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INCLUSIVE RADIATIVE PION AND MUON CAPTURE IN $N = Z$ NUCLEI

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A B S T R A C T

Sum rule techniques are used to predict average excitation energy and total capture rates for inclusive radiative pion and muon capture in $N = Z$ nuclei. The processes under consideration are shown to exhibit different excitation features because of the strong dependence on the orbit of the captured particle. Results are given for ^{16}O and ^{40}Ca . A semi-quantitative agreement with experimental data is found, amenable to the approximations in the treatment of nuclear dynamics.

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1. INTRODUCTION

Inclusive radiative pion and muon capture are best considered simultaneously because of the similarity of the transition operators and of the technique used for their evaluation, i.e. sum rules. Energy-weighted sum rules (EWSR) of validity for $N = Z$ nuclei have been introduced elsewhere¹⁾ for the full plane-wave operator of muon capture.

By this procedure the total capture rate is parametrized in terms of a mean energy and of an enhancement factor [almost equal to the dipole one in the $SU(4)$ limit]. The advantage of EWSR lies in the fact that the role of correlations is less crucial than in non-energy weighted sum rules (NEWSR); none of them is, however, separately predictive. To actually calculate the mean excitation energy, the ratio of EWSR and NEWSR must be explicitly evaluated in some model as is done, for instance, for heavy nuclei in the Fermi gas model described elsewhere²⁾. The EWSR approach has been applied also to radiative pion capture by Lipparini et al.³⁾. The obvious advantage of radiative pion over muon capture consists in the fact that the nuclear excitation spectrum is actually observed. Therefore, to within minor details, the radiative rate can be directly predicted from the measured average excitation energy.

In the present work the problem will be reconsidered and the mean energy will be calculated from the ratio of energy-weighted to non-energy-weighted sum rules in the nuclear model discussed below, for both processes.

As concerns radiative pion capture, this quantity can be compared to the experimental one.

Since the mean energy is much more free from uncertainties than is the rate (see below), this constitutes a further check on the reliability of our theoretical inputs. Then the predicted rates will be compared with experiments for both radiative pion and muon capture.

The simultaneous consideration of these two processes will help in understanding the importance of the orbit of the captured particle for the excitation spectrum.

2. CAPTURE RATES

a) The (π^-, γ) rate for a transition of the pion in the Bohr state $n\ell$ from a nuclear state $|i\rangle$ to all final accessible states $|f\rangle$ is given in the impulse approximation by⁴⁾

$$\Lambda_{\pi, n\ell}^{\gamma} = \left(1 + \frac{m_{\pi}}{M}\right)^2 \frac{1}{2J_i+1} \frac{1}{2\ell+1} \sum_{\substack{M_i, M_f \\ \lambda, m}} \frac{k}{m_{\pi}} \int d\Omega_k |\langle f | O^- | i \rangle|^2 \quad (1)$$

where

$$O^- = \sum_{j=1}^A \tau_j^- \left[A \underset{\sim j}{\sigma} \cdot \underset{\sim \lambda}{\epsilon} + B \underset{\sim j}{\sigma} \cdot \underset{\sim \lambda}{\epsilon} \underset{\sim j}{q} \cdot \underset{\sim j}{k} + C \underset{\sim j}{\sigma} \cdot \underset{\sim j}{k} \underset{\sim j}{q} \cdot \underset{\sim \lambda}{\epsilon} - \right. \\ \left. - i D \underset{\sim \lambda}{\epsilon} \cdot \underset{\sim j}{q} \times \underset{\sim j}{k} + E \underset{\sim j}{\sigma} \cdot \underset{\sim j}{q} \underset{\sim j}{q} \cdot \underset{\sim \lambda}{\epsilon} \right] e^{-i \underset{\sim j}{k} \cdot \underset{\sim j}{r}} \Phi_{n\ell m}^{\pi}(\underset{\sim j}{r}) \quad (2)$$

Here m_{π} and M are the pion and nucleon masses; σ_j and r_j the spin and spatial coordinate of the nucleon j ; ϵ_{λ} and k the photon polarization and momentum; $\underset{\sim j}{q} = -i\nabla_j$ is the pion momentum which is assumed to operate on the pion wave function only; and τ_j^- is defined by $\tau^- |p\rangle = |n\rangle$. The recommended values for the coefficients in Eq. (2) are $A = -0.0326 m_{\pi}^{-1}$, $B = 0.007 m_{\pi}^{-3}$, $C = -0.027 m_{\pi}^{-3}$, $D = -0.014 m_{\pi}^{-3}$, $E = 0.028 m_{\pi}^{-3}$.

