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QUARKS AND LEPTONS AS COMPOSITE STATES OF CONFINED
 $O(n)$ PREONS

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ABSTRACT

We explore the possibility that quarks and leptons are composite states of preons, bound by an $O(n)$ gauge force. Solutions to the consistency conditions proposed by 't Hooft are found. The simplest solution consists in precisely n families of quarks and leptons, which can be interpreted as composite states of one preon and $O(n)$ glue. The breaking of the gauge symmetry associated to the observed interactions requires technicolour, besides $O(n)$ forces.

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The complexity of quark and lepton spectrum, and the existence of more than one replica of the u, d, e, ν_e structure have led many authors ^{1),2)} to suggest that the fermions we see now are composite states of more fundamental, preon, fields. If this is so, we know already that the inverse radius of quarks and leptons, Λ , must be very large, compared to the present energy scale. We estimate that a composite structure of radius Λ^{-1} would give rise to diagonal and non-diagonal anomalous magnetic moments of order :

$$\kappa_{f \rightarrow f'} \approx \frac{e}{\Lambda} \left(\frac{m_f + m_{f'}}{\Lambda} \right) \quad (1)$$

($f, f' = e, u, s, d, \dots$). The mass of f or f' must appear, because a non-vanishing κ requires the breaking of the global chiral symmetry in the light fermion system, and, as a consequence, two powers of Λ^{-1} are needed to give the physical dimensions. The most severe restriction arises from the non-observation of $\mu \rightarrow e \gamma$. We find :

$$\Lambda \approx \left[\frac{96 \pi^3 \alpha}{G^2 B(\mu \rightarrow e \gamma)} \right]^{1/4} > 1.7 \cdot 10^2 \text{ TeV} \quad (2)$$

for ³⁾ : $B(\mu \rightarrow e \gamma) < 1.9 \times 10^{-10}$. Less severe restrictions are given by the μ and e ($g-2$) and by the limits on $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ or $K_L \rightarrow \mu^+ \mu^-$.

In view of the result (2), one is immediately confronted with one basic problem, namely to find a suitable dynamics capable of binding preons into fermions with masses essentially negligible with respect to their inverse radius. The investigation of this problem was started by 't Hooft ⁴⁾, who assumed that the binding force is provided by non-Abelian gauge forces (meta-colour) which become strong at energies of order Λ . If binding is not accompanied by the spontaneous breaking of the chiral symmetry, then massless spin- $\frac{1}{2}$ bound fermions would arise naturally. Furthermore, the composition of the massless bound states with respect to the symmetries external to meta-colour (and which we call metaflavour) is restricted by interesting consistency conditions ⁴⁾, which we shall discuss later on. 't Hooft studied explicitly the case where the metacolour group is a vector-like $SU(3)$, preons transforming like triplets. He found no solution to the consistency conditions.

In this paper, we show that a more favourable situation occurs if the metacolour group is $O(n)$ (with n odd) preons transforming as the n dimensional, vector, representation. We find non-trivial solutions for the bound states; the simplest solution contains n replicas of the same metaflavour structure which we could identify with a sequence of n families; in addition, non-trivial, three preon, solutions are allowed.

Furthermore, within $O(n)$, a qualitative argument can be given, to indicate that binding can be obtained without spontaneous chiral breaking. We shall illustrate in what follows the above results, and consider in the end the gauging of the metaflavour symmetry (to reproduce the observed interactions) and the problem of symmetry breaking.

We assume that preons are described by left-handed Weyl spinors which are n dimensional $O(n)$ vectors, with an additional, metaflavour, index α :

$$\chi_{i,d} \quad \begin{matrix} i = 1, \dots, n \\ d = 1, \dots, N \end{matrix} \quad (3)$$

The symmetry group of the spinors in Eq. (3) is $O(n) \otimes SU(N)$. $SU(N)$ is the metaflavour symmetry, for the moment a global, ungauged symmetry. We cannot enlarge $SU(N)$ with a further $U(1)$ factor, since the corresponding current has Adler-Bell-Jackiw (ABJ) metacolour anomalies⁵⁾. Metacolour currents are free from anomalies, instead, since (3) gives a real representation of $O(n)$. No preon mass is allowed, in the exact $SU(N)$ limit.

