SMALL DISTANCE BEHAVIOUR IN EINSTEIN THEORY OF GRAVITATION

V. de Alfaro *)

Istituto di Fisica Teorica, Torino

S. Fubini and G. Furlan **)

CERN -- Geneva

A B S T R A C T

We present a preliminary discussion of some consequences of a fully dilatational invariant formulation of the Einstein theory of gravitation. Arguments are given to show that the small distance behaviour of the theory can be greatly improved, leading to a situation very similar to usual renormalizable theories.

*) And Istituto Nazionale di Fisica Nucleare, Sezione di Torino.

**) Permanent address : Istituto di Fisica Teorica, Trieste, Italy, and Istituto Nazionale di Fisica Nucleare, Sezione di Trieste.

1. Gravitation is attracting more and more attention both to achieve a satisfactory formulation as a quantum field theory and to understand its relevance to elementary particle physics. This attitude has been stimulated by the recent successful formulation of unified theories of weak and electromagnetic interactions, where a primary rôle is played by the local gauge principle and by spontaneous symmetry breaking. The possibility of exploiting these ingredients for the Einstein theory of gravitation requires an improved understanding of its connection to gauge theories and of its scaling properties.

In a previous work ¹⁾ we have discussed in detail the second point and examined the invariance properties of the Einstein action under the dilatation transformation $x^\mu \rightarrow x^\mu + \delta x^\mu$, $\delta x^\mu = -\epsilon x^\mu$. Let us first of all recall the old example of a scalar, self coupled, massless field with Lagrangian density (Euclidean formulation)

$$\mathcal{L} = -\frac{1}{2} (\partial_\mu \phi)^2 - \lambda \phi^4. \quad (1)$$

The action for this theory is invariant under the dilatation transformation

$$\delta x^\mu = -\epsilon x^\mu, \quad \delta \phi = \epsilon (x \cdot \partial + 1) \phi \quad (2)$$

which ascribes dimensionality -1 (in units of length) to the scalar field ϕ .

In the same way one can easily get convinced that the Einstein action ^{*)}

$$A_E = -\frac{1}{4} \int d^4x \sqrt{g} (R + \Lambda) \quad (3)$$

or equivalently ²⁾

$$A_E = \frac{1}{4} \int d^4x \sqrt{g} \left\{ g^{\mu\nu} (\Gamma_{\mu\lambda}^\sigma \Gamma_{\nu\sigma}^\lambda - \Gamma_{\mu\nu}^\sigma \Gamma_{\sigma\lambda}^\lambda) - \Lambda \right\} \quad (3')$$

is dilatation invariant for transformations of the field $g_{\mu\nu}$

$$\delta g_{\mu\nu} = \epsilon (x \cdot \partial + 2) g_{\mu\nu}, \quad \delta g^{\mu\nu} = \epsilon (x \cdot \partial - 2) g^{\mu\nu} \quad (4)$$

corresponding to dimensionality -2(2) for $g_{\mu\nu}(g^{\mu\nu})$.

This assignment of dimensionality to the field $g_{\mu\nu}$ differs from the conventional one (based on the geometric interpretation) where $g_{\mu\nu}$ is taken to have

*) For sake of generality we have included a cosmological term Λ (dimensionless in our formulation).

dimensionality zero. This is not justified, in our opinion, from a field theoretical point of view. Our formulation guarantees that the action can be written in a fully scale invariant form with no need for the introduction of a dimensional constant, like the Newton constant in the standard approach. The Newton constant, $G_N = \ell^2/4\pi$, appears in the definition of the vacuum state namely

$$\langle 0 | g_{\mu\nu}(x) | 0 \rangle = \frac{1}{\ell^2} \delta_{\mu\nu} \quad (5)$$

which leads to the expansion

$$g_{\mu\nu}(x) = \frac{1}{\ell^2} \delta_{\mu\nu} + \frac{1}{\ell} \varphi_{\mu\nu}(x) \quad (6)$$

where $\varphi_{\mu\nu}$ is the quantum field.

Newtonian gravity is thus related to the spontaneous breaking of dilatation invariance *).

2. Let us consider the analogy with the scalar theory of Eq. (1). There are two kinds of results for such a model that we want to recall:

- a) Classical solutions exist, for instance of the power type in $r \equiv \sqrt{x^2}$ whose power is determined by the dimensionality of ϕ . (In general these solutions are characterized by invariance under subgroups of the $O(4,2)$ conformal group.)
- b) The small distance behaviour of the theory, as given order by order by the perturbation series in λ , can be guessed on the basis of naïve power counting. The free propagator is fixed by translation and dilatation invariance arguments. This, in addition to the dimensionality -4 of the interaction, ensures that higher order terms have a manageable small distance behaviour. In particular the first correction to the scalar propagator involves the matrix element of the operator $\delta A / \delta \phi(x)$ (with dimensionality -3), namely

$$\langle 0 | \frac{\delta A}{\delta \phi(x)} \frac{\delta A}{\delta \phi(y)} | 0 \rangle \approx \frac{1}{[(x-y)^2]^3} \quad (7)$$

*) In Ref. 1) we have investigated the possibility of classical solutions, different from the constant one, leading to different definitions of the vacuum state. For sake of definiteness we shall here stick to the "Newtonian" structure of the vacuum defined in Eq. (5).

The renormalization procedure actually introduces a mass scale; however, such a deviation from the dilatation invariant limit is of the logarithmic type only.

If we now move to the scale invariant theory of gravitation as given in Eq. (3) it has been shown that classical solutions still exist, for instance of a power form dictated by dimensional arguments (i.e. $g_{\mu\nu} \propto \delta_{\mu\nu}/r^2$) in complete analogy with the $\lambda\phi^4$ model. This property may be taken as a further hint that arguments based on simple dimensional estimates can represent a valuable guidance to establish the small distance behaviour of the theory, apart from logarithmic terms.

If this point of view is correct, one would expect that the hopeless infinities one meets in the standard treatment of quantum gravity are not inherent to the theory. They rather follow from the use of the expansion (6) beyond its natural limit of validity.

Since in our approach the parameter ℓ is not naturally provided by the Lagrangian but characterizes a particular solution valid at large distances, it will not be surprising to find that the above expansion in ℓ is inadequate to reproduce the correct behaviour of the theory at small distances. The aim of this work is to present an argument in favour of this point of view: the heart of the matter is the use of an improved perturbation expansion which formally does not spoil the scale invariant properties of the theory.

The idea that the small distance properties of quantum gravity could be much better than what is indicated by standard perturbation theory has been strongly advocated by A. Salam and we refer to his inspiring talk in 1974 ³⁾. The main new point of our work is the use of dilatation invariance as the guiding principle.

3. As a first indication let us examine the heuristic model obtained by assuming the ansatz

$$g_{\mu\nu}(x) = f(x) \delta_{\mu\nu} \quad (8)$$

(Notice that such a $g_{\mu\nu}$ retains its form under the transformations of the 15-parameter conformal group so that the ensuing expression of the model will exhibit explicit conformal invariance.)

The action takