MONTE CARLO CALCULATION OF PHOTON, ELECTRON, AND
POSITRON PRODUCTION FROM PRIMARY PROTON BEAMS

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ABSTRACT

Combining the Monte Carlo calculations of the hadron cascade and electromagnetic cascade within a nuclear target irradiated by a high-energy proton beam, we calculate the yields of secondary photons, electrons, and positrons from the target. These yields are useful for the design and optimization of secondary photon, electron, and positron beams. Furthermore, the calculations allow one to study the filling of electron or positron storage rings operating together with a proton synchrotron or storage ring such as the CERN SPS, the BNL ISABELLE, or the proposed Serpukhov UNK.

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1. INTRODUCTION

Secondary hadron production by beams of high-energy protons interacting with nuclear targets is well understood and is usually described by empirical formulae or by theoretical models such as the thermodynamic model¹⁾. The design and optimization of secondary hadron beams around high-energy accelerators can be done using these descriptions of secondary hadron production. Since the construction of the 400 GeV proton synchrotrons at Fermilab and CERN, it has been realized that these accelerators, in addition to producing secondary hadron beams, are able to produce photon, electron, and positron beams of the highest energy at present available, and experiments with such beams are operating. However, the yields of these particles are not yet as systematically understood as are the corresponding hadron yields.

Electron or positron production from a proton target has been proposed²⁾ as an elegant method of filling electron or positron storage rings operating together with a proton synchrotron. Such a scheme could be an interesting alternative to the construction of an electron linac or synchrotron as an injector for the storage ring. Projects for colliding beams of electrons or positrons and protons which have been discussed are the CHEEP electron-proton facility at the CERN-Super Proton Synchrotron (SPS)³⁾, an electron-proton or positron-proton version of ISABELLE⁴⁾, and the electron-proton version of the Serpukhov UNK⁵⁾.

In the present paper we describe the combination of Monte Carlo calculations of the hadron cascade and the electromagnetic cascade in a target irradiated by a high-energy proton beam (Section 2). Using the combined cascade calculations, we calculate the yields of secondary photons, electrons, and positrons from such a target (Section 3).

2. PHOTON, ELECTRON, AND POSITRON PRODUCTION BY PROTON BEAMS -- A COMBINED MONTE CARLO CALCULATION OF HADRON AND ELECTROMAGNETIC CASCADES

2.1 Calculation of the hadron cascade using the Monte Carlo program KASPRO

The Monte Carlo method used in the computer program KASPRO^{6,7)} is designed to give optimal results when calculating secondary particle fluxes from extended

targets (as opposed to point-like targets). The particle flux from a target is an average quantity which is not influenced by the fluctuations in single collisions. Exact energy-momentum conservation in each event is not essential, nor is it at all necessary to sample realistic events. This is especially important when one considers events that happen rarely, such as antiproton production, whereby it becomes advisable to use methods that rely on variance reduction and particle weighting. Such methods are employed in KASPRO for particle production in highly disfavoured regions of phase space; namely, at large longitudinal or transverse momenta. In each collision KASPRO samples only a limited number of secondaries which are weighted in such a way that correct single-particle distributions, average multiplicities, particle production ratios, as well as energy and momentum conservation, are reproduced in the average over many collisions.

However, if one kind of secondary particle is the most interesting in a particular calculation -- such as π^0 production in our case -- the number of these particles can be strongly emphasized (i.e. increased) with a simultaneous reduction of the weight associated with such particles. In this manner the statistical error is considerably reduced and the results are still valid. The method can be used to enhance the production of particles in disfavoured regions of phase space as well. The actual sampling methods that are used in KASPRO have been described in detail elsewhere⁷⁾ and will not be repeated here.

