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COMPUTATIONAL ASPECTS OF POSITRON IMAGING
USING MULTIWIRE PROPORTIONAL CHAMBERS IN NUCLEAR MEDICINE

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ABSTRACT

The application of multiwire proportional chambers to fields outside high energy physics is becoming increasingly widespread. In Nuclear Medicine, for the imaging of positron-emitting radionuclides, the high density proportional chamber offers large solid-angle acceptance and good spatial resolution in the final image. The medical data analysis (emission computed tomography) is rather different to that used in high energy physics, being based on reconstruction from projections rather than combinatorial or track-following methods. This paper will review some of these differences, and describe the mathematical basis and implementation of an algorithm for emission computed tomography with a proportional chamber positron camera.

1. INTRODUCTION

The use of proportional chambers in fields other than high energy physics is becoming increasingly widespread, and one important application is to the imaging of positrons. Positron-emitting radionuclides are used as a probe in diagnostic nuclear medicine for the non-invasive investigation of the functional behaviour of internal organs such as the brain, liver and heart.¹⁾

After emission from the nucleus into surrounding tissue, the positron, antiparticle of the electron, rapidly thermalises and annihilates with an electron to produce two 511 keV gamma rays with an opening angle close to 180° . The intensity profile of these photons reflects the internal distribution of the radionuclide; a pair of two-dimensional, position-sensitive detectors that record in coincidence the locations of the two photons from a single annihilation (an event) may be used as a positron imaging system.²⁾ A pair of multi-wire proportional chambers equipped with carefully-constructed lead-sandwich converters^{3,4)} to make them reasonably sensitive to 511 keV photons have been operated in coincidence as a positron camera⁵⁾ (Fig. 1).

The positron-emitting radionuclide is introduced into the patient by injection or inhalation, and a certain fraction concentrates within the organ under study, presenting a diffuse source of photon pairs to the external imaging system. By neglecting differences of the pair opening angle from 180° , and assuming the positron annihilates close to its point of emission, a coincident photon pair defines a line through the radionuclide distribution, somewhere along which the annihilation occurred. After correction for photon attenuation, the sum of sufficient lines that have the same position and direction, is proportional to the total radionuclide activity along that line. Repeating this summation process at other positions and in other directions, a set of radionuclide line integrals is produced from the detection of individual positron annihilation events. A group of line integrals with the same direction forms a projection of the radionuclide distribution; for example,

the projection $p(x,y)$ along the z -direction of a function $f(x,y,z)$ is given by :

$$p(x,y) = \int_{-\infty}^{\infty} f(x,y,z) dz \quad (1)$$

Projections in other directions are defined by the appropriate coordinate rotation.

The problem is therefore to recover the original, three-dimensional radionuclide distribution by combining two-dimensional projections obtained from summing a large sample of individual positron annihilation events. These events are taken to be simple straight lines and their recognition and reconstruction is a trivial process. The mathematical treatment enters with the reconstruction from projections.

In contrast, in high energy physics, the complexity of events is much greater and sophisticated pattern recognition and geometrical reconstruction methods are required for individual events.⁶⁾ In general, a direct determination of the position of the interaction point (vertex) is usually possible for each event. A sample of events then reveals the three-dimensional distribution of target nuclei (c.f. the positron radionuclide distribution), but of course a high energy physics experiment is usually designed to study more complex parameters concerned with the underlying interaction dynamics.

In medical positron imaging, the problem of determining the distribution of radionuclide is thus transformed into a problem of reconstruction from projections. An algorithm that performs this reconstruction will be the subject of the remainder of this paper.

2. RECONSTRUCTION FROM PROJECTIONS

The three-dimensional Fourier transform of the unknown radionuclide distribution $f(x,y,z)$ is defined by :

$$F(k_x, k_y, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) e^{-2\pi i (xk_x + yk_y + zk_z)} dx dy dz \quad (2)$$

Setting $k_z = 0$ and using equation (1), this becomes :

$$\begin{aligned} F(k_x, k_y, 0) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p(x, y) e^{-2\pi i (xk_x + yk_y)} dx dy \\ &= P(k_x, k_y) \end{aligned} \quad (3)$$

where $P(k_x, k_y)$ is the two-dimensional Fourier transform of the projection $p(x, y)$. This result, equation (3), is known as the projection theorem⁷⁾, and states that the Fourier transform of the projection of a distribution along any axis is identical to the Fourier transform of the distribution itself on a plane perpendicular to the direction of the projection. It has been shown here for the special case of the z-axis, but clearly generalises to any direction. The Fourier transform of the unknown distribution may therefore be built up from the Fourier transforms of its projections; an inverse Fourier transform in three dimensions then yields the required distribution.

