METHODS OF DETERMINING THE ELECTRON DRIFT VELOCITY IN DRIFT CHAMBERS

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ABSTRACT

A displacement method for measuring electron drift velocities in adjustable-field drift chambers has been developed and its results have been compared with those of a track-reconstruction method, which uses a least-squares track-fitting procedure and computes drift velocities by iteration. The reconstruction method has been adapted for use with tracks of unknown momentum curved by a magnetic field.

Drift velocities in a set of chambers fitted with double sense wires and operating in magnetic fields between zero and 0.8 T have been determined by the two methods. The results agree within the limits of error and show a non-linear time-distance relation.

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1. INTRODUCTION

The time-distance relation (TDR) for a drift chamber fitted with double sense wires may be written as

$$x_h = x_k + (-1)^k \int_0^{t_d} v(t) dt = x_k + (-1)^k \int_T^{t_0} v(t) dt. \quad (1)$$

Here x_h is the coordinate of a hit in the k th cell, x_k is that of the k th sense wire, $v(t)$ is the instantaneous electron-drift velocity and t_d is the drift time.

T is a complementary drift time defined by

$$T = t_0 - t_d,$$

where t_0 is the net delay between the triggering signal and that formed on the sense wire. Equation (1) is often written in terms of a mean drift velocity $w(t_0, T)$ between the position of the hit and the sense wire as

$$x_h = x_k + (-1)^k (t_0 - T) w(t_0, T). \quad (2)$$

To establish the TDR, t_0 and either $v(t)$ or $w(t_0, T)$ must be determined. The drift-time spectrum yields approximate values of these quantities [see, for example, Cheng et al.¹⁾], but special methods have been developed for their accurate measurement. These methods rely either on displacement or on some form of track reconstruction.

In the displacement methods²⁻⁴⁾ a chamber and a pencil beam, which may in turn be defined by other chambers, are moved with respect to one another and the resulting change in drift time is observed. Two beams of known separation have also been used^{4,5)}. Displacement methods are especially suited to studying drift velocities as a function of different parameters; they become impracticable in complex detection systems.

When three or more chambers are available the TDR can be obtained by track reconstruction using successive approximations. The method described by Twijnstra⁶⁾ and Spierenburg⁷⁾ is an example. Here hits are registered in several planes and trajectories are fitted to them using assumed drift velocities. By an iterative procedure chamber parameters and track parameters are then adjusted alternately until the fit between the track and the hit positions is optimized.

The problems encountered when operating drift chambers in magnetic fields also affect the drift-velocity measurements. Displacement measurements in magnetic fields are described in Refs. 2, 4, 8, and 9, but the track-reconstruction method of Refs. 6 and 7 used straight tracks.

We have employed both the displacement and the reconstruction method in the course of an experiment performed with the Omicron Spectrometer on the backward scattering of pions¹⁰). Figure 1 is a layout showing four adjustable-field drift chambers D1 to D4. They are in a magnetic field of about 1 T and are traversed by the scattered pions. Details of the chambers and their operating conditions are published in Ref. 9. For the displacement measurements a test beam of known momentum was directed through the scintillators S1 and S2 and so traversed the chambers. The reconstruction method was applied to tracks of particles backscattered by the target TA.

2. THE DISPLACEMENT METHOD

At least three chambers are required for our version of this method. Two of these are fixed and define the trajectories used to test the others. The trajectories may be straight or curved. The chambers are irradiated in a given beam and hits are registered on tape. The chambers under test are then displaced and a further set of hits is recorded. For the analysis pairs of drift-time intervals called "time-windows" are specified in the fixed planes. Each pair defines a set of trajectories and a programme selects tracks which follow these trajectories from the events recorded.

The displacement of a chamber by a distance s must be parallel to its plane and perpendicular to the wires of the cells tested. If T and T' are the complementary drift times measured in a chamber for a given trajectory before and after displacement then, by Eqs. (1) and (2)

$$s = x_h - x'_h = \int_T^{T'} v(t) dt = (T' - T) w(T', T) . \quad (3)$$

The velocity measured is the mean value over the interval $T' - T$. By using several pairs of windows several values of $w_i(T'_i, T_i)$ are obtained simultaneously, and one can therefore study w in different parts of a cell in one measurement.

We used time-windows in D1 and D3 of Fig. 1 to test D2 and D4 and vice versa. Figure 2 shows the drift-time distributions in a cell of D2 when three pairs of windows were defined in D1 and D3. In Fig. 3 the results of a series of measurements of $w(T', T)$ in D2 are plotted as a function of $\bar{T} = (T' + T)/2$. The chambers were in a magnetic field of 0.79 T parallel to the vertical sense wires. The drift field was tilted by $\arctan(1/3)$ to compensate the Lorentz force on the electrons.

$v(t)$ is obtained by requiring that

$$\sum_i (s - s_i)^2 = \sum_i \left[s - \int_{T_i}^{T'_i} v(t) dt \right]^2 \quad (4)$$

is a minimum and by assuming for $v(t)$ the polynomial

$$v(t) = \sum_{m=0}^M a_m t^m. \quad (5)$$

The precision required for the computed value of s_i defined by Eq. (4) determines the minimum value of M . Generally $M = 2$ or $M = 3$ suffice to reduce the distribution of the residuals to a Gaussian of $\sigma \sim 100 \mu\text{m}$.

It is an important feature of the method that it does not require knowledge of either the TDR in the defining chambers, or the particle momentum, or the analysing field. However, it assumes that these quantities are unchanged by the displacement. The value of t_0 needed to construct the TDR has to be determined separately, e.g. by an electronic measurement.

The displacement has to be known precisely, especially when the trajectories are neither parallel to each other nor perpendicular to the plane of the chamber. To avoid corrections we found it necessary to keep any displacements perpendicular to the plane of the chamber below $200 \mu\text{m}$.

To observe a variation of $v(t)$ within a cell of length L the displacement s must be substantially smaller than L but large enough to permit its precise measurement. Positions of the chambers were measured with the survey rig of the Omicron

Spectrometer, which has an r.m.s. error of about 40 μm for relative distances. L is 13 mm for D1 and 25 mm for the other chambers. We chose $s = L/3$ and took care to space the intervals (T'_i, T_i) evenly through the cell to avoid weighting $v(t)$.

The principal error of our measurement is that of the time interval $T' - T$. For chamber D2 this is indicated in Fig. 2. For D3 and D4 it is enlarged by multiple scattering. Taking account also of the error in the measurement of s we estimate the relative error in $w(T', T)$ to be about 3%.

The results of the displacement measurements are discussed in Section 4.

3. THE RECONSTRUCTION METHOD

Hits produced in three or more drift-chamber planes by the passage of fast particles are recorded and tracks are fitted to them using assumed values of x_k , t_0 and w . If the fitted track intersects a given plane at x_r while the position of the corresponding hit calculated by (2) is x_h , then we define the residual as

$$\Delta x(T) = x_r - x_h = x_r - x_k - (-1)^k (t_0 - T) w(t_0, T). \quad (6)$$

The residuals obtained for a given T and for a number of tracks will have a Gaussian distribution. Let $\overline{\Delta x}$ be its mean and σ^2 its variance. σ^2 is given by both random and systematic errors, while $\overline{\Delta x}$ in a large sample is given by systematic errors. The effect of these on $\overline{\Delta x}$ is obtained from Eq. (2) as

$$\overline{\Delta x}(T) = \Delta x_k + (-1)^k \left[w \Delta t_0 + (t_0 - T) \Delta w \right], \quad (7)$$

where the deltas indicate systematic errors.

The effects of specific errors in the chamber parameters on $\overline{\Delta x}(T)$ are discussed in detail in Ref. 7. As an example, we may note that, according to Eq. (7), errors in x_k have the same effect in odd and even cells while the effect of other errors, such as Δw , depends on the parity of the cell.

Figures 4 and 5 are scatter plots of Δx which show some of these effects. In the absence of systematic errors the plotted points should be grouped in a band around $\Delta x = 0$ for all T . Instead Fig. 4, which refers to a single cell, shows a non-linear dependence of $\overline{\Delta x}$ on T and so suggests a T -dependent error Δw .

Figure 5 shows the effect of an error Δw on a region of eighteen cells belonging to a plane as function of x_h .

In correcting systematic errors we note that variations in the relative distance of sense wires are usually small compared with position errors Δx_s of entire planes. Hence Eq. (7) may be written as

$$\overline{\Delta x}(T) = \Delta x_s + (-1)^k [w \Delta t_0 + (t_0 - T) \Delta w] . \quad (8)$$

Let $\overline{\Delta x} O(T)$ and $\overline{\Delta x} E(T)$ be mean residuals in odd and even cells, respectively.

