ON THE REDUCTION IN SPACE RESOLUTION OF TRACK DETECTORS
CAUSED BY CORRELATIONS IN THE COORDINATE QUANTIZATION

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ABSTRACT

Using particle track detectors, there are in general two different kinds of contribution to the error in the measured coordinates, one due to the quantization of the track detector coordinate scale and the other due to atomic processes of stochastic nature in the detector gas. When several detector planes are used, the first kind of errors are often correlated between neighbouring planes. This implies that the frequently used assumption that the errors on the measured track parameters decrease with the number of coordinates N like $1/\sqrt{N}$ is not necessarily valid. The Monte Carlo method has been used to study the deviations from the $1/\sqrt{N}$ law in two detector set-ups: one is an idealized detector with full correlation between all detector planes, and the other is a more realistic set-up in which the correlation is only partial.

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1. INTRODUCTION

The measured space coordinate of a charged particle trajectory can in general only take on certain discrete values that correspond to integer multiples of a given quantum length. When a multiwire proportional chamber (MWPC) is used for the measurement, the integer multiple is the number of the wire hit, and the quantum length is the wire distance. If the current division method is used for the read-out along a wire, the multiple is an ADC digit, and the quantum length is the wire length divided by the number of ADC bins used. In the case of a drift chamber, the multiple is a read-out clock digit, and the quantum length is the electron drift velocity divided by the clock frequency.

Furthermore, every track detector has an intrinsic limit to its space resolution which is due to processes of a stochastic nature such as electron diffusion and fluctuations of the primary ionization density in the detector gas.

The quantization of the coordinate scale and the stochastic processes both contribute to the uncertainty in the measurement of the track position. The stochastic error is often assumed to have a Gaussian distribution, characterized by the root mean square σ_s . The error due to the quantization has a distribution which is uniform over the quantum length d and zero outside, and has a r.m.s. of $\sigma_q = d/\sqrt{12} \approx 0.29d$. Assuming that the two errors are independent, the distribution of the global error is a convolution of the Gaussian and the uniform distribution. In practice it is often assumed that the uniform distribution of the error due to the quantization can be approximated by a Gaussian distribution having a r.m.s. of $d/\sqrt{12}$. The distribution of the global error is then a convolution of two Gaussians, which is a Gaussian with a r.m.s. error of $\sqrt{\sigma_s^2 + \sigma_q^2} = \sqrt{\sigma_s^2 + d^2/12}$.

When using an ensemble of single-plane detectors to detect several coordinates on a track, with the aim of determining the trajectory of the particle, it is usually assumed on the basis of a theorem in statistical theory that the errors in the fit parameters of the track (such as the track position or curvature) decrease like $1/\sqrt{N}$ for large N , where N = number of measured coordinates on the track. An important condition for this law to be valid is, however, that the

errors in the different detector planes are not correlated. In practice this condition is usually fulfilled for the stochastic errors but not always for the quantization errors. As an example of a case with full correlation between the quantization errors, one may take the measurement of a straight track by an ensemble of MWPCs having the same wire pitch in all planes and all wires aligned in straight rows along the direction of the track. In a drift chamber ensemble of the same basic geometry, full correlation between the quantization errors is obtained if in addition to the above conditions one single clock is used to read out the different planes for the drift-time measurements.

As will be shown below, in the case of full correlation the errors of the track fit parameters decrease with N less rapidly than $1/\sqrt{N}$ when the quantization error σ_q is bigger than the stochastic error σ_s . Cases of full correlation are, however, rare in practice. On the other hand, partial correlations may occur more frequently in practice than is generally assumed. It is not unusual, for example, to have correlations ranging over only a few neighbouring planes in a detector. Also, such partial correlations lead to a deviation from the $1/\sqrt{N}$ behaviour, as will be shown below for a specific case.

2. THE CASE OF FULL CORRELATION

We will first study the case of full correlation described in the previous section. In this case the true particle trajectory has the same position within the same quantization step d in all detector planes, but the measured coordinates may be different when going from one plane to another as a result of the stochastic processes.

To formalize the problem, let ϕ be the true coordinate of the track in a plane and ψ the coordinate disturbed by the stochastic process. The observed coordinate η is then ψ truncated by the quantization procedure.

If the stochastic process is unbiased, then the expectation value $E(\psi) = \phi$, the variance $V(\psi) = \sigma_s^2$, and the probability density function for ψ , given a certain value of ϕ , is

$$f(\psi|\phi) = \frac{1}{\sigma_s \sqrt{2\pi}} \exp - \left[\frac{(\psi - \phi)^2}{2\sigma_s^2} \right]. \quad (1)$$

The quantization procedure is assumed to be such that

$$\eta = [\psi/d], \quad (2)$$

i.e. η is an integer number, positive or negative.

The probability density function for η , given a certain value of ϕ , is then

$$p(\eta|\phi) = \int_{d \cdot \eta}^{d \cdot (\eta+1)} f(\psi|\phi) d\psi. \quad (3)$$

As estimator for the true position of the track ϕ we choose

$$\hat{\phi}_N(\bar{\eta}) = d \left[\frac{1}{N} \sum_{i=1}^N \eta_i + \frac{1}{2} \right], \quad (4)$$

where $\bar{\eta} = \{\eta_1, \eta_2, \dots, \eta_N\}$ is the set of measurements of the same position in the N different detector planes.

The probability density function for $\hat{\phi}$, given a certain value of ϕ , is then

$$g(\hat{\phi}|\phi) = \sum_{\bar{\eta}} \left\{ \delta[\hat{\phi} - \hat{\phi}_N(\bar{\eta})] \prod_{i=1}^N p(\eta_i|\phi) \right\}. \quad (5)$$

We have used the Monte Carlo technique to evaluate $E(\hat{\phi}|\phi)$ and $V(\hat{\phi}|\phi)$ as a function of the exact track position ϕ within the quantization bin, for different ratios of the stochastic error σ_s to the quantization bin width d . The results are shown in Figs. 1 and 2, respectively, over only half the range of a quantization bin ($0 < \phi/d < 0.5$), which is sufficient, as the problem is symmetrical around the midpoint ($\phi/d = 0.5$) of the bin.

For the case $\sigma_s = 0$, η_i will be the coordinate of the same bin in all detector planes irrespective of where ϕ is within the bin. Therefore $E(\hat{\phi}|\phi)$ has a bias that grows linearly from 0 to $d/2$ when ϕ moves from the bin centre to its borderline ($\phi/d = 0$). For $\sigma_s \neq 0$, η_i will however assume the coordinate of the adjacent bin in some of the detector planes, namely in those where the stochastic

error was big enough to push ψ over the borderline between the bins. For very large σ_s , ψ will be spread over several bins. This process reduces the bias in $E(\hat{\phi}|\phi)$. As can be seen from Fig. 1, when $\sigma_s > \sigma_q \approx 0.29d$, i.e. when ψ starts to spread over several bins, the bias is less than about 10%.

For $\sigma_s = 0$ the variance $V(\hat{\phi}|\phi)$ is zero for all positions of ϕ within the bin, since η_i will be the coordinate of the same bin in all planes irrespective of where ϕ is positioned within the bin [except when it is on the borderline, in which case $V(\hat{\phi}|\phi)$ will be $d^2/4$]. For $\sigma_s \neq 0$, ψ will be pushed over to the adjacent bin in some detector planes, and this will give rise to non-zero contribution to the variance. For $\sigma_s > \sigma_q \approx 0.29d$, ψ will be distributed over several bins and $V(\hat{\phi}|\phi)$ is largely determined by σ_s^2 .

Using the Monte Carlo technique a sample of events were generated, choosing for each event a new value of ϕ from a uniform probability distribution in the range $\phi \in [0, d]$. This was done in order to study the combined influence of the bias in $E(\hat{\phi}|\phi)$ and the variance $V(\hat{\phi}|\phi)$ averaged over the quantization bin. Different numbers of planes (N) were used in order to investigate the $1/\sqrt{N}$ behaviour. Figure 3 shows how the standard deviation $\sigma(\hat{\phi}) = \sqrt{V(\hat{\phi})}$, normalized to the quantization bin width d , depends on the stochastic error σ_s , also normalized to d .

