{\text{c.m.s.}} = \frac{\lambda(M^2, s, t)}{4M^2} & \text{or} \\ \left(k - \frac{k_1 + k_2}{2}\right)^2, & \end{cases} \quad (6.7a)$$

$$\kappa_{12}^2 = \begin{cases} |\vec{k}_1 - \vec{k}_2|^2 & \text{in the c.m.s. or} \\ (k_1 - k_2)^2, & \end{cases} \quad (6.7a')$$

$$(6.7b')$$

and introduce factorized *Ansätze* analogous to Eq. (6.4). For the Gaussian *Ansatz*, for instance,

$$|\Gamma|^2 = \exp(-\kappa^2 R^2 \alpha) \exp(-\kappa_{12}^2 R^2 \beta). \quad (6.8)$$

In a nonrelativistic harmonic-oscillator potential wave functions do factorize as in Eq. (6.8). In the general three-body problem we have no reason to assume this kind of factorization. Yet, all we want to implement is the assumption that the relative momenta between any two quarks is of order R^{-1} , and Eq. (6.8), while restrictive, does reflect this assumption.

$$F(x) = x \int ds dt \theta(s) \theta(x(1-x)M^2 - xs - (1-x)t) |\Gamma(s, t)|^2 A(x, s, t), \quad (6.9)$$

$$A(x, s, t) = [x(M^2 - s) - t] \text{Abs}\Pi'(s) |\Pi(t)|^2 + 2x \text{Abs}M'(s) \text{Re}[\Pi^*(t)M(t)] + (1-x) \text{Abs}\Pi'(s) |M(t)|^2.$$

The corresponding expression for a nucleon is even more complicated, and requires more information on the quark spin structure. The obvious candidate information is SU(6) symmetry. In this paper we do not explore the effects of spin correlations to their bitter end. The motivation for this shortcoming is practical. The main impact of spin complications on the shape of structure functions [compare Eqs. (6.1) and (6.9)] is the fact that $F(x) \sim x^2$ for uncorrelated spins, while $F(x)$ contains terms both linear and quadratic in x in the correlated-spin case. We have estimated the possible effects of spin correlations in our predictions by using two extreme inputs: $F_\pi(x, q_0^2)$ and $F_N(x, q_0^2)$ both given by the spin-uncorrelated expressions ($\sim x^2$), or both given by the same expressions with one fewer power of x . While the structure functions themselves are clearly different in both cases, the ratios of their moments $F_\pi^n(q_0^2)/F_N^n(q_0^2)$ are not so different. It is these ratios that we eventually use to predict $F_\pi(x)$ at large q^2 , which turns out to be sensitive to spin correlations at the 10–20% level. This is also the level of uncertainty introduced by our different *Ansätze* for wave functions and propagators.

FIG. 13. Notation for the description of a nucleon structure function.

To discuss spin correlations in a spinless meson or spin- $\frac{1}{2}$ nucleon made of spin- $\frac{1}{2}$ quarks we must employ the following procedure. Write a dispersion relation analogous to Eq. (2.14) for the $M(k^2)$ term in the quark propagator of Eq. (2.13). Make an *Ansatz* analogous to Eq. (6.2) for the $\text{Abs}M(k^2)$. Rewrite, in the meson case, Eq. (2.15) as

VII. COUNTING RULES IN THE $x \rightarrow 1$ LIMIT

In this paper we limit ourselves to the study of the leading-twist contribution to structure functions. Higher-twist effects affect structure functions as corrections of the form

$$\left(1 + \sum_i b_i [\bar{m}^2/Q^2(1-x)]^i\right),$$

where $\bar{m}$ is a typical hadronic scale and the b_i are numbers expected to be of order unity. The discussion of the $x \rightarrow 1$ limit of leading-twist structure functions is thus to some extent academic. Higher-twist effects will mask the behavior of the leading-twist contribution as $x \rightarrow 1$ and it is not clear that the data and the leading-twist structure function can be compared as $x \rightarrow 1$. Yet, next-to-leading-twist effects will it is hoped be better understood in the near future, and the $x \rightarrow 1$ leading-twist limit is interesting in its own right. Moreover, the “memory” of the behavior as $x \rightarrow 1$ is not completely lost at finite $1-x$. Thus, we devote a section to this subject, though we are primarily interested in the shape of structure functions at intermediate x .

Consider the correlated-spin purely quark-anti-

FIG. 14. (a) Dominant QCD diagram in the $x \rightarrow 1$ limit of a meson structure function. (b) A similar "uncorrelated-spin" many-body diagram. (c) Dominant QCD diagram in the $x \rightarrow 1$ limit of a nucleon structure function.

quark contribution to a meson structure function, in leading order of perturbation theory for the quark propagators, as given by Eq. (2.12). The leading behavior in $(1-x)$ as $x \rightarrow 1$ originates in the term linear in t in the $g(x, s, t)$ of Eq. (2.12). Explicitly,

$$\lim_{x \rightarrow 1} F(x) = \int ds \delta(s - m^2) \int_{-\infty}^{M^2 - s/(1-x)} \frac{(-t)dt}{(t - \mu^2)^2} |\Gamma(s, t)|^2 \cong \int_{-\infty}^{M^2/(1-x)} \frac{1}{t} |\Gamma(m^2, t)|^2 dt. \quad (7.1)$$

$$\lim_{x \rightarrow 1} F_{\pi}(x) = \int_{-\infty}^{M^2 - s_0/(1-x)} \int_{s_0}^{[M^2 - t](1-x)} ds |\Gamma(s, t)|^2 A(1, s, t) \quad (7.4a)$$

$$\rightarrow (1-x) \int_{-\infty}^{M^2 - s_0/(1-x)} dt |\Gamma(s_0, t)|^2 A(1, s_0, t) \left[(M^2 - t) - \frac{s_0}{1-x} \right]. \quad (7.4b)$$

Now, as in Eq. (7.1), the result depends on the $t \rightarrow -\infty$ dynamics: $M^2 - s_0/(1-x) \rightarrow -\infty$, as $x \rightarrow 1$. For correlated spins, the dominant behavior originates in the $A(1, s_0, t) \sim -t$ term, and for $\Gamma \sim t^{-1}$ as in QCD, we recover the standard $(1-x)^2$

The upper limit of integration tends to $-\infty$: the result depends on the $t \rightarrow -\infty$ as a limit of $\Gamma(m^2, t)$. In QCD, where the interactions are well behaved at short distances, $\Gamma(m^2, t)$ has been argued to be calculable in this limit. The result, dominated by the one-gluon-exchange diagram in Fig. 14(a), is $\Gamma(m^2, t) \sim t^{-1}$, up to logarithms. The limiting behavior of the pion structure function, first derived by Ezawa¹³, is

$$\lim_{x \rightarrow 1} F_{\pi}(x) \propto (1-x)^2. \quad (7.2)$$

The discussion is more complicated if we depart from the simple-pole approximation to quark propagators. The $x \rightarrow 1$ limit of the correlated-spin expression Eq. (6.9) is

$$\lim_{x \rightarrow 1} F(x) = \int_{-\infty}^{M^2} dt \int_0^{(M^2 - t)(1-x)} ds |\Gamma(s, t)|^2 A(1, s, t) \quad (7.3a)$$

$$\rightarrow (1-x) \int_{-\infty}^{M^2} dt |\Gamma(0, t)|^2 A(1, 0, t)(M^2 - t) \propto (1-x). \quad (7.3b)$$

The result is the same for uncorrelated spins Eq. (2.15), with $A \equiv |\Pi(t)|^2 \text{Abs } \Pi'(s)$. In writing Eq. (7.3b) we have assumed that the integrand vanishes fast enough as $t \rightarrow -\infty$ for the relevant values of $(M^2 - t)(1-x)$ in Eq. (7.3a) to be small in the $x \rightarrow 1$ limit. The asymptotic QCD expressions as well as all of our low-momentum *Ansätze* of Sec. VI satisfy the necessary conditions. The limiting behavior Eq. (7.3b) is surprising in two respects. First, it appears to be independent of the detailed behavior of the bound-state dynamics: independent of $|\Gamma(s, t)|^2$. Second, the power behavior is different from the standard QCD result Eq. (7.2). To understand the origin of this mismatch, let us introduce a quantity s_0 , the minimum squared invariant mass of the recoiling constituents. So far, we have set $s_0 = 0$; we will presently develop prejudices on the actual value of s_0 . For $s_0 > 0$, Eq. (7.3) is modified to

behavior. As we shall see in more detail below, the uncorrelated-spin contribution with many recoiling quanta is subdominant in the $x \rightarrow 1$ limit.