The production of π^0 's in proton-nucleus collisions is described in the calculation by the following formula:

$$E \frac{d^2N}{dp_L p_T dp_T} = a_1 \exp(-a_2 p_L^2 / E_{cm}^2) [\exp(-a_3 p_T^2) + a_4 \exp(-a_5 p_T)] , \quad (1)$$

defined in a nucleon-nucleon centre-of-mass system with parameters as given in Table 1; E , p_L , p_T refer to energy and longitudinal and transverse momentum of the produced pion in the c.m. system, and E_{cm} is the total c.m. energy. Only the shape of the momentum and angular distribution is determined from Eq. (1). The

absolute normalization of the distribution is determined from the inelasticity for π^0 production, $k_{p\pi^0} = 0.15$, which determines the fraction of the primary energy carried away by π^0 mesons. The inclusive distribution given by Eq. (1) has the property of Feynman scaling:

$$E \frac{d^3N}{d^3p} = f(x, p_T) \quad (2)$$

(where $x = 2p_L/E_{cm}$), which is known to agree rather well with experimental data in the proton energy range of interest for our calculations ($E_{lab} = 20$ to 3000 GeV), especially in the fragmentation region from where the majority of high-momentum, small-angle π^0 's result. In the central region, a slow increase of particle production with primary energy has been found in proton-proton collisions at the CERN Intersecting Storage Rings (ISR). This increase is partly described by particle production according to the thermodynamic model¹⁾, but this occurs in a kinematic region that is not expected to be dominant for our application. The inclusive particle production formulae in KASPRO can easily be changed should this appear to be desirable for future applications, or when more accurate data on π^0 production in hadron-nucleus collisions become available.

2.2 Calculation of the electromagnetic cascade using the Monte Carlo program EGS

The Monte Carlo program EGS was developed jointly at SLAC and at HEPL (Stanford University) in order to provide a means for solving problems involving electromagnetic cascades. In particular, EGS is capable of simulating showers in any material, and for electrons, positrons, or photons incident on virtually any geometry. To accomplish this, a companion code called PEGS (Preprocessor for EGS) is used in order to set up the media data needed by EGS, and the two codes essentially constitute what is commonly referred to as Version 3 of the EGS Code System⁸⁾.

In EGS the actual physical processes are simulated as closely as possible. However, in anticipation of the fact that variance reduction techniques could make some calculations more efficient, a weighting parameter is included as part

of the state function description of each particle that is transported, and we make use of this capability in the present calculation that couples EGS with the hadronic cascade code KASPRO.

At very high energies (above 10,000 GeV) the gross behaviour of the electromagnetic shower will probably change as a result of a reduction in the bremsstrahlung and pair production cross-sections. This so-called "suppression effect"^{9,11)} has been discussed in the EGS manual⁸⁾ and elsewhere¹²⁾, and a safe upper limit of 100 GeV has been established for using EGS. Nevertheless, we have used the EGS program with KASPRO at incident proton energies up to 3000 GeV since there will most likely not be very many photons at extremely high energies, and since we have also ignored other high-energy phenomena (e.g. photohadron production) as well.

2.3 The combination of KASPRO and EGS

Both KASPRO and the EGS Code System are at present operational on the IBM-370/168 computer at CERN. To use EGS most easily, the user generally writes a "User Code", as described in the EGS manual⁸⁾. This consists of a MAIN program and the subroutines HOWFAR and AUSGAB, the latter two determining the geometry and output (scoring), respectively. Subroutine SHOWER is called as many times as necessary in the MAIN program of the User Code in order to initiate an electromagnetic cascade history. The parameters of SHOWER, including the weight factor, determine the state of the incident particle.

In coupling KASPRO and EGS, we have elected to make KASPRO part of a more general EGS User Code, such that each time a π^0 is produced in a hadron-nucleus collision, subroutine SHOWER is called with parameter values associated with the photon decay mode of the π^0 . EGS follows the electromagnetic cascade until all particles have reached a cut-off energy of 1 GeV, or until they exit the target. By having subroutine AUSGAB call the analysis routine in KASPRO, the electrons, positrons, and photons that leave the target are sorted into the desired histograms for subsequent output.

3. RESULTS

The combined program calculates the particle fluxes from the target in the form of one- or two-dimensional distributions. For example,

$$N(p, \theta) = \frac{d^2 N}{dp d\Omega} (\text{GeV}^{-1} \text{ sr}^{-1} \text{ inc}^{-1}) , \quad (3)$$

and

$$N'(p, \theta) = \frac{d^2 N}{(dp/p) d\Omega} [(1\% \Delta p/p)^{-1} \text{ sr}^{-1} \text{ inc}^{-1}] , \quad (4)$$

where p and θ , the laboratory momentum and angle of the produced particle, are each calculated in the form of two-dimensional histograms (50×50 bins). The sizes of the Δp and $\Delta \theta$ bins have to be chosen by the user.

One obtains one-dimensional distributions by integrating over momentum or solid angle; that is,

$$N(p) = \frac{dN}{dp} \quad \text{or} \quad N'(p) = \frac{dN}{dp/p} \quad (5)$$

and

$$N(\theta) = \frac{dN}{d\Omega} , \quad (6)$$

respectively. In addition, the program calculates the total flux within ten different angular cones according to

$$N(p)_\theta = 2\pi \int_0^{\theta=n\Delta\theta} \frac{d^2 N}{dp d\Omega} \sin \theta d\theta \quad (7)$$

or

$$N'(p)_\theta = 2\pi \int_0^{\theta=n\Delta\theta} \frac{d^2 N}{(dp/p) d\Omega} \sin \theta d\theta , \quad (8)$$

where $n = 1, 2, \dots, 10$.

Distributions (3) to (8) are of use in designing secondary electron, positron, or photon beams at proton accelerators. In order to give examples for the fluxes to be expected and their dependence on the target material and thickness, calculated results are shown in Figs. 1a to 1c for 400 GeV/c protons incident on Be

and W targets of various thicknesses^{*)}. Smooth curves were drawn by eye through each of the histograms, with one histogram shown on each figure to demonstrate the statistical nature of the data. In order to understand the results for different targets, we first note the differences in the hadronic absorption length λ_{abs} and the radiation length λ_{rad} of the two materials; namely,

$$\begin{aligned}\lambda_{\text{abs}}(\text{Be}) &= 36.7 \text{ cm}, & \lambda_{\text{rad}}(\text{Be}) &= 35.3 \text{ cm}, \\ \lambda_{\text{abs}}(\text{W}) &= 10.3 \text{ cm}, & \lambda_{\text{rad}}(\text{W}) &= 0.35 \text{ cm}.\end{aligned}\tag{9}$$

The number of π^0 's produced in the target depends on the hadronic absorption length, a thickness of one λ_{abs} being optimum. The conversion of photons (from π^0 decay) into electrons and positrons depends on the radiation length of the material. In beryllium the radiation length is of the same order of magnitude as the absorption length, so that e^+e^- production increases with target thickness as long as the target thickness is not much larger than λ_{abs} . Pair conversion in the second half of the Be target is less than optimal, since the photons see less than one radiation length of material. In tungsten (see Fig. 1b), just the opposite happens. The production of e^+e^- only increases for target thicknesses up to about 1 cm, which is much less than one absorption length. The short radiation length (0.35 cm) provides the reason; namely, the electrons and positrons are degraded in energy with the development of the electromagnetic cascade. The optimum target length for W depends strongly on the secondary momentum of interest. For small secondary momenta the production is still rising with target thickness at 2 cm. At large secondary momenta the 0.5 cm target starts to become more efficient than the 1 cm target, and the production from the 2 cm target is already strongly suppressed.

Because of these effects the optimum production from a Be target is larger than from a W target. The production from the Be target can be further improved by adding about one radiation length of a heavy material, such as tungsten,

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^{*)} As expected the e^+ and e^- yields were found to be the same in these calculations; hence, the results were averaged.

downstream. As seen in Fig. 1c, the tungsten converts the photons, produced by π^0 's made in the beryllium, into e^+e^- pairs.