An obvious implication of this theorem is that sufficient projections should be available to construct $F(k_x, k_y, k_z)$ to the required precision. From equation (3) it is clear that unmeasured projections will correspond, in Fourier space, to planes in which the frequency distribution $F(k_x, k_y, k_z)$ is unknown. However, if sufficient projections are available, the unknown distribution should be recoverable.

This is normally the case for the X-ray transmission scanners that have been commercially available since 1972.⁸⁾ To reconstruct the linear attenuation coefficients for a single transverse section, the latest generation scanners may use as many as one thousand projections, from a full 360° range

of views. To recombine these projection data, efficient reconstruction algorithms have been derived ^{9,10)}, based mainly on the original work of Radon ¹¹⁾.

Reconstruction for the proportional chamber positron camera depicted in Fig. 1 presents an additional complication. Many projections are measured, but along directions grouped within a limited angular range, instead of being uniformly distributed through 360° . Providing this angular range is not too small, a modified algorithm of the Fourier deconvolution type ^{12,13)} can be used. Additional information about the spatial extent of $f(x,y,z)$ is needed to supplement the limited projection data. The algorithm has been developed for the CERN proportional chamber positron camera prototype, ¹⁴⁾ but it is applicable to any similar system. It will be described in the next section.

In high energy physics, the information provided by proportional chambers about individual events is essentially in the form of projections. Could a reconstruction from projections algorithm be used on an event by event basis to recover the three-dimensional track distribution? There are three reasons why this would not be possible: firstly, very few projections (e.g. two to four) are usually available for each event; secondly, the chambers do not really measure line integrals - overlapping tracks give a single pulse; and finally, it is important to associate the individual points into tracks for each event, in order to extract the relevant parameters. This association is best made on the projections where simple geometrical constraints (e.g. linearity) often apply. ⁶⁾

3. A LIMITED ANGLE RECONSTRUCTION ALGORITHM

The procedure for reconstructing an unknown distribution $f(x,y,z)$ from a limited set of projections may be separated into three distinct steps: (1) back-projection, (2) frequency-space filtering and (3) estimation of unmeasured projections using additional information.

3.1 Back-projection

This procedure is represented mathematically ⁷⁾ by :

$$\hat{f}(x,y,z) = p(x,y) \quad (4)$$

for the projection $p(x,y)$. It corresponds to projecting each line integral back through the region in which the unknown distribution is to be reconstructed, repeating the process for all projections. The result is a function $\hat{f}(x,y,z)$ that forms a first estimate for the unknown distribution $f(x,y,z)$. In practice, back-projection may be performed on an event-by-event basis, thereby avoiding the need to resort the randomly acquired data into projections. Nevertheless, the intersection of each event line with the three-dimensional grid $\hat{f}(x,y,z)$ is a time consuming process (see Table I) when many events are involved. Although the simplicity of the procedure makes it ideally suited for implementation on a special hardware processor, access is required to a large quantity (typically 2 Mbytes) of random access memory.

3.2 Frequency Space Filtering

The back-projected image $\hat{f}(x,y,z)$ is a distorted estimate of the true distribution $f(x,y,z)$. This can be seen from Fig. 2, which shows the result of back-projecting annihilation events from a single point source of activity placed at the mid-point of a dual detector, stationary system. The square-section, cone-shaped distribution that replaces the original point source is the camera point response function, denoted by $h(x,y,z)$. A more general distribution $f(x,y,z)$ may be considered as an assembly of point sources, each of which, after back-projection, must be replaced by the cone-shaped distribution $h(x,y,z)$, assuming of course that $h(x,y,z)$ is independent of the position of the original source. The overall back-projected distribution is represented mathematically as a convolution of $f(x,y,z)$ with $h(x,y,z)$:

$$\hat{f}(x,y,z) = f(x,y,z) * h(x,y,z) \quad (5)$$

where $*$ denotes convolution. An elegant way to solve this equation for $f(x,y,z)$ is to take the three dimensional Fourier transform of both sides so that the convolution operation is converted into a simple point-wise product (the Fourier convolution theorem¹⁵⁾) :

$$\hat{F}(k_x, k_y, k_z) = F(k_x, k_y, k_z) \cdot H(k_x, k_y, k_z) \quad (6)$$

where $F(k_x, k_y, k_z)$ is given by equation (2), and $\hat{F}(k_x, k_y, k_z)$ and $H(k_x, k_y, k_z)$ are similarly defined as the three-dimensional Fourier transforms of $\hat{f}(x,y,z)$ and $h(x,y,z)$ respectively. A formal solution to equation (6) is :

$$F_R(k_x, k_y, k_z) = \hat{F}(k_x, k_y, k_z) / H(k_x, k_y, k_z) \text{ for } H(k_x, k_y, k_z) \neq 0 \quad (7)$$