We now proceed to discuss the nonuniform limits $s_0 \rightarrow 0$, $x \rightarrow 1$. We can read off the upper inte-

gration limit in Eq. (7.4b) that for $s_0 \neq 0$ we have to specify the integrand as a function of t , only in the limit $t \rightarrow -\infty$. Rather than giving a general analysis, we concentrate on an example that contains all of the physical information we would like to extract. Recall our *Ansatz* Eq. (6.2) for the absorptive part of a propagator. To simplify the expressions (in a way that does not modify the conclusions as $x \rightarrow 1$), take $\gamma^2 \ll M^2$, in which case

$$|\Pi(t)|^2 \sim \frac{1}{(t - \mu^2)^2 + \gamma^4}. \quad (7.5)$$

Perturbative QCD states that at large t , $|\Gamma(s_0, t)|^2 \sim 1/t^2$. We expect the parameter that specifies

the range over which Γ changes appreciably, to be of order R^{-2} , with R the radius of the bound state. The following is a simple *Ansatz* compatible with the QCD limiting behavior:

$$|\Gamma(s_0, t)|^2 \sim \frac{1}{(t - R^{-2})^2 + O(R^{-4})}. \quad (7.6)$$

In a pion, we expect $\mu^2, R^{-2} \gg M^2$. The conclusions as $x \rightarrow 1$ are not changed by the following simplifications: neglect γ in Eq. (7.5), neglect the $O(R^{-4})$ term in Eq. (7.6) and take μ^2 and R^{-2} to be of the same general magnitude $\mu^2 = R^{-2} = \Lambda^2 \gg M^2$. With all these simplifications, Eq. (7.4) becomes, in the correlated-spin case for which $A(1, s_0, t) \sim -t$,

$$\lim_{x \rightarrow 1} F(x) \sim \int_{-\infty}^{M^2 - s_0 / (1-x)} dt \frac{1}{(t - \Lambda^2)^4} (-t) \left[M^2 - t - \frac{s_0}{1-x} \right] \text{Abs} \Pi'(s_0) - \frac{(1-x)^2}{[s_0 + \Lambda^2(1-x)]} \begin{cases} (1-x)^2 & \text{for } s_0 \neq 0, \\ (1-x) & \text{for } s_0 = 0. \end{cases} \quad (7.7)$$

Should we have chosen a power behavior $\Gamma^2(t \rightarrow -\infty) \sim t^{-\alpha}$, the result would have been

$$\lim_{x \rightarrow 1} F(x) \sim \frac{(1-x)^\alpha}{[s_0 + \Lambda^2(1-x)]^{\alpha-1}} \begin{cases} -(1-x)^\alpha & \text{for } s_0 \neq 0, \\ -(1-x) & \text{for } s_0 = 0 \end{cases}. \quad (7.8)$$

The s_0 dependence of Eqs. (7.7) and (7.8) dramatizes the announced infrared sensitivity of the counting rules, that must thus be taken *cum grano salis*. Notice that the power behavior for $s_0 = 0$ is $(1-x)$, independently of the bound-state dynamics (the explicit form of Γ). Since s_0 plays an essential role in the $x \rightarrow 1$ limit, we are led to the crucial question of the relative magnitudes of s_0 and $\Lambda^2 \sim (\text{few hundred MeV})^2$. The quantity s_0 is the onset of the cut (or the position of the pole) in the renormalized quark propagator at the low-momentum scale q_0^2 at which we are trying to estimate the leading-twist structure function [see Eq. (6.1)]. In QCD perturbation theory, and for a quark whose mass is not strictly zero, s_0 diverges as the perturbatively calculated running coupling constant $\bar{\alpha}_s$ diverges at a scale of $\sim \Lambda$. We believe the low-momentum singularity in the running coupling constant (and the consequent singular behavior of s_0) to be spurious, a consequence of our limited understanding of the infrared properties of QCD. It is much more reasonable to expect $\bar{\alpha}_s$ not to become singular at a finite q_0^2 , and to be of order unity at a typical hadronic scale. In this case, and for light (u, d) quarks, whose current-algebra masses are of the order of a few MeV, $(s_0)^{1/2}$ will be of the order of a few times 10 MeV, not hundreds of MeV's and certainly not infinity. Thus we expect

$$\frac{s_0}{\Lambda^2} \sim \left(\frac{\text{few tens MeV}}{\text{few hundreds MeV}} \right)^2 < \frac{1}{10}. \quad (7.9)$$

For s_0/Λ^2 in this range the $x \rightarrow 1$ limiting behavior in Eq. (7.7) changes from $(1-x)$ to the standard QCD result $(1-x)^2$, above $x = 0.9$. In practice, we are not interested in such large values of x , and in the numerical calculations we have set $s_0 = 0$, except for strange-quark propagators.

To complete the study of the pion structure function, consider the uncorrelated-spin diagrams of Fig. 14(b). A similarly lengthy discussion results in the substitution of Eq. (7.8) by

$$\lim_{x \rightarrow 1} F(x) \propto \frac{(1-x)^{1+\alpha}}{[s_0 + \Lambda^2(1-x)]^\alpha} \begin{cases} -(1-x)^{\alpha+1} & (s_0 \neq 0), \\ -(1-x) & (s_0 = 0). \end{cases} \quad (7.10)$$

Notice that in the $s_0 \neq 0$ case, this contribution is subdominant. For $s_0 \rightarrow 0$ the result coincides with the correlated-spin result.