The pion wave function, which in principle should be obtained by solving the Klein-Gordon equation in the appropriate optical potential generated by the nucleus, is approximated by a distorted hydrogenic wave function, with standard values for the strong interaction distortion factors $C_{n\ell}$ defined through $\langle \phi_{n\ell}^2 \rangle = C_{n\ell} \langle \phi_{n\ell}^2 \rangle_{\text{(hydrogenic)}}$. The branching ratio of radiative capture to the total absorption $\lambda_{n\ell}^a$ is defined as

$$R = \sum_{n\ell} \omega_{n\ell} \frac{\Lambda_{\pi, n\ell}^{\gamma}}{\lambda_{n\ell}^a} \quad (3)$$

$\omega_{n\ell}$ being the relative probability for pion capture from an $n\ell$ orbit. Furthermore, the assumption that $\Lambda_{\pi,n\ell}^{\gamma}/\lambda_{n\ell}^a$ is independent of n for fixed ℓ is made and the obvious condition $\sum_{\ell} \omega_{\ell} = 1$ is imposed. For the nuclei we are considering, the following values will be used:

$$^{16}\text{O} \quad \omega_s = 0.08, \omega_p = 0.92, \lambda_{1s}^a = 7.50 \pm 0.50 \text{ keV}, \lambda_{2p}^a = 4.60 \pm 0.79 \text{ eV}, \\ C_{1s} = 0.4, C_{2p} = 1.45.$$

$$^{40}\text{Ca} \quad \omega_p = 0.7, \omega_d = 0.3, \lambda_{2p}^a = 2.00 \pm 0.25 \text{ keV}, \lambda_{3d} = 0.7 \pm 0.3 \text{ eV}, \\ C_{2p} = 1.4, C_{3d} = 1.4.$$

The main features of radiative pion capture are:

- i) The dominance of the Kroll-Rudermann $\vec{\sigma} \cdot \vec{\epsilon}$ term (pion-nucleon s-wave) also in higher pion-nucleus orbits as explicitly shown in Ref. 3, in agreement with Delorme and Ericson⁵⁾;
 - ii) the capture from higher pion-nucleus orbits with increasing atomic number, which in turn means a different excitation operator, enhancing higher multipolarities.
- b) The (μ, ν) total capture rate can be written, according to the standard treatment⁶⁾, as

$$\Lambda_{\mu} = \frac{\nu^2 \langle \Phi_{\mu}^2 \rangle}{2\pi} \left[G_V^2 M_V^2 + 3G_A^2 M_A^2 + (G_P^2 - 2G_A G_P) M_P^2 \right] + \Lambda' \quad (4)$$

where

$$M_{\nu, A, P}^2 = \frac{1}{2J_i + 1} \sum_{M_f, M_i} \int \frac{d\Omega_2}{4\pi} | \langle f | O_{\nu, A, P}^- | i \rangle |^2 \quad (5)$$

with

$$O_V^- = \sum_{J=1}^A \chi_J^- e^{-i\vec{\nu} \cdot \vec{r}_J} \quad O_A^- = \sum_{J=1}^A \chi_J^- \frac{\vec{\sigma}_J}{\sqrt{3}} e^{-i\vec{\nu} \cdot \vec{r}_J} \quad O_P^- = \sum_{J=1}^A \chi_J^- \frac{\vec{\sigma}_J \cdot \vec{r}_J}{\sqrt{3}} e^{-i\vec{\nu} \cdot \vec{r}_J}$$

Standard values for the average $1s$ muon wave function⁷⁾ and for the effective coupling constants, $G_V = 1.02 G$, $G_A = -1.46 G$, $G_P = -0.59 G$, with $G = 1.02 \times 10^{-5}/M^2$, have been used.