For $\sigma_s > \sigma_q \approx 0.29d$ and $N = 1$, the standard error $\sigma(\hat{\phi})/d$ is seen to be approximately equal to σ_s/d . For $N = 10$ and $N = 50$, $\sigma(\hat{\phi})/d$ is reduced by factors of $\sqrt{10}$ and $\sqrt{50}$, which is apparent from Fig. 3b where each curve has been multiplied by $\sqrt{N}$ to illustrate this fact. For $\sigma_s > \sigma_q$, ψ spreads over several quantization bins and the effect of the quantization must therefore be small.

For $\sigma_s < \sigma_q$ and $N = 1$ the standard error $\sigma(\hat{\phi})$ tends towards $0.29d$ when σ_s goes to zero. It is important to note that this is also true for $N = 10$ and $N = 50$, i.e. for small σ_s/d the error $\sigma(\hat{\phi})$ does not decrease like $1/\sqrt{N}$. This is an effect due to the correlated quantization. The smaller the value of σ_s the weaker the decrease with N . The minima for the curves $N = 10$ and $N = 50$ indicate that for a given value of d there is an optimal value of σ_s , which seems to be about $0.29d \approx \sigma_q$. For smaller σ_s , ψ will not spread over several quantization bins,

and the bias illustrated in Fig. 1 will in fact have the effect of increasing $\sigma(\hat{\phi})$ with the decreasing σ_s . For larger σ_s the effect of the stochastic scattering of ψ will dominate, which leads to $\sigma(\hat{\phi})$ increasing with σ_s .

3. REAL CASES WITH PARTIAL CORRELATIONS

In the preceding section we treated the case of full quantization correlation between the coordinate planes. This case rarely appears in reality since there exist several potential randomizing factors in real detector assemblies. One such factor is the curvature of a charged particle trajectory in the presence of a magnetic field. Some correlation does, however, remain, for example between neighbouring planes in the case described in Section 2 also if the tracks are moderately curved.

For real cases with partial correlation the error $\sigma(\hat{\phi})$ and its N-dependence can only be estimated after considering all relevant details of the detector in a complete Monte Carlo simulation. The simple case of Section 2, however, sets interesting limits to the phenomenon since it represents the extreme case of full correlation. The $\sigma(\hat{\phi})/d$ curves corresponding to the other extreme case, that of no correlation at all in the quantization, are obtained for $N = 10$ and $N = 50$ by dividing the $N = 1$ curve in Fig. 3 by $\sqrt{10}$ and $\sqrt{50}$, respectively. These curves are shown as dotted lines in Fig. 3. The partial correlation case must thus lie between the full line curve and the dotted line curve. From this it follows that independently of how strong or weak the correlation is, it can have a noticeable effect ($> 10\%$) on $\sigma(\hat{\phi})$ only if $\sigma_s/d \lesssim 0.29$. Furthermore, since in the region $\sigma_s/d < 0.29$ the effect on $\sigma(\hat{\phi})$ decreases for increasing σ_s/d , one may expect that the limit in σ_s/d above which the effect on $\sigma(\hat{\phi})$ is small will decrease below 0.29 when the correlation decreases.

4. THE CASE OF THE LEP JET DETECTOR

The LEP Jet Detector is a large cylindrical drift chamber assembly, 6 m long, with 3.6 m out diameter and 0.2 m inner diameter, surrounded by a superconducting solenoidal magnet which produces an axial field of 1.5 T. An over-all view of the detector is shown in Fig. 4. The detector is a hypothetical case study made by

the ECFA-LEP Working Group, and its calculated properties have been described in reports of the Working Group¹⁾. In the Monte Carlo program written to simulate the detector performance, the drift time digitalization in the different cylindrical drift chambers was governed by one single clock. The detection wires are axial and the drift is in the azimuthal direction. The effect of the magnetic field on the drift direction was neglected. Two different geometrical configurations of the wires in the plane perpendicular to the axis were tried. In one configuration, here called A, the distance in azimuthal arc length between two wires is approximately the same at all radii and equal to about 5 cm. In the other configuration, here called B, the wires are placed on radial lines that increase in number at larger radii to keep the wire density approximately the same as in A.