The standard counting-rule QCD prediction for the $x \rightarrow 1$ behavior of a nucleon structure function, based on the analysis of the dominant diagram of Fig. 14(c), is

$$\lim_{x \rightarrow 1} F_N(x) \propto (1-x)^3. \quad (7.11)$$

The power behavior Eq. (7.11) reflects¹³ the fact that more than one propagator in Fig. 14(c) is driven far off shell in the $t \rightarrow -\infty$ limit, at fixed s . Let us introduce again a minimum squared invariant mass s_0 of the spectators in scattering off a nucleon [$s_0 = 4m^2(Q_0^2)$, with m the renormalized mass of a light quark]. As in the discussion of meson structure functions, the QCD result, in this case Eq. (7.11), is reproduced for $s_0 \neq 0$ and reflects the underlying dynamics. In the $s_0 \rightarrow 0$

limit the result again coincides with Eq. (7.11), and is independent of the detailed bound-state dynamics. To convince oneself of this surprising result, consider the analog of Eq. (7.3a) for a nucleon. As we discussed in Sec. VI, the analog of $\Gamma(s, t)^2 \text{Abs}\Pi(s)$ in a nucleon involves the summation over the at-least-two-body phase space of the recoiling spectators. The phase space of two particles of negligible mass and total squared invariant mass s vanishes as s^2 close to $s=0$. Thus, the right-most integrand in Eq. (7.3a) now behaves as s^2 and the integral itself as $(1-x)^3$. In our approach, the counting rule for the $(1-x)$ power behavior in the $s_0 \rightarrow 0$ limit directly reflects the number of constituents, rather than the related number of off-shell propagators as $x \rightarrow 1$. In general, for a target with N constituents

$$\lim_{x \rightarrow 1} F(x) \sim (1-x)^{2N-3}.$$

In conclusion, we expect the leading-twist $F_\pi(x, Q_0^2)$ to vanish as $(1-x)$ for large x and as $(1-x)^2$ for extremely large x ($1-x \ll 0.1$). The leading-twist $F_N(x, Q_0^2)$ vanishes as $(1-x)^3$ all the way to $x=1$. It should be clear from the above discussion that these results are not completely general. A counterexample: if μ and γ in Eq. (7.5) are strictly zero, $F_\pi(x=1)$ is a constant.

VIII. THE EVOLUTION OF STRUCTURE FUNCTIONS WITH q^2

Let a nonsinglet leading-twist structure function $F(x, q_0^2)$ for a pion or nucleon be computed as in Sec. VI, for a given common *Ansatz* of quark propagators and bound-state vertex functions. The standard procedure to predict $F(x, q^2)$ would be as follows.

- (i) Compute the x moments $F^n(q_0^2)$.
- (ii) Rescale the moments with the classic QCD evolution equations to obtain $F^n(q^2)$. The necessary machinery has been developed to second-order renormalization-group-improved perturbation theory.
- (iii) Reconstruct $F(x, q^2)$ from its moments. This procedure has three disadvantages.
 - (i) q_0^2 must be chosen as an arbitrary parameter.
 - (ii) The dynamical QCD scale Λ , defining the coupling constant in a given renormalization scheme, must be chosen as a second arbitrary parameter.
 - (iii) One must hope for the best and trust perturbation theory down to a small momentum scale q_0^2 .

We will presently propose a trick to avoid all of the above problems. To avoid them, a price must be paid. We will not be able to predict *independently* the pion and nucleon structure functions at a momentum scale q^2 . We will predict

the pion $F_\pi(x, q^2)$ from the measured nucleon structure function $F_N(x, q^2)$. But, since we are primarily interested in understanding the different shapes of different particle structure functions, this is a price we are prepared to pay.

Let $F_\pi^n(F_N^n)$ be the n th moment of a leading-twist pion (nucleon) nonsinglet structure function. The q^2 evolution of these moments, to all orders of QCD perturbation theory, is governed by equations of the type

$$\begin{aligned} F_\pi^n(q^2) &= F_\pi^n(q_0^2) R^n(\bar{\alpha}(q^2), \bar{\alpha}(q_0^2)), \\ F_N^n(q^2) &= F_N^n(q_0^2) R^n(\bar{\alpha}(q^2), \bar{\alpha}(q_0^2)), \end{aligned} \quad (8.1)$$