Concerning the second piece of Eq. (4), Λ' representing recoil corrections, the estimate of Ref. 6 has been followed.

At this point it is probably worth stressing the analogy between the two processes, as has already been done extensively in the literature [see, for example, Ebert and Meyer-Ter-Vehn⁸⁾].

In radiative pion capture the excitation operator is predominantly axial in character, except for the D term. In muon capture there is a larger vector contribution (G_V term) which however represents only 20% of the rate, so that the excitation is again primarily axial. The main difference comes from the wave function which is strictly $1s$ for the muon and, to a very good approximation, $r_j^{\ell} Y_m^{\ell}$ for radiative pion capture in the light nuclei we are considering. The excitation operators to be compared are hence essentially $\sigma_j \exp(-i\mathbf{v} \cdot \mathbf{r}_j)$ and $\sigma_j \exp(-i\mathbf{k} \cdot \mathbf{r}_j) r_j^{\ell} Y_m^{\ell}$.

3. PREDICTIONS FOR MEAN ENERGIES AND TOTAL RATES

Given the transition matrix element of interest, $\Sigma_f |\langle f | O^3 | i \rangle|^2$, an isospin rotation^{1,6)} admissible in $N = Z$ nuclei transforms it into $\frac{1}{2} \Sigma_f |\langle f' | O^3 | i \rangle|^2$, the index 3 standing for the 3rd component of the isospin operator, and f' for the corresponding isobaric states in the parent nucleus.

A natural definition of mean energy is

$$\bar{E} = \frac{\sum_{f'} E_{f'} |\langle f' | O^3 | i \rangle|^2}{\sum_{f'} |\langle f' | O^3 | i \rangle|^2} = \frac{S_1}{S_0} = \frac{\langle i | [O^3, H] O^{3\dagger} | i \rangle}{\langle i | \{O^3, O^{3\dagger}\} | i \rangle} \quad (6)$$

For its actual calculation the nuclear Hamiltonian and the nuclear ground state are necessary. In the following we will use the D1 interaction⁹⁾ for the former,

which is known to reproduce very well the ground state properties of these nuclei, i.e. mean square radius, binding energy, etc. The form of such a Hamiltonian is

$$H = \sum_i \frac{h_i^2}{2M} + \sum_{\beta=1}^2 e^{-\left(\frac{r}{\mu_\beta}\right)^2} \left(W_\beta + M_\beta P_{12}^x - H_\beta P_{12}^y + B_\beta P_{12}^z \right) + t_3 \rho^{1/3}(\underline{r}) \delta(\underline{r}) (1 + P_{12}^\sigma) + i W_{\ell s} (\underline{\sigma}_1 + \underline{\sigma}_2) \cdot (\underline{\nabla} \times \delta(\underline{r}) \underline{\nabla}) \quad (7)$$

With respect to the Skyrme Hamiltonian used in Ref. 3, this force has a finite range, and consequently is expected to reproduce more correctly the behaviour of the strength at high momentum transfer.

The ground state $|i\rangle$ is constructed as a Slater determinant built with harmonic oscillator wave functions. According to the Thouless theorem¹⁰⁾, the evaluation of the double commutator $\langle i | \left[[O^3, H], O^{3\dagger} \right] | i \rangle$ on an uncorrelated ground state, accounts for the effect of dynamical RPA correlations in the excitation spectrum. On the contrary, the evaluation of the non-energy weighted strength (expressed by the anticommutator $\langle i | \{O^3, O^{3\dagger}\} | i \rangle$) depends more critically on the presence of ground state correlations. Therefore our description of the ground state is quite adequate for the calculation of S_1 , while it gives only an approximate estimate of S_0 . The inclusion of dynamical correlations, in the case of isovector excitations, is expected to lower this estimate. An improvement, which is beyond the scope of the present work, might consist in the calculation of $\bar{E}'$, defined as $\sqrt{S_1/S_{-1}}$, where S_{-1} is the inverse energy-weighted sum rule. Such an approach would have the advantage that dynamical RPA correlations are much more easily calculated in S_{-1} ¹¹⁾ than in S_0 .