We shall restrict n to be odd, to allow for composite, $O(n)$ singlet, states with half-integer spin. Furthermore, if metacolour has to be infra-red divergent (i.e., asymptotically free), we must require⁶⁾ :

$$n > 2 + \frac{2N}{11} \quad (4)$$

The simplest, $O(n)$ singlet, composite states can be constructed out of one preon, whose metacolour charge is screened by the metagluon field. This is an important difference, with respect to QCD, and it is possible only for n odd. Three or more preon states can also be constructed for any odd n . It is not clear that these states are bound, since they could dissociate into three screened preons : they are similar to the exotic (e.g., $q\bar{q}q\bar{q}$) states of QCD. We leave this as an open question. It is interesting to observe that spinless, $X-X$, states are exotic in the same sense. Even though they may still be bound, it is conceivable that the weaker preon attraction, with respect to the QCD case, would not be sufficient to make the scalars acquire a vacuum expectation value. Binding without chiral symmetry breaking could thus be obtained.

We have reported in the Table the $SU(N)$ properties of the one- and three-preon, $O(n)$ singlet, states. By convention, the helicity of these states is taken to be $-\frac{1}{2}$. Since we are allowing for arbitrary internal configurations, both helicities of preons can be freely used. Representations with upper and lower $SU(N)$ indices appear, because $O(n)$ does not distinguish between X and its charge conjugate. For convenience, traces have not been removed, so that the last two tensors are reducible.

For each representation r , the complex conjugate, $\bar{r}$, is also possible with the same helicity. We shall be interested in the so-called indices, $\ell(r)$, defined, for a given helicity, as the number of states transforming like r minus the number of those transforming like $\bar{r}$. Therefore the Table is a complete list of one and three preon states. Exact $SU(N)$ means $\ell(r)$ massless states, transforming like r ($\ell > 0$) or $\bar{r}$ ($\ell < 0$).

The indices $\ell(r)$ must satisfy appropriate consistency conditions ⁴⁾. The first states that if we give an $O(n)$ invariant, Majorana, mass m to say the preon $\chi_{i,1}$, the composite states containing the heavy preon must be able to acquire a mass invariant under the residual $SU(N-1)$ symmetry, i.e., they must form a $SU(N-1)$ vector-like set. This is the only way the $SU(N)$ theory can reduce to the $SU(N-1)$ theory, when $m \rightarrow \infty$ (decoupling condition). The second condition states that the ABJ anomaly of the $SU(N)$ currents, computed with preons, be the same as that computed with the massless bound states running in the triangle loop. A discussion of the meaning of the anomaly condition can be found in Refs. 4), 7).

Notice that the enforcing of the decoupling condition makes the anomaly of the bound states already independent from N ^{*)}. The preon anomaly simply specifies the N independent value of the bound state anomaly. The decoupling condition, for the states of the Table, implies :

$$\begin{aligned} a &= b \\ d &= e \\ c + a + e &= 0 \end{aligned} \quad (5)$$

The $SU(N)$ anomaly of the representation r is proportional to

$$\text{Tr} [\lambda^a(r) \lambda^b(r) \lambda^c(r)] = C(r) d^{abc} \quad (6)$$

where $\lambda^a(r)$ are the $SU(N)$ generators in r , so normalized that $C=1$ for the fundamental representation. Using the values of $C(r)$ given in the Table, the anomaly condition gives :

$$27a + 9e + f = n \quad (7)$$

and we get the following $SU(N)$ composition of bound states :

*) This is because the decoupling requirement implies that we go from $SU(N)$ to $SU(N-1)$ by removing a vector-like (and therefore anomaly free) set of states. This observation makes also clear that the N independence of the total $SU(N)$ anomaly is a weaker requirement, and indeed it can be checked that there are sets of states which have a N independent anomaly but do not obey Eq. (5).

$$\text{bound states } (h = -1/2) = n T_\alpha + l_1 (T_{[\alpha\beta\gamma]} \oplus T_{\{\alpha\beta\gamma\}} \oplus T^{\{\alpha\beta\gamma\}} \oplus 27 T^\alpha) + l_2 (T_{[\beta\gamma]}^\alpha \oplus T_{\{\beta\gamma\}}^\alpha \oplus T^{\{\alpha\beta\gamma\}} \oplus 9 T^\alpha) \quad (8)$$

l_1 and l_2 are integers, and we have explicitly indicated the multiplicity of the $\bar{N}$ representation relative to the others.