We should mention that targets made of light materials such as Be are also more favourable if the radiation heating by the combined hadronic and electromagnetic cascades is considered. The production of e^+ , e^- , and γ 's depends on many things: primary momentum, target size and material, secondary momentum, secondary angle, and angular and momentum acceptance of secondary beams. Therefore, we have not attempted to give results that would allow secondary beam design in all possible situations. It is recommended instead that the KASPRO/EGS program be run for each particular beam design in order to determine the optimum particle fluxes.

In addition to momentum and angular distributions, the emittance area of the secondary beam is of interest for injection of hadron-produced electrons or positrons. In Figs. 2a to 2d we plot the quantity

$$F(p,L) = \left. \frac{dN}{dp/p} \right|_L, \quad (10)$$

which is the number of particles (e^+ or e^-) per incident proton per 1% $\Delta p/p$ within the emittance area L (in rad·m). The particle production is taken in the forward direction inside a cone with a given opening angle θ . The incident beam was Gaussian in shape in both transverse planes x and y ; that is,

$$N(x) \approx \exp \left(- \frac{x^2}{2(\Delta x)^2} \right) \quad (11)$$

with Δx = r.m.s. radius = 1 mm. The momentum distribution of electrons or positrons, $N'(p) = dN/(dp/p)$ [Eq. (5)], was reasonably flat in terms of the secondary momentum p , so that the results are given for rather large momentum bins. Within the bins no significant variation is expected from one secondary momentum to another.

The primary proton momentum of $p_0 = 400$ GeV/c was chosen as being typical for the FNAL, the CERN SPS, and the BNL ISABELLE accelerators. In Figs. 2a to 2d

we also give results for one target irradiated with 3000 GeV/c protons, typical for the Serpukhov UNK⁵⁾. The secondary momentum range for the 3000 GeV/c curves is about twice as large as for the 400 GeV/c ones.

As an application of these curves (see Fig. 2b) let us consider the number of 12 GeV/c positrons or electrons, within a momentum bite of 1% $\Delta p/p$ and an emittance interval 1 to $1.5\pi \times 10^{-6}$ rad·m, which could be injected into an e^+ or e^- storage ring operating together with a proton accelerator. For 400 GeV/c protons we get 2×10^{-4} secondaries (per incident proton) from a 40 cm Be followed by a 0.3 cm W target. For 3000 GeV/c primaries striking a 30 cm Be plus 0.3 W target, we get 1×10^{-3} secondaries per proton. If the proton accelerator gave a burst of 5×10^{13} protons every 10 seconds, we would need 500 bursts of 400 GeV/c protons in order to fill a storage ring with 5×10^{12} electrons or positrons; or 83 min filling time. In the case of 3000 GeV/c protons we need only 100 bursts, or a filling time of 17 minutes. The secondary momentum dependence of these numbers is rather weak.

In filling storage rings in this manner not only the total number but also the time structure of the secondaries is of importance. The latter will be identical to that of the proton beam incident on the target. However, judging only from the number of secondaries produced, we conclude from our calculations that electron or positron storage rings could be filled using hadron beams, at least in the momentum range of several TeV/c. For the design of such a scheme, further optimization calculations will be required.

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Table 1

Parameters in the particle production model [Eq. (1)]
for different target materials. The parameters for
other materials are determined by interpolation

A	a_1	a_2	a_3	a_4	a_5
H	4.94	33.8	6.11	0.69	4.12
Be	1.81	33.4	3.01	5.12	7.34
Al	1.54	35.5	3.7	3.03	4.94
Cu	2.36	37.2	5.83	0.76	3.22
Pb	2.36	37.2	5.83	0.76	3.22

Figure captions

- Fig. 1 : The momentum distribution [Eq. (7)] of secondary e^+ or e^- produced into an opening angle of 9 mrad by 400 GeV/c protons incident on various targets: a) Be; b) W; c) Be followed by W.
- Figs. 2a-2d : The number of electrons or positrons produced in various targets within a 1% $\Delta p/p$ momentum interval at secondary momentum p , and within an emittance area L (abscissa).

Fig. 1a

Fig. 1b

Fig. 1c

Fig. 2a

Fig. 2b

Fig. 2c

Fig. 2d