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TRANSIENT ELECTROMAGNETIC FIELDS EXCITED BY BUNCHES OF CHARGED PARTICLES
IN CAVITIES OF ARBITRARY SHAPE

by

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ABSTRACT

The electromagnetic fields excited by arbitrarily shaped bunches of charged particles travelling through accelerating structures with cylindrical symmetry are calculated by a numerical method solving Maxwell's integral equations in the time domain. The computer program based on this method calculates transient electromagnetic fields as well as the total energy radiated and the energy gain of particles inside the bunch. The shape of the accelerating structure may be defined by the user and can be approximated in a mesh of up to 50'000 nodes. The electric field-lines of a Gaussian bunch passing through a LEP cavity are shown.

1. INTRODUCTION

Calculations of electromagnetic fields excited by relativistic bunches of charged particles travelling through accelerating structures have been made¹⁻¹⁰⁾ for two reasons: The first one is a prediction of the total energy loss due to the excitation of RF cavities by the beam (higher-order mode losses) for planning RF systems for new accelerators. The second is a prediction of the energy loss of particles inside the beam as a function of their position (usually called wake field) because the wake field determines the current dependent bunch shape.

Up to now, rigorous results for the energy loss and the wake field are known only for two rather idealized structures: the parallel-plate capacitor and the pill-box cavity^{1,2,4,7,8)}.

For realistic cavities, the resonant modes are used for such calculations^{5,6)}. These modes are usually calculated with computer programs such as KN7C⁹⁾ and SUPERFISH¹⁰⁾.

The calculation of high-order mode losses is meanwhile well established and can be considered a solved problem. The calculation of wake fields, however, is still an open problem since the idealized structures are not good models for realistic cavities and since the mode analysis concept is still connected with an uncertainty due to the missing contributions of the free charges to the electromagnetic field⁶⁾.

These problems could be solved by a method giving the electromagnetic field for all times in the presence of free moving charges without any assumptions or restrictions. In order to stay within realistic ranges of computer sizes and calculation times, we first have to restrict the generality to the case of cylindrical symmetry.

Such a numerical tool for the computation of transient electromagnetic fields is described. Further integrations over the calculated fields yield the energy loss of particles inside the bunch as well as the total energy radiated. After a comparison with results of other methods, some results (wake fields, radiation losses) for the LEP cavity will be given.

2. THE NUMERICAL METHOD

We consider a closed cavity with cylindrical symmetry. A bunch of charged particles (charge density $\lambda(r, z)$) travelling at the speed $v = \beta c$ through the cavity produces a current density $J_z(r, z, t) = \beta c \lambda(r, z - \beta c t)$. Maxwell's equations have to be solved only for the field components $H_\phi(r, z, t)$, $E_r(r, z, t)$ and $E_z(r, z, t)$ as all the others vanish. In the r - z plane, a grid may be defined (r_i, z_j) to approximate the cavity shape. By separating the time dependence and the space dependence and by replacing the field components by their values on the grid points, we get the vector functions $h(t) = \{H_\phi(r_i, z_j, t)\}$, $e(t) = \{E_r(r_i, z_j, t), E_z(r_i, z_j, t)\}$ and $j(t) = \{J_z(r_i, z_j, t)\}$. Using the "finite integration method"¹³⁾, Maxwell's equations may be written as a set of matrix equations:

$$D_h (-\mu_0 \dot{h}) = R_e e, \quad \left(- \iint_A \mu_0 \frac{\partial \vec{H}}{\partial t} \cdot d\vec{A} = \oint_{(A)} \vec{E} \cdot d\vec{s} \right), \quad (1)$$

$$D_e (\epsilon_0 \dot{e} + j) = R_h h, \quad \left(\iint_A \left(\epsilon_0 \frac{\partial \vec{E}}{\partial t} + \vec{J} \right) \cdot d\vec{A} = \oint_{(A)} \vec{H} \cdot d\vec{s} \right). \quad (2)$$

D_h and D_e are diagonal matrices. R_h and R_e are simple sparse matrices.

In order to eliminate the continuous time dependence, the r - z grid is complemented by a time grid with a time step size Δt . The time derivatives are then replaced by¹¹⁾:

$$\dot{e}(\Delta t n) = \frac{1}{\Delta t} \left\{ e\left(\Delta t n + \frac{\Delta t}{2}\right) - e\left(\Delta t n - \frac{\Delta t}{2}\right) \right\}, \quad (3)$$

$$\dot{h}\left(\Delta t n + \frac{\Delta t}{2}\right) = \frac{1}{\Delta t} \left\{ h(\Delta t n + \Delta t) - h(\Delta t n) \right\}. \quad (4)$$

Using an index instead of the time argument, Maxwell's equations can be rewritten as:

$$h^{n+1} = h^n - Y_0 c \Delta t D_h^{-1} R_e e^{n+1/2}, \quad (5)$$

$$e^{n+1/2} = e^{n-1/2} + Z_0 c \Delta t \left(D_e^{-1} R_h h^n - j^n \right). \quad (6)$$

with $Z_0 = \sqrt{\mu_0/\epsilon_0}$, $Z_0 Y_0 = 1$.

