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ANOMALY NON-MATCHING IN COMPOSITE MODELS

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ABSTRACT

Recent results of 't Hooft and others on anomaly non-matching in composite models are extended to include "exotics" among the possible composite states. The previous conclusions are unchanged.

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In order to have a sensible theory in which preons are confined to make quarks and leptons, the values of anomalies should not depend on whether they are calculated in terms of the fundamental fields or in terms of composite states. 't Hooft¹⁾, and more recently several others^{2),3)}, have explored the circumstances under which models can satisfy this requirement. They have found that, when the requirement known as "Appelquist-Carrazzone-Symanzik decoupling"¹⁾ (described below) is also imposed, no theories of the form $SU_C(N) \times SU_L(n) \times SU_R(n) \times U(1)$ are acceptable, although with certain "colour" (confining) groups other than $SU(N)$ solutions do exist^{3),4)}. This result is of great importance, since among its consequences is the inevitability of chiral symmetry breaking for the strong interactions. Thus it deserves being put on the strongest possible ground. This note addresses the question of whether the result can be evaded when in addition to the simplest colour singlet composite fermions (e.g., fff) one allows colour singlet exotics (e.g., fff $\bar{f}$). The conclusion is that the result cannot be evaded by this means.

We consider the case of $SU(N)$ of "colour" (for the case of quarks and leptons being composed of preons one does not want to be restricted to $SU(3)$) and $SU_L(n) \times SU_R(n) \times U(1)$ as the flavour group. The representations under flavour of the composites are conveniently labelled by their Young tableaux, which for colour non-exotics contain a total of N boxes, and specify their transformations under $SU_L(n)$ and $SU_R(n)$. For instance $(\square\square\square, 1)$ is the $SU_C(3)$ non-exotic which is completely symmetric under $SU_L(n)$ and a singlet under $SU_R(n)$. For a given composite representation R , the index $\ell(R)$, is defined to be the difference between the number of left- and the number of right-handed particle states in that representation of flavour. Presumably the representation content and thus the indices of the bound states is a dynamical question. The analysis of 't Hooft asks whether there exists any set $\{\ell(R)\}$ which is physically acceptable.

The net contribution of representation R to an anomaly is $\ell(R) A(R)$, where $A(R)$ is the anomaly of a single member of the representation. Thus the requirement that the anomalies must be the same whether evaluated in terms of the fundamental, $(1, \square)$ and $(\square, 1)$, or composite states means that summing over all composite representations R :

$$\sum_R \ell(R) A(R) = A(\square) \quad (1)$$

Thus one has a constraint on the $\ell(R)$'s. Since $A(R)$ depends on the flavour group, this equation is n -dependent. 't Hooft argued¹⁾, however, that the dynamics binding the fundamental objects into composites, and thus determining the $\{\ell(R)\}$, should not depend on the number of fundamental fermions. Thus he seeks

a set of integer $\ell(R)$'s which satisfy Eq. (1) for any n . A stronger set of constraints, which also implies that Eq. (1) should be satisfied in an n -independent way, are the so-called Appelquist-Carrazzone-Symanzik (ACS) equations^{1),2)} These are an additional set of equations for the $\ell(R)$ which follow from requiring the composite theory to be smooth and sensible as masses of the fundamental flavours are varied from zero to infinity.

It has by now been shown^{1),2)} that when the composites are restricted to non-exotics, there is no solution to the combined ACS equations and the anomaly Eq. (1) for any N , except for the special case $n = 2$. Now let us show that removing the restriction to non-exotics does not permit a solution, either.

First we show that in the non-exotic case in which the representations are formed from a total of N boxes there is no integral $\{\ell(R)\}$ solution to the anomaly Eq. (1) even if the ACS equations are satisfied. This can be seen by focusing on the n -independent part of Eq. (1), i.e., the equation obtained by setting $n = 0$. In that case only representations of the form $(N \text{ boxes}, 1)$ need be considered since the anomaly of any "mixed" representation is proportional to n . For example the $SU_L(n)$ anomaly of $(N-1 \text{ boxes}, \square)$ is proportional to the number of times $A_L(N-1 \text{ boxes})$ appears, which is just the dimension of $\square$, i.e., n . Furthermore we will show below that the n -independent part of the anomaly of any representation of the form $(N \text{ boxes}, 1)$ is zero or $\pm N^2$ *). However, the total anomaly of the fundamental representation is just 1 per colour, hence N in this normalization, and there is no way to add together multiples of $\pm N^2$ to obtain N . Thus there is no integer-valued choice for the $\{\ell(R)\}$ which satisfies Eq. (1) for $n = 0$. We then show that the same result holds for exotic representations.