The inverse Fourier transform of (7) yields the filtered reconstruction $f_R(x,y,z)$. A closed-form expression for $H(k_x, k_y, k_z)$ has been derived for the limited angle positron camera¹⁶⁾, assuming an analytic expression for the point response function. The resulting expression exhibits a cone of unmeasured frequencies ($H(k_x, k_y, k_z) = 0$) corresponding to the missing projections. At these frequencies, $F_R(k_x, k_y, k_z)$ is also set to zero. High frequency noise amplification is reduced by incorporating a roll-off function into the filter.¹⁷⁾

3.3 Estimation of Unmeasured Frequencies

An estimate of $F_R(k_x, k_y, k_z)$ at those frequencies for which $H(k_x, k_y, k_z)$ is zero (corresponding to unmeasured projections) can be made by incorporating additional information about $f(x,y,z)$ into the algorithm. The function $f(x,y,z)$ represents the distribution of the radionuclide within the human body, and hence it must have finite extent. The Fourier transform $F(k_x, k_y, k_z)$ is therefore an entire function, and this information may be used to continue $F_R(k_x, k_y, k_z)$ to frequencies at which it is not measured. An iterative algorithm proposed by Papoulis¹⁸⁾ has been used for continuation of $F_R(k_x, k_y, k_z)$, involving repeated transformation from real space to Fourier space. A noticeable improvement in the reconstructions is achieved¹⁹⁾.

4. SUMMARY OF THE ALGORITHM

The reconstruction process has been summarized in Fig. 3 for a point source of radionuclide activity mid-way between the two detectors; the activity distribution on a horizontal plane through the source is shown to the right. The width of the original source in (3a) is due to the experimental detector resolution. After back-projection, the familiar cone-shaped blurring (3b) appears. The back-projection of individual events is performed by a separate program because it is time-consuming and usually done only once for a given data set. A second program performs the filtering and continuation to unmeasured frequencies, indicated schematically in Fig. 3c and 3d. Sharp truncation of the filter at high frequency causes ringing and amplification of noise; a smoother roll-off (Fig. 3d) reduces these effects, but at the expense of some spatial resolution. Computer timings for the various stages are summarized in Table I.

As in high energy physics, the effect of limited event statistics can cause interpretation problems for the results. This is particularly important in nuclear medicine, where the images are often severely statistics-limited, the filtering process amplifies noise, and the results of misinterpretation could be more serious.

5. CONCLUSIONS AND FUTURE DEVELOPMENTS

This paper has outlined some of the differences of proportional chamber data processing in high energy physics and nuclear medicine. An algorithm has been presented that performs this processing for positron imaging, and examples of its use may be found elsewhere.^{5,13,14)}

It is clear that the limited angular acceptance is a considerable restriction on the quality of the final image. The analytic continuation is expensive on time, noise sensitive and can never replace actual measurement of the missing projections. The construction of a four chamber imaging system should overcome this restriction, and the development of a suitable filtering algorithm will be necessary.

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FIGURE CAPTIONS

1. A dual detector positron camera.
2. Point response function for a limited angle, square-section, positron camera.
3. A summary of the filtering process.

Procedure	Timings*
Back-Projection	120 secs.
Filtering	15 secs.
Continuation to unmeasured frequencies ¹⁸⁾ (per iteration)	8 secs.
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TOTAL (10 iterations)	215 secs.

TABLE I : * Timings are for an IBM 370/168 assuming 10^6 positron annihilations and a $64 \times 64 \times 16$ element reconstruction matrix.

POSITRON ANNIHILATION

Figure 1

POINT RESPONSE FUNCTION

Figure 2

FOCUSED TOMOGRAPHY

ORIGINAL

(a) POINT SOURCE OF ACTIVITY

BLURRED

(b) BACK - PROJECTION

RINGING

(c) BAND - LIMITED FILTER

SMOOTHED

(d) HANNING FILTER

Figure 3