At the present stage we still regard our position as a reasonable starting point for an actual evaluation of $\bar{E}'$ and consequent prediction of the rates. With these provisions using energy conservation, one immediately obtains from Eq. (6),

$$\bar{E}'_{\mu} = m_{\mu} - \epsilon_{\mu} + E_c - (M_n - M_p) - \nu = \frac{\nu^2}{2M} \frac{(1 + \kappa(\nu))}{(1 - \delta(\nu))} \quad (8a)$$

$$\bar{E}'_{\pi, \ell} = m_{\pi} - \epsilon_{\pi, \ell} + E_c - (M_n - M_p) - k = \frac{(\kappa^2 + \ell(2\ell+1) \cdot \langle r^{2\ell-2} \rangle / \langle r^{2\ell} \rangle) (1 + \kappa_{\ell}(k))}{2M (1 - \delta_{\ell}(k))} \quad (8b)$$

The factors ϵ_{μ} and $\epsilon_{\pi, \ell}$ are the binding energies of the muon and the pion, respectively; the terms E_c and $(M_n - M_p)$ appear because of the isospin rotation by which the actual ν and k are related not to the excitation energy E relative to the ground state of the final nucleus, but to the "fictitious" one in the parent, related to the previous one by $E' = E + E_c - (M_n - M_p) + \Delta M$ (ΔM standing for the mass difference between final and initial ground states). As regards the pion, the index ℓ refers to the fact that it can be captured from different orbits, in contrast to the muon which is captured only with $\ell = 0$. First, *the above equations are implicit ones that can be solved to predict the average neutrino and photon energy.* To that aim, the quantities κ , representing the enhancement of S_1 due to the exchange potential and δ accounting for correlations have been calculated and reported in Table 1. It must be stressed that, contrary to the case of the dipole photoabsorption, the factor κ of the energy-weighted sum rule is influenced also by the Bartlett potential.

It is interesting to compare the different roles played by the seemingly similar excitation operators in the two cases for the mean energy. For orientation purposes, let us just consider the dominant $\sigma_j \exp(-i\nu r_j)$ and $\sigma_j \exp(-ikr_j) r_j^{\ell} Y_{\ell}^{\ell}$. The enhancement factor κ is similar in the two cases. As regards the commutator of the excitation operator with the kinetic part of the Hamiltonian, the different form of the wave function yields in the case of radiative pion capture an "effective" momentum transfer squared $k^2 + |\nabla\phi|^2/\phi^2$, which is manifestly bigger than the photon momentum. This is in turn reflected in the fact that correlations in the NEWSR are much less important (with increasing ℓ) in the case of radiative pion capture with respect to muon capture, for which the momentum transfer is strictly the neutrino momentum.

Therefore, for all but the very lightest nuclei for which a sizeable percentage of (π^-, γ) capture also has $\ell = 0$, mean excitation energies are markedly different in the two cases. In a certain sense, the difference between the actual wave function of the captured particles overcomes the similarity between the same axial excitation operators governing the elementary process. Results are collected in Table 1.

4. RESULTS AND CONCLUSIONS

Our formulation, valid for all $N = Z$ nuclei, is applied to spin-saturated nuclei, well described by the Hamiltonian we use, i.e. ^{16}O and ^{40}Ca .

Mean energies and total rates, predicted through the ratio of EWSR and NEWSR, are given in Table 2. For (π^-, γ) capture, the average excitation energy is obviously defined as $\bar{E}'_{\pi} = \sum_{\ell} \omega_{\ell} \bar{E}'_{\pi, \ell}$. It is apparent, first of all, that $\bar{E}'_{\pi}$ is markedly bigger than $\bar{E}'_{\mu}$, both lying higher up than the giant dipole resonance excitation energy. This reflects the obvious fact that higher multipolarities are excited, building up in the case of radiative pion capture the observed quasi-free spectrum.

Further, it is worth stressing the different sensitivity of $\bar{E}'$ and of the rates to our theoretical inputs because of the phase-space factors.

As a matter of fact, $\Lambda_{\mu} \sim (m_{\mu} - \bar{E}')^2 S_0$ and $\Lambda_{\pi}^{\gamma} \sim (m_{\pi} - \bar{E}') S_0$. It is also evident that *the mean energy in the case of radiative pion capture is a more reliable quantity than the rate itself because the wave function distortion factors and the coupling constants do factorize in the ratio*. In this case, in which such a quantity is experimentally available, our theoretical predictions are somewhat lower than experimental number. This is not unexpected, as anticipated, because of the absence of dynamical correlations in our calculation of S_0 . This feature is of course further reflected in the rates, where theoretical predictions are in turn *higher* than experimental data.

Our results agree with recent more sophisticated calculations of the rate of radiative pion capture in ^{16}O ¹²⁾. The rates of both processes have also been fitted in the NEWSR approach ¹³⁾.

Helium-4 has been omitted in our treatment because of the substantial effect of SU(4) breaking in its ground state. A naive application of our procedure with an SU(4) singlet ground state would give us, for instance, for muon capture a *lower* rate (instead of a *higher* one as expected from our arguments about the calculation of the mean energy) as obtained with a different procedure but with the same assumption about the ground state in Ref. 6.

To summarize, we have compared radiative pion and muon capture predictions via sum rules, mean energies, and total rates. The results are in semiquantitative agreement with experiment. Discrepancies can be attributed to an approximate treatment of nuclear dynamics. No conclusions can be drawn about the validity of the impulse approximation effective Hamiltonian. Radiative pion and muon capture have markedly different excitation features, to be traced back to the excitation operators which are only superficially similar.

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Table 1

Enhancement factor κ_ℓ , correlation factor δ_ℓ , and mean energy $\bar{E}'_\ell$ of the axial excitation operator χ_ℓ (see text) for various values of the angular momentum ℓ as a function of the momentum transfer. The last two columns give the predictions of the mean energy for the processes under consideration. Energy and momentum transfer in MeV.

	q = v, k	70	80	90	100	110	120	200	$\bar{E}'_\mu$	$\bar{E}'_\pi$
^{16}O	κ_0		0.46	0.46	0.46	0.45	0.45	0.42		
	δ_0		0.79	0.75	0.70	0.66	0.62	0.34		
	$\bar{E}'_0(q)$		23.7	25.2	25.9	27.5	29.3		24	28
	κ_1		0.42	0.41	0.40	0.39	0.38	0.32		
	δ_1		0.33	0.38	0.38	0.38	0.37	0.22		
	$\bar{E}'_1(q)$			30.6	32.7	35				34
^{40}Ca	κ_0	0.48	0.48	0.48						
	δ_0	0.81	0.77	0.72						
	$\bar{E}'_0(q)$	20.3	22.9	22.8					22	
	κ_1			0.42	0.41	0.40	0.39	0.35		
	δ_1			0.52	0.50	0.49	0.46	0.25		
	$\bar{E}'_1(q)$			28.9	30.4	32.7	33.8			34
	κ_2			0.33	0.32	0.31	0.30	0.26		
	δ_2			0.29	0.28	0.27	0.26	0.17		
	$\bar{E}'_2(q)$			32.1	33.3	34.6	36.1			35

Table 2

Theoretical and experimental quantities pertaining
to radiative pion and muon capture in ^{16}O and ^{40}Ca

Radiative pion capture					Muon capture	
Nucleus	$\bar{E}'_{\text{exp}}$ (MeV)	$\bar{E}'_{\text{th}}$ (MeV)	R_{exp} (%)	R_{th} (%)	$\Lambda_{\mu}^{\text{exp}}$ (s^{-1})	$\Lambda_{\mu}^{\text{th}}$ (s^{-1})
^{16}O	37	34	2.24 ± 0.48	3.3	$0.98 \pm 0.05 \times 10^5$ $0.97 \pm 0.03 \times 10^5$	16×10^5
^{40}Ca	39	34	1.82 ± 0.15	2.9	$25.5 \pm 0.5 \times 10^5$	42×10^5