To see that the n -independent part (called $\bar{A}(R)$) of the anomaly of any representation R of the form $(N \text{ boxes}, 1)$ is zero or $\pm N^2$, first use the formula [Eq. (14) of Ref. 5)] for the anomaly of the N -box symmetric representation:

$$A \left(\overline{\square}^N \right) = \frac{(n+N)! (n+2N)}{(n+1)! (N-1)! (n+2)} = N^2$$

for $n = 0$. Now in general [this is most evident from the supergroup method of Ref. 2)]:

$$A \left(\left. \begin{array}{c} \overline{\square}^{N-m} \\ \square^m \end{array} \right\} \right) = (-1)^m A \left(\overline{\square}^N \right)$$

*) The argument here is for the $(SU_L(n))^3$ anomaly for concreteness. The same sort of argument applies for the $(SU_L(n))^2 \cdot U(1)$ anomalies as well.

Next we show that all other N-box representations have $\bar{A} = 0$. First cross the "elbow" representations of Eq. (2) with $\square$. Since $A(\square \otimes \text{anything})$ is always proportional to n , $\bar{A}(\square \otimes \text{anything}) = 0$ and we find

i.e., $\bar{A}(\text{diagram}) = 0$: we have shown that $\bar{A}$ of any elbow with one "interior" box vanishes. Similarly crossing any elbow with diagram and diagram proves that $\bar{A}$ of elbows with two interior boxes vanish. Proceeding by induction, assume elbows with m interior boxes have vanishing $\bar{A}$. Then by contraction with every $(m+1)$ -box representation one sees that every elbow with $(m+1)$ interior boxes has vanishing $\bar{A}$. (By virtue of the fact that the m -interior-box $\bar{A}$'s are zero, the only term in each contraction equation which is not a priori zero is the one for which all the $m+1$ boxes are interior.)

Thus, since all $\bar{A}(\text{N boxes})$ are either zero or $\pm N^2$, there can be no integer-valued solution to Eq. (1) for $n = 0$.

What happens if we include colour exotics, i.e., representations formed by contracting an N -box representation with one or more of $(\square, \bar{\square})$, $(\bar{\square}, \square)$, $(\square \times \bar{\square}, 1)$ or $(1, \square \times \bar{\square})$? As noted previously only a representation of the form $(\dots, 1)$ has a non-zero $\bar{A}$, so we only need to consider exotics formed by $\{N \text{ boxes}\} \otimes \square \otimes \bar{\square}$, i.e., $\{N + 1\} \otimes \bar{\square}$. The Young Tableau of $\bar{\square}$ is an $n-1$ box column. Thus contracting $\{N + 1\} \otimes \bar{\square}$ yields an object with a total of $N + n$ boxes -- except that columns containing n boxes represent a singlet and are therefore removed from the tableau. So that in general we have a $(cn + N)$ -box object (where c is an integer) whose $\bar{A}$ is either 0 or $\pm(cn + N)^2$, if it turns out to be an elbow configuration. But for $\bar{A}$, $(cn + N)^2 \rightarrow N^2$ and we

see we have not enlarged our set of possible values of $\tilde{A}$. Clearly, adding any number of exotics is equivalent. We thus conclude that the 't Hooft anomaly-matching equations cannot be solved by means of allowing composites in exotic as well as non-exotic representations.

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REFERENCES

- 1) G. 't Hooft, Cargèse Summer Institute Lectures, (1979).
- 2) Y. Frishman, A. Schwimmer, T. Banks and S. Yankielowicz, Weizmann Institute preprint WIS-80/27 (1980).
- 3) R. Barbieri, L. Maiani and R. Petronzio, CERN preprint TH-2900 (1980).
- 4) A. Schwimmer, private communication.
- 5) J. Banks and H. Georgi, Phys. Rev. D14 (1976) 1159.