A previously established event-generation program based on the two-jet model of Feynman and Field was used to generate particle trajectories. In the subsequent calculation the coordinate of each track in each cylindrical drift chamber was reconstructed (ϕ), then randomized (ψ) using a Gaussian distribution with a standard deviation of σ_s , and finally quantized (η) using the common clock and the drift distance to the nearest sense wire divided by the drift velocity. A circular arc trajectory was fitted to these quantized coordinates, and the track momentum was derived from the curvature of the arc. The error in this individual momentum determination was then obtained by comparison with the momentum used to generate the original particle trajectory.

Figure 5 shows the results of these calculations. The relative momentum error $\Delta p/p$ is plotted as a function of $1/\sqrt{N}$, where N is the total number of cylindrical drift chamber layers in the detector. Figure 5a contains tracks with momenta between $2 < p < 5$ GeV/c, implying values of the track sagitta s between 81 and 33 mm; in Fig. 5b, $5 < p < 10$ GeV/c and $33 > s > 16$ mm; in Fig. 5c, $10 < p < 20$ GeV/c and $16 > s > 8$ mm; and in Fig. 5d, $20 < p < 50$ GeV/c and $8 > s > 3$ mm. The stochastic error is everywhere $\sigma_s = 0.2$ mm, and in each figure there are curves corresponding to three different quantization steps: $d = 5$ mm, 1 mm, and 0.2 mm, corresponding to σ_s/d values of 0.04, 0.2, and 1.

If there were no quantization correlation, $\Delta p/p$ would behave as $1/\sqrt{N}$ and all lines would point to $\Delta p/p = 0$ for $1/\sqrt{N} = 0$ in Fig. 5. As seen in all four figures, the points on the $d = 0.2$ mm line do indeed lie on a straight line through the origin. (The fluctuations around this line indicate the magnitude of the statistical errors in the calculation.) As $\sigma_s/d = 1 \gg 0.29$ this is expected to happen independently of how strong or weak the correlation is in the particular case studied. For $d = 5$ mm, on the other hand, the deviations from the straight line are quite large at high N . Note that the wire configuration B with the wires on radial lines shows a stronger effect than the non-radial wire configuration A. This is to be expected as the tracks, although curved, are essentially directed radially, which leads to a higher degree of correlation in the radial configuration. Furthermore, the effect is more pronounced for tracks of high momenta. This too is to be expected, as a high-momentum track has a small sagitta which in turn should enhance the correlation.

There may be several reasons why the deviations from the $1/\sqrt{N}$ law are more pronounced at big N than at small. One reason can be seen from Fig. 3, where the $N = 50$ line deviates more from the linear $1/\sqrt{N}$ behaviour than does the $N = 10$ line. However, in Fig. 5 the $N = 17$ and $N = 34$ points do lie approximately on a line through the origin also for $d = 5$ mm. For $N = 17$ and $N = 34$, the radial distance δ between the drift chamber planes is 100 mm and 50 mm, respectively. With increasing N this distance gets smaller, and eventually becomes of the same order as the quantum length 5 mm. In this case the correlation between neighbouring planes may be expected to become quite strong, and adding more measurement planes between those already existing does not decrease $\Delta p/p$.

It may also be noted that at the highest momenta (Fig. 5d) the size of the sagitta ($8 > s > 3$ mm) is of the same order as the large quantum length ($d = 5$ mm). This is another critical criterion in cases of high intrinsic correlation and low σ_s/d ratio. For sufficiently small sagittas the whole track may, in such cases, appear as a straight line; consequently the upper limit to the momentum uncertainty will grow infinitely large.