where $\bar{\alpha}_s$ is the running QCD coupling constant, also a function of the momentum scales and the dynamically defined scale Λ^2 . The q_0^2 moments in Eqs. (8.1) are the target-dependent matrix elements of operators that we have discussed at length. But the functions R^n in Eq. (8.1) are target independent, computable in terms of diagrams involving only quarks and gluons. Essentially R embodies the renormalization effects of viewing a quark with probes of different wavelengths. It follows from Eq. (8.1) that the relation

$$F_\pi^n(q^2) = F_N^n(q^2) \frac{F_\pi^n(q_0^2)}{F_N^n(q_0^2)} \quad (8.2)$$

is correct to leading twist, all orders of $\bar{\alpha}_s$. This will be our master formula to predict $F_\pi(x, q^2)$ at large q^2 in terms of the observed nucleon structure function at the same q^2 . The moments at q_0^2 are determined by our model, discussed in Sec. VI, where the existence but not the numerical value of q_0^2 had to be assumed.

There is yet another significant advantage to the use of Eq. (8.2). When we compute $F_\pi(x, q_0^2)$, or its moments, with the different *Ansätze* for quark propagators and bound-state vertex functions described in Sec. VI, we find results that vary significantly from *Ansatz* to *Ansatz*. We cannot decide with confidence which *Ansatz* is best, because the observed structure function and the leading twist one need not be very similar at small q_0^2 . Moreover we do not have sufficient confidence in truncated perturbation theory to use Eqs. (8.1) to compute $F_\pi(x, q^2)$, and compare it with data at large q^2 , where the comparison is sensible. The advantage of Eq. (8.2) is that the ratios $F_\pi^n(q_0^2)/F_N^n(q_0^2)$ vary from *Ansatz* to *Ansatz* much less than the individual pion or nucleon moments. Thus the results for $F_\pi(x, q^2)$ in terms of $F_N(x, q^2)$ depend significantly only on the input parameters that are common to all *Ansätze*. These are the dimensionless quantities $(MR)_\pi$ and $(MR)_N$.

Let us now turn our attention to the effect of higher-twist corrections in the comparison to the data. Let $F^n(q^2)$ continue to be a leading-twist

moment and let $F^n(\text{EXP}, q^2)$ be the moments of the observed structure function. For moments of moderate order, the theoretical expectation is

$$F_N^n(\text{EXP}, q^2) = F_N^n(q^2) \left[1 + O\left(n \frac{\bar{m}^2(N)}{q^2}\right) \right]. \quad (8.3)$$

Combining this expression and Eq. (8.2), we obtain

$$F_r^n(\text{EXP}, q^2) = F_N^n(\text{EXP}, q^2) \frac{F_r^n(q_0^2)}{F_N^n(q_0^2)} \times \left[1 + O\left(n \frac{\bar{m}^2(\pi) - \bar{m}^2(N)}{q^2}\right) \right]. \quad (8.4)$$

Even the most casual reader believes the $O(1/q^2)$ instead of $O(1/q_0^2)$ correction in Eq. (8.4) to be a misprint. This is not true. The $F_N^n(q_0^2)$ computed in our covariant parton model are the leading-twist moments. Corrections of $O(1/q_0^2)$ are present in their comparison to $F_N^n(\text{EXP}, q_0^2)$ but not in Eqs. (8.2) and (8.4). Because of the n/q^2 higher-twist corrections, we can only predict $F_r^n(q^2)$ with a given precision for the first few moments. Thus, we can only trust the comparison of $F_r(x, q^2)$ (as reconstructed from its moments) with experiment, away from the $x \rightarrow 0$ or $x \rightarrow 1$ limits or, more precisely, in the sense of a Δx smearing of order $\Delta x \sim (\text{few hundred MeV}/q)^2$.

IX. DEEP-INELASTIC SCATTERING VERSUS LEPTON-PAIR PRODUCTION

We decided in the previous section to use in the prediction of meson structure functions $F_{r,K}(x, q^2)$ the observed nucleon structure functions $F_N(x, q^2)$ as part of the input. The best measured nonsinglet nucleon structure function³ is $x F_3(\nu N)$, the weak-interaction $V-A$ interference term. In the present state of experimental affairs, meson structure functions are extracted from data⁴ on lepton-pair production: meson + nucleon $\rightarrow \mu^+ \mu^-$ + anything. The double differential distribution of $\mu^+ \mu^-$ invariant masses and longitudinal momenta can be expressed as a product of meson and nucleon structure functions. In QCD the structure functions describing lepton scattering and pair production are identical in the leading-twist, leading-logarithm approximation. But the $O(\alpha_s)$ corrections to their comparison beyond leading logarithms have been computed and found to be gigantic.¹⁴ The largest pieces of these corrections are separable into the effect they have on the meson and the nucleon structure functions describing lepton-pair production. More precisely, the moments $F_r^{(n)}(q^2)$ extracted from lepton-pair production (LPP) and from deep-inelastic scattering (DIS) experiments satisfy, up to an n -independent factor, a relation¹⁴

$$F_r^n(\text{LPP}) = F_r^n(\text{DIS}) \left\{ 1 + \frac{\alpha_s}{3\pi} \left[\frac{8\pi^2}{3} + 2 + g(n) \right] \right\}, \quad (9.1a)$$

$$g(n) = -3 \sum_{j=1}^{n-1} \frac{1}{j} - \frac{4}{n} - \frac{6}{n+1} + 2 \sum_{k=1}^{n-1} \frac{1}{k} \sum_{j=1}^k \frac{1}{j} + 2 \sum_{k=1}^{n-1} \frac{1}{k} \sum_{j=1}^k \frac{1}{j}, \quad (9.1b)$$

where $g(n) \sim 2 \ln^2 n$ for large n . Even for small n the coefficient of α_s in Eq. (9.1) is large, making the use of QCD perturbation theory extremely suspicious. It is $F_r^n(\text{DIS})$ that we are able to predict from $F_N^n(\text{DIS})$, to all orders of QCD perturbation theory. But it is $F_r^n(\text{LPP})$ that we would like to compare with existing data. Thus we seem to have to face untrustworthy $O(\alpha_s)$ corrections. Fortunately, we are saved once more by our interest on the shape, rather than the absolute magnitude, of structure functions. The shape of a structure function is governed by the ratios of its different moments, not their absolute magnitude. For moment ratios Eq. (9.1) becomes

$$\frac{F^{n+1}(\text{LPP})}{F^n(\text{LPP})} = \frac{F^{n+1}(\text{DIS})}{F^n(\text{DIS})} \left\{ 1 + \frac{\alpha_s}{3\pi} [g(n+1) - g(n)] \right\} \quad (9.2)$$

when we have written the result in consistent $O(\alpha_s)$ perturbation theory [the result is also correct to all orders in the large n -independent corrections of Eq. (9.1), if these corrections exponentiate]. The shape correction coefficients $g(n+1) - g(n)$ are small for the moderate n of interest to us.

In comparing our predictions to meson structure functions extracted from LPP data, we make the shape corrections of Eq. (9.2) explicitly. Their effect on a typical pion structure function is shown,

FIG. 15. Comparison of shapes of a pion structure function as extracted from lepton-pair production data (LPP) or deep-inelastic lepton scattering data (DIS) at the same q^2 . The relative normalization is arbitrary.

with arbitrary normalization, in Fig. 15, for $\Lambda = 500$ MeV, $q^2 = 20$ GeV². The corrections are small except at large x , where the structure function itself is small. The LPP structure function is always harder (bigger at large x) than its DIS counterpart.

X. PREDICTIONS FOR THE PION STRUCTURE FUNCTION AND COMPARISON WITH DATA

As discussed in the previous two sections, we compute the moments of a pion structure function at large q^2 from Eq. (8.2). The input $F_N(x, q^2)$ is the deep-inelastic structure function $x F_3(\nu N)$. Small shape corrections [Eqs. (9.2)] are included to obtain the pion structure function entering the description of $\pi N \rightarrow \mu^+ \mu^-$ data. For the relevant neutrino scattering and μ -pair production experiments $|q^2| \sim 20$ GeV². In the rest of this section, we refer only to q^2 in this range. At this momentum scale and in the range $0.03 < x < 0.6$, the neutrino data are well fit by a function of the form¹⁹

$$\begin{aligned} x F_3(x, q^2) &\sim x^\alpha (1-x)^\beta, \\ \alpha &= 0.51 \pm 0.02, \\ \beta &= 3.03 \pm 0.09. \end{aligned} \quad (10.1)$$

Our predictions for $F_\pi(x, q^2)$ are also well represented by a simple function of the form