Then for a given T

$$\Delta x_s = [\overline{\Delta x} O(T) + \overline{\Delta x} E(T)]/2 \quad (9)$$

and for $T = t_0$

$$\Delta t_0 = [\overline{\Delta x} E(t_0) - \overline{\Delta x} O(t_0)]/2w . \quad (10)$$

To correct the drift velocities we assume that they vary with T and put

$$w(t_0, T) = \sum_{m=0}^M b_m T^m . \quad (11)$$

Then by Eq. (8)

$$\overline{\Delta x}(T) = \Delta x_s + (-1)^k \sum_{m=0}^M \left[(b_m \Delta t_0 + t_0 \Delta b_m) T^m - \Delta b_m T^{m+1} \right] . \quad (12)$$

We let

$$\overline{\Delta x}(T) = \Delta x_s + (-1)^k \sum_{m=0}^{M+1} A_m T^m \quad (13)$$

and determine the A_m by fitting a polynomial of order $M + 1$ to the scatter plot.

Then the corrections Δb_m are found by comparing coefficients in Eqs. (12) and (13).

For a given set of $\Delta x(T)$ the corrections (9), (10), and (12) are calculated by the program PC RECON SOURCE.

In each iteration, tracks are fitted to a sample of events and the fitted tracks are used to correct the chamber parameters. In the absence of a magnetic field a least-squares procedure is used to fit straight lines to a set of hits. In a magnetic field a reconstruction routine using Runge-Kutta stepwise integration¹¹⁾ yields estimates of the five parameters characterizing each trajectory.

These include its unknown momentum. A number of reconstructed trajectories then permit the calculation of $\overline{\Delta x}(T)$ for each plane and the correction of drift-chamber parameters as for straight tracks. Iterations are continued as long as there are significant variations of the parameters. The result of this procedure is seen in Figs. 6 and 7, in which the data of Figs. 4 and 5 have been corrected by the elimination of systematic errors. The remaining $\sigma \sim 300 \mu\text{m}$ of the scatter plots is mainly given by multiple scattering.

The accuracy attainable by this method depends on many factors, such as the number of drift planes, the order M of the polynomial (11) used, the number of available events, and on random effects. In magnetic fields field-errors and approximations used in track reconstruction may further limit the precision.

Since these factors are difficult to evaluate with confidence we give some examples to illustrate the convergence and the accuracy obtained under different conditions. Table 1 lists the mean variation of $\overline{\Delta x}$ in the final iteration and the r.m.s. error of the residual found in the reconstruction of trajectories of momentum p in a magnetic field B . N is the number of planes used, M the order of the polynomial (11), n the number of tracks in the sample and j the number of iterations. Drift chambers D1 to D4 were used for the reconstruction of straight tracks. For curved tracks an additional coordinate was given by a multiwire proportional chamber of 1 mm wire spacing placed close to the target TA. t_0 was a fitted parameter for straight tracks; for curved tracks it was obtained from the drift-time spectra and held constant. The effect of positioning errors was tested by introducing a simulated error $\Delta x_s \leq 2 \text{ mm}$ in one plane. It was found that the value of w was not significantly affected.

Figure 8 is a plot of the mean residual $\overline{\Delta x}$ as a function of T after five iterations. It shows little correlation of $\overline{\Delta x}$ with T except close to the field wire in a region containing a small fraction of the events. The effect of varying the order M of the polynomial (11) was tested on the sample of straight tracks and was found to change the estimate of w by less than 1% in any part of the cell. The absence of any marked correlation between $\overline{\Delta x}$ and T proves, in fact, that the remaining scatter of the residuals is due to random effects.

The error of the TDR at T is equal to $\overline{\Delta x}(T)$ and the relative error in w is given by

$$\Delta w/w(T) = \overline{\Delta x}(T)/w(T) (t_0 - T) . \quad (14)$$

Using the r.m.s. error $\sqrt{\langle \overline{\Delta x} \rangle^2} = 220 \text{ } \mu\text{m}$, together with $w = 50 \text{ } \mu\text{m/ns}$ and $(t_0 - T) = 200 \text{ ns}$ in mid-cell, one finds a relative error in w of 2%. The error becomes large as T approaches t_0 , since the TDR is insensitive to the drift velocity close to the sense wire.

The drift velocities obtained by the reconstruction method using $M = 1$ for chamber D2 are shown together with the results of the displacement method in Fig. 3.