In Fig. 5 the intermediate case $d = 1$ mm, i.e. $\sigma_s/d = 0.2$, is also shown. The corresponding lines show the same qualitative behaviour as the $d = 5$ mm lines, but the effect is weaker, as expected, since the value of $\sigma_s/d = 0.2$ is not far from the limit $\sigma_s/d = 0.29$, above which the effect should be negligible. We may conclude that the observed deviations from the $1/\sqrt{N}$ behaviour in Fig. 5 are consistent with being due to partial correlations in the quantization of the read-out system since the degree of the effect varies with track curvature, wire arrangement, distance between detection planes, and quantum length according to this hypothesis.

Finally, we may also note that since in a real LEP Jet Detector the number of wire planes would hardly be made to exceed 170 (1 cm between the wires), and since the quantization length would not need to be longer than 1 mm, the correlation effects would not cause any noticeable effect in the particular case studied here although the value of σ_s/d would be 0.2.

5. SUMMARY AND CONCLUSION

There are in general two different kinds of contribution to the uncertainty in the measurement of a space coordinate on a charged particle trajectory. One is due to the quantization into discrete steps of the coordinate scale of the track detector; the other is due to stochastic processes such as electron diffusion and fluctuations in the primary ionization density in the detector gas. When combining measurements from N detector planes in order to reconstruct a particle trajectory, it is generally assumed that the measurements in the different planes are independent and that, as a consequence, the uncertainty in the determined track parameters decreases like $1/\sqrt{N}$ for large N . However, in many practical cases the coordinate quantization in neighbouring detector planes is correlated in space. In cases in which the stochastic r.m.s. error σ_s is less than or equal to 0.29 times the quantum length d , such correlation will lead to a decrease with N of the track parameter errors which is less rapid than $1/\sqrt{N}$.

The exact behaviour of the errors with N depends on the strength of the correlation in each particular case. For the special case where the distance between the detector planes is of the same order or smaller than the quantum

length d , strong deviations are to be expected. Finally, if a magnetic field is used to determine the momentum of the particle, the uncertainty in this determination can become very big in a detector with correlated planes and a $\sigma_s \lesssim 0.29d$ if the sagitta of the curved trajectory is of the same order as or smaller than the quantum length d .

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REFERENCE

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Figure captions

- Fig. 1 : The bias in the mean value of the estimator $\hat{\phi}$ as a function of the true coordinate ϕ over half the range of a quantization bin for different ratios of the stochastic r.m.s. error σ_s to the quantum length d .
- Fig. 2 : The standard deviation of the estimator $\hat{\phi}$ as a function of the true coordinate ϕ over half the range of a quantization bin for different ratios of σ_s to d .
- Fig. 3 : The standard deviation of the estimator $\hat{\phi}$ averaged over all positions of ϕ within the quantization bin as a function of the ratio σ_s to d for $N = 1, 10$, and 50 planes. In Fig. 3b the curves have been multiplied by $\sqrt{N}$ to show the validity of the $1/\sqrt{N}$ law in the region $\sigma_s/d > 0.29$.
- Fig. 4 : Side (a) and front (b) views of the LEP Jet Detector (from ref. 1).
- Fig. 5 : Momentum resolution of the LEP Jet Detector as a function of $1/\sqrt{N}$, where N is the number of equidistant cylindrical drift chamber planes in the 1.8 m radius detector (inner radius 0.1 m). The corresponding distance δ between the drift chamber planes is indicated on the lower scale. Curves for three different quantum lengths $d = 0.2$ mm, 1 mm, and 5 mm are shown. The stochastic error is everywhere $\sigma_s = 0.2$ mm. A and B correspond to two different wire configurations described in the text. Figure 5a shows $\Delta p/p$ averaged over momenta in the range 2-5 GeV/c; Fig. 5b in 5-10 GeV/c; Fig. 5c in 10-20 GeV/c; and Fig. 5d in 20-50 GeV/c.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5