$$\nu W_2^\pi(x, q^2) \propto F_\pi(x, q^2) \sim x^a (1-x)^b. \quad (10.2)$$

For all of our different input *Ansätze* of quark propagators and vertex functions $a \sim 0.5$, $b \sim 1$. A predicted pion structure function $\sim (x)^{1/2} (1-x)$ is much wider and "harder" (larger at large x) than the observed nucleon structure function $\sim (x)^{1/2} \times (1-x)^3$. This reflects the fact that the input $F_\pi(x, q_0^2)$ is also much wider and harder than $F_N(x, q_0^2)$. Before discussing any details we pause to understand this general fact. Recall that we had already guessed it in unrealistic nonrelativistic models (Sec. IV) and field theoretical models (Sec. V). But, why do our complicated covariant-parton-model expressions systematically behave in a similar manner? There are two reasons. The first is the counting rules described in Sec. VII. For small $(1-x)$, $F_\pi(x, q_0^2) \sim (1-x)$ and $F_N(x, q_0^2) \sim (1-x)^3$, the exponent is twice the number of valence constituents minus three. The memory of this limiting behavior is not completely lost at finite $1-x$. The second reason is that $(M^2 R^2)_N \gg 1 \gg (M^2 R^2)_\pi$, but the consequences of this require a more detailed explanation. The structure function is always an integral of the type Eq. (2.15) over the squared four-momenta t and s of the struck quark and the spectators. The region of integration in the (t, s) plane is limited by the

FIG. 16. The domain of integration in the (s, t) plane in the covariant-parton-model calculation of a nucleon structure function. For a pion structure function M^2 would be much smaller in the scale of this figure, set by R^2 , with R the pion or nucleon radius.

conditions

$$s \geq 0, \quad (10.3a)$$

$$M^2 x(1-x) \geq xs + (1-x)t. \quad (10.3b)$$

These are depicted in Fig. 16. For $x=0$, the second condition is simply $t < 0$, while as $x \rightarrow 1$, s is limited to a wedge $0 < s < O(1-x)$. The envelope of the straight lines [Eq. (10.3b)] is $|\vec{k}_{\text{REL}}|^2|_{\text{c.m.s.}} \propto \lambda(M^2, s, t) = 0$, and is also shown in Fig. 16 as a dashed curve. For a nucleon structure function, the effect of quark propagators is to enhance the integrand in regions shown as shaded domains in the figure. For $\gamma^2 \sim R^{-2}$ in Eq. (6.2) these regions are narrow on the scale of M_N^2 and the propagators produce a quasielastic enhancement somewhere in the vicinity of $x = \frac{1}{3}$. For a pion $M^2 \ll R^{-2}$ and the integrand in Eq. (2.15) is a very slowly varying function in the region $s, t \sim M_\pi^2$ shown in the figure. The dynamics affects the pion structure function very little, the effect of a quasielastic enhancement is minor, and the results are dominated by the effect of kinematical boundaries Eqs. (10.3). The resulting $F_\pi(x, q_0^2)$ is much wider and harder than $F_N(x, q_0^2)$.

We now describe the actual way our numerical calculations are done. Recall that we are limited by higher-twist effects to interpret our comparison to experiments in the sense of predictions for the first few moments of structure functions. In terms of the structure functions themselves, this means our predictions are relevant at intermediate values of x , away from the kinematical boundaries. In practice, and in view of these limitations, we compute a and b in Eq. (10.2) as follows. First, we use the fit Eq. (10.1) and our expressions Eqs. (8.2) and (9.2) to compute the first four theoretical moments of $F_\pi(x, q^2)$, $(x^{-1}, x^0,$

x, x^2). Second, we take the same moments of the *Ansatz* Eq. (10.2). Third, we make the best fit of a and b to the theoretically computed moments. Finally, we check that a few more moments are also reasonably described by Eq. (10.2). In practice this very rough procedure works much better than one would expect, in the sense that the exact inverse Mellin transform is well described by an expression like Eq. (10.2), or, equivalently, the theoretical large n moments are very close to the moments of $x^a(1-x)^b$. The reason is the following. The values of a and b fit from the low moments are always close to $a=0.5$, $b=1$. Imagine taking the expression Eq. (8.2) in the large- n limit. It is a trivial exercise to prove in this limit that if

$$\lim_{x \rightarrow 1} F_N(x, q^2) \sim (1-x)^b,$$

$$\lim_{x \rightarrow 1} F_N(x, q_0^2) \sim (1-x)^{b_0},$$

$$\lim_{x \rightarrow 1} F_\pi(x, q_0^2) \sim (1-x)^{b_0},$$

then

$$\lim_{x \rightarrow 1} F_\pi(x, q^2) = (1-x)^b, \quad b = b_0 + \beta - \beta_0 \approx 1.$$

In the last expression we have used $b_0=1$, $\beta_0=3$ as discussed in Sec. VII, and $\beta \approx 3$, see Eq. (10.1). The behavior of $F_\pi(x, Q^2)$ as $x \rightarrow 0$ can also be readily extracted from Eq. (8.2). The $x \rightarrow 0$ power behavior is dominated by the right-most singularity of $F_\pi^n(Q^2)$ in the complex n plane. For $F_N(x, Q^2) \sim (x)^{1/2}$ at small x , $F_\pi(x, Q^2) \sim (x)^{1/2}$ as well. This is because in our models $F_\pi(x, Q_0^2)$ and $F_N(x, Q_0^2)$ both behave as $x(x^2)$ in the correlated- (uncorrelated-) spin situations. Thus $F_\pi(x, q^2) \sim x^{1/2}$ at small x and $F_\pi(x, q^2) \sim (1-x)$ at large x . It so happens that the "interpolation" $F_\pi(x, q^2) = x^{1/2}(1-x)$ is also a good description of the intermediate- x region.

In Fig. 17 we compare the data⁴ on $F_{\pi^-} - F_{\pi^+}$ with the theoretically predicted shape. The parameters in the quark propagator in Eq. (6.2) are $\gamma=1/2R_p$, with $R_p \approx 0.8$ F, and $m=336$ MeV. The vertex functions Γ are Gaussian as in Eqs. (6.4a) and (6.8). The continuous (dashed) curve in Fig. 17 corresponds to the uncorrelated- (correlated-) spin situations described in Sec. VI. The normalizations are arbitrary. As explained in Sec. VI, the spin correlations in the proton are not treated with precision. They do not affect the large- x behavior of the structure function, see Sec. VII. They affect the small- x behavior and the dashed curve is an estimate of their maximum effect. The correct correlated-spin predictions should have a shape somewhere in between the dashed and the continuous curve in Fig. 17. We shall

FIG. 17. Comparison of data (Ref. 4) and theoretical predictions for a nonsinglet pion structure function. Quark masses are "typical constituent masses", $m=336$ MeV. Normalization is arbitrary.

presently see that the uncertainties in the bound-state dynamics are of the same order of magnitude. The results for parametrizations of Γ that are exponential or power behaved, as in Eqs. (6.4b) and (6.4c) are practically indistinguishable from the results for a Gaussian Γ , and are not shown. Figure 18 is meant to give an idea of the theoretical uncertainty stemming from our ignorance of the parameters describing the bound state. The curves labeled A are the same as in Fig. 17. The curves labeled B correspond to a choice of

FIG. 18. Various theoretical predictions of a nonsinglet pion structure function, indicating the sensitivity to the parameters entering the description of the bound-state dynamics. R is the proton radius. Normalization is arbitrary.

parameters $\gamma = m = 1/R_p$. The curves labeled C are an extreme case for which $\gamma = m \ll M_\pi$. In all cases the vertex function Γ is Gaussian, and the dependence on its form is smaller than the spread of the curves shown in the figure. The continuous (dashed) curves correspond to uncorrelated- (correlated-) spins. As repeatedly advertised, the results are not very sensitive to the details of bound-state dynamics, provided the same type of parametrization is used in each case to describe the nucleon and the pion.

XI. KAON STRUCTURE FUNCTIONS

A kaon nonsinglet structure function has two distinct contributions: a strange-quark contribution, and a light- [u, d] quark contribution. In a naive kaon (nonrelativistic as in Sec. IV or point-like as in Sec. V) the strange-quark contribution would be harder (peaked at larger x) than its light-quark counterpart, a reflection of the naive expectation that the heavier quark carries a larger fraction of momentum.

The comparison of pion and kaon structure functions has three advantages relative to the comparison of nucleon and pion structure functions.

(i) Pion and kaon structure functions are extracted from the same type of experiment, that is, lepton-pair production. Thus, their absolute normalizations, and not only their relative shapes, can be studied within consistent QCD perturbation theory.

(ii) The spin correlations in mesons are sufficiently simple for one to be able to fully analyze the spin-correlated and -uncorrelated scenarios.

(iii) The comparison of kaon and pion structure functions is sensitive to the details of the bound-state models, particularly to the quark masses. Thus one can extract quark masses from this comparison.

We will devote a subsequent publication to the detailed analysis of kaon versus pion structure functions.

XII. CONCLUSIONS

We have shown that we can estimate the shape of a pion structure function from the shape of a nucleon structure function at the same momentum scale. The method is powerful from the point of view of QCD perturbation theory: correct to all orders of α_s , leading twist. The uncertainties due to our ignorance of the QCD bound-state dynamics are not large, provided similar dynamics describe nucleons and pions. The kaon structure functions, unlike the pion's, are very sensitive to the parameters, particularly the strange- and light-quark masses.

Clearly, we cannot claim that our understanding of the shapes of structure functions is a conclusive text of QCD, since we have not solved the bound-state problem. All we can claim is consistency between the naive *Ansatz* that there exists a momentum scale at which mesons and baryons are valence dominated, leading-twist perturbative QCD, and the observed shapes of structure functions.

The methods we have developed provide a simple framework in which to analyze observables other than structure functions. Two examples are elastic form factors and large transverse-momentum distributions.

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