Starting with $e^{0.5} = h^0 = j^0 \equiv 0$, the transient electromagnetic field may be calculated for all times in an alternating sequence of products of matrices with vectors already known. Since it is easy to recalculate the single sparse matrices R_e and R_h in each step, they need not be stored in memory. Hence this procedure occupies computer space only for h and e .

Some restrictions^{12,13,14,16)} have to be mentioned concerning the choice of Δt and $j(t)$. There is an upper limit for Δt which causes no problems for relativistic bunches. The Fourier transformation of $j(t)$ at each grid point must decay rapidly above the maximum sample frequency $\omega_z = \pi c/\Delta z$ of the mesh. This excludes the treatment of fast point charges ($\beta > 0.01$) and short range step currents. It is, however, quite straightforward to set up the calculation for realistic bunch shapes occurring, for example, in electron-positron storage rings.

3. RESULTS

A computer program named BCI has been written which can process arbitrarily shaped bunches and structures with cylindrical symmetry using a mesh of up to 50'000 nodes.

Although the main advantage of BCI is the ability to compute transient effects and electromagnetic fields, it may first be checked by a comparison of results for the total energy loss with results of other methods. The computation of the total energy loss due to the excitation of cavities by the bunch (beam loading, higher order mode losses) is meanwhile well established and has been checked by measurements⁵⁾. As the total loss of a Gaussian bunch traversing a pill-box cavity can be determined analytically, we compare BCI results with analytical ones for two different geometries. Results are shown in Table 1.

Table 1.

The total energy loss of a Gaussian bunch ($\sigma_{r.m.s.} = 2.5$ cm) due to a pill-box cavity computed analytically and numerically

gap length/m	radius/m	$k_{tot}/(V/pC)$
0.10	0.05	0.168 analytically
		0.169 BCI (mesh: 61 x 121)
0.05	0.10	0.270 analytically
		0.270 BCI (mesh: 81 x 41)

Some significant results for a realistic LEP cavity are given below (cavity shape see Fig. 4, exact geometrical parameters see Ref. 15).

The total energy loss of a Gaussian bunch has been computed by integrations over the field energy in the cavity after the passage of the bunch.

Figure 1 shows the total loss parameter as a function of the r.m.s. bunch length between 2 cm and 20 cm.

When a bunch traverses a cavity, particles gain or lose energy due to scattered electromagnetic fields. BCI can compute such results as a function of the position of the particles inside the bunch and as a function of time. For the following results, a single cell of a LEP cavity was used as well as the beam parameters:

$\sigma_{r.m.s.} = 4.5$ cm (Gaussian bunch)

$N_p = 1.45 \cdot 10^{12}$ (number of particles).

Fig. 1 The loss parameter $k_{tot}(\sigma)$ for a single cell of the LEP cavity and a Gaussian bunch (r.m.s. length σ).

Figure 2 shows the energy loss of particles as a function of their position along the bunch at four different time points. After the bunch has passed the cavity, the final energy distribution (from now independent of time) is usually called wake field. The total energy gain of the particles including the accelerating field may easily be obtained by adding the wake field and the accelerating voltage. For LEP, stage 1, the parameters are:

$$V_{rf} = 0.508 \text{ MV/cell} ; \quad f_{RF} = 353.4 \text{ MHz} ; \quad \phi_s = 130.6^\circ .$$

Since these parameters include fundamental beam loading, the voltage and the phase to be used have to be recalculated to the stage before the bunch traverses the cavity:

$$\bar{V}_{rf} = 0.533 \text{ MV/cell} ; \quad \bar{\phi}_s = 128.3^\circ .$$

Fig. 2 The transient (time dependent) energy gain of particles inside a Gaussian bunch due to a single cell of a LEP cavity.

- 1: bunch shape.
- 2: the bunch (centre is 4σ in front of the cavity centre.
- 3: the bunch is in the middle of the cavity.
- 4: the bunch is 4σ behind the cavity centre.
- 5: the bunch is 8σ behind the cavity centre, i.e. has left the cavity (curve 5 is usually called wake field).

Figure 3 shows the steady-state energy balance inside the bunch including the accelerating voltage and the wake field after the passage. Results such as the time dependent energy gain, wake field and total energy gain may be used to study the reaction on the bunch shape which was assumed to be rigid so far.

The transient electric field lines of a Gaussian bunch travelling through a LEP cavity in the absence of accelerating fields are shown in Fig. 4. Additional results may be obtained, such as electric and magnetic fields or wall currents at any point in the cavity as a function of time.

- bunch shape
- total energy gain
- wake field
- .-.- accelerating voltage

Fig. 3 Wake field and total energy gain of particles inside a Gaussian bunch after the passage of a single cell of an accelerating LEP cavity.

Fig. 4 Electric field lines of a Gaussian bunch ($\sigma_{r.m.s.} = 4.5$ cm) travelling through a single cell of a LEP cavity in the absence of accelerating fields.

4. CONCLUSIONS

The computer program BCI treats arbitrarily shaped bunches and computes transient results like energy gain and electromagnetic fields. The influence of all cylindrically symmetric accelerator components may be studied, such as bellows and steps in the cross-section.

Future extensions are planned to include structures with finite wall conductivity and three-dimensional structures which are not cylindrically symmetric.

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