The complex of states with indices l_1 and l_2 have some remarkable properties. They form a $SU(N)$ anomaly free set, for any N ; they are massless in the exact $SU(N)$ limit; they can be given a preon interpretation. For $l_2 = -3l_1$ we obtain the pure three-preon set :

$$T_{\alpha,\beta,\gamma} \oplus 3 T_\alpha^{\beta,\gamma} \quad (9)$$

(no symmetry among equivalent indices). As an illustration, we take ^{*)} $N=10$ and specify the properties of preons under the $SU(5)$ group ⁸⁾, according to : $10 = 5 \oplus \bar{5}$. The set (9) reduces to the vector-like $SU(5)$ multiplet :

$$4 (\underline{5} + \overline{\underline{10}} + 5 \underline{5} + 3 \underline{5} + 2 \underline{40} + 3 \underline{45} + 3 \underline{70}) \oplus \text{complex conjugate}$$

For special values of n , the anomaly condition can be saturated by three preon states only. For $n=9$ the general solution is given by the states :

$$T_{\{\alpha\beta\gamma\}} \oplus T_\alpha^{\beta,\gamma} \quad (10)$$

plus the set (9) with arbitrary index.

The observed interactions are obtained by gauging all or part of the metaflavour group. With the preons (3), the gauging of the full $SU(N)$ is forbidden by anomalies. A simple possibility is to choose $N=16$, $l_1 = l_2 = 0$, and to gauge the anomaly free Pati-Salam subgroup ¹⁾ $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, with the preon assignment :

$$\underline{16} = (4, 2, 0) \oplus (\bar{4}, 0, 2) \quad (11)$$

*) The most economical choice of $N=5$ and standard $SU(5)$ quantum numbers leads to bound states which are not vector-like under colour $SU(3)$.

(fractionally charged particles). We then find n families of bound states which replicate the structure (11). Because of (4) we need at least five metacolours, i.e., at least five families of quarks and leptons. This is an interesting phenomenon. The families are a consequence of the exact, hidden, metacolour symmetry, rather than that of an explicit (horizontal) symmetry. Chiral breaking will eventually remove the degeneracy of families but this does not imply massless Goldstone bosons, or the need of gauging the family group, since the $O(n)$ symmetry remains unbroken in any case.

Gauging only a subgroup of $SU(N)$, one could remain with some residual, unwanted, global symmetry. In the case of Eq. (11), the residual symmetry is a $U(1)$ associated to the fermion number ^{*)}

$$F = 3B + L \quad (12)$$

This causes no problem, since we do not need to break F , at this level. Unwanted, conserved, quantum numbers would have remained, if we had gauged a smaller group, e.g., $SU(3)_{\text{col}} \otimes SU(2)_L \otimes U(1)$.

The hardest part of the programme is the realization of the chiral symmetry breaking. The possibility that we see is that some of the scalar composites, which do not acquire a vacuum expectation value in the metacolour dynamics, do so at an energy scale where a part of the metaflavour interaction becomes strong. This means that something similar to the technicolour mechanism ⁹⁾ has to be introduced. It seems possible to us, however, that the addition of technipreons to Eq. (11) may give rise not only to the breaking of $SU(4) \otimes SU(2)_L \otimes SU(2)_R$, but also to normal fermion masses, thus avoiding the need of an extended technicolour ¹⁰⁾. This is because the global chirality of normal fermions could be explicitly broken by their residual metacolour interactions.

In conclusion, the idea that preons are bound into quarks and leptons by a new gauge force, strong at Λ , can be made to work. The limit given in (2) suggests that metacolour may become visible at accessible energies, where leptons would start to behave as strongly interacting particles, and be copiously produced in hadronic collisions ¹¹⁾. At lower energies, it seems almost inevitable that technicolour also shines.

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*) As noted before, the $U(1)$ associated to a common phase rotation of the multiplet (11), which is not an acceptable symmetry, is removed by the metacolour anomaly.

Composition	SU(N) representation r	SU(N) anomaly $C(r)$	Index $\ell(r)$
X	T_α	1	f
$X-X-X$	$T[\alpha\beta\gamma]$	$\frac{N^2-9N}{2} + 9$	a
	$T\{\alpha\beta\gamma\}$	$\frac{N^2+9N}{2} + 9$	b
	$T\{[\alpha\beta]\gamma\}$	$N^2 - 9$	c
	$T^\alpha_{[\beta\gamma]}$	$\frac{N^2-7N}{2}$	d
	$T^\alpha_{\{\beta\gamma\}}$	$\frac{N^2+7N}{2}$	e

SU(N) properties of composite, one and three-preon states. [...], {...} and {[...]} denote, respectively, antisymmetric, symmetric and mixed representations. $C(r)$ and $\ell(r)$ are defined in the text. Traces have not been subtracted from the tensors of the last two lines.

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