4. RESULTS AND CONCLUSIONS

A comparison of the two methods of determining electron-drift velocities was made for the even cells of chamber D2 operating without a magnetic field and in a field of 0.79 T parallel to the sense wires of the plane tested. Table 2 lists the values of the velocity $w(T', T)$, defined by Eq. (3), for various values of the drift time $(t_0 - T)$. w_D are velocities obtained by displacement and w_R those found by reconstruction. By calculating w_R for a fixed interval the errors are somewhat larger than given by Eq. (14). The results obtained by the two methods lie within the combined limits of error. Both show a variation of $w(T', T)$ with drift time for the chambers which were operating with a mixture of 70% argon, 27% isobutane, and about 3% methylal. The drift field was 1.44 kV/cm and the anode potential about 2 kV.

The relative variation $\Delta w/w$ of about 6% over the length of the cell in the absence of a magnetic field is compatible with our previous measurements, which were not sensitive to the small non-linearity of the TDR produced by this variation. The effect cannot be attributed to oblique incidence such as that observed by Breskin et al.³⁾ and the modification of the drift field by the double sense wire cannot fully account for it. Palladino et al.¹²⁾ have noted that a loss of electrons during drifting, combined with a high threshold for the detection of the

anode pulse, can produce an apparent variation of w with drift time, and our observation may be due to this effect.

Against this the low mean velocity of $47 \mu\text{m/ns}$ and its 20% relative variation observed in a chamber operating in a field of 0.79 T are anomalous and at variance with the results of Ref. 3 and our earlier findings reported in Ref. 9. While we have no reason to doubt the values reported here, which were obtained by two independent methods, we believe that they are due to a malfunctioning, possibly a contamination of the chamber gas. This view is supported by the fact that mean velocities higher by about 15% were recorded under similar conditions in earlier measurements. The results quoted here merely illustrate the agreement between our two methods and incidentally show the need to monitor drift velocities during an experiment.

Comparing the two methods of measuring drift velocities one may claim that the displacement method using multiple software windows is simple and accurate, provided the precautions mentioned in Section 2 are observed. However, the method necessitates a separate series of measurements and cannot be used unless the detection system allows the accurate displacement of some of its elements. The more complicated reconstruction method has been shown to be capable of furnishing drift velocities from tracks of unknown momentum curved in a magnetic field. It has been verified by the displacement method, and in its present form it can be used to construct a TDR applicable to a set of experimental data from a sample of the events and without any subsidiary measurements. It even permits the analysis of data from experiments in which a change of drift velocity has occurred.

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Table 1

Convergence and residual r.m.s. error of track reconstruction

p (MeV/c)	B (T)	Δx_k (mm)	N	M	n	j	$\overline{\Delta x}(j) - \overline{\Delta x}(j-1)$ (μm)	$\sqrt{\langle \overline{\Delta x} \rangle^2}$ (μm)
240	0	0	4	2	4×10^4	8	10	100
240	0	1	4	2	4×10^4	8	25	
240	0	2	4	2	4×10^4	8	23	
100-218	0.79	0	5	1	650	5	40	220
100-218	0.79	2	5	1	650	5	100	

Table 2

Comparison of drift velocities
obtained by displacement and reconstruction

$t_0 - T$ (ns)	B = 0		B = 0.79 T	
	w_D ($\mu\text{m}/\text{ns}$)	w_R ($\mu\text{m}/\text{ns}$)	w_D ($\mu\text{m}/\text{ns}$)	w_R ($\mu\text{m}/\text{ns}$)
89			52.2 $\pm$ 1.6	51.1 $\pm$ 2.5
130	54.1 $\pm$ 1.6	53.2 $\pm$ 1.6		
180	51.5 $\pm$ 1.5	51.6 $\pm$ 1.2		
189			46.7 $\pm$ 1.4	48.3 $\pm$ 1.2
271	50.3 $\pm$ 1.5	50.7 $\pm$ 0.8		
289			44.6 $\pm$ 1.3	45.4 $\pm$ 0.8
307	51.9 $\pm$ 1.6	50.7 $\pm$ 0.6		
358	51.6 $\pm$ 1.6	49.5 $\pm$ 0.6		
389			41.3 $\pm$ 1.2	42.6 $\pm$ 0.6

Figure captions

- Fig. 1 : Layout of drift chambers in the Omicron Magnet.
- Fig. 2 : Drift time spectra obtained by the displacement method.
- Fig. 3 : Drift velocities in chamber D2 operating in a magnetic field of 0.79 T.
- Fig. 4 : Scatter plot of residuals as function of complementary drift time before correction.
- Fig. 5 : Scatter plot of residuals for a plane showing the effect of an error in the drift velocity.
- Fig. 6 : Scatter plot of residuals as function of complementary drift time after correction.
- Fig. 7 : Scatter plot of residuals for a plane after correction.
- Fig. 8 : Mean deviations as function of complementary drift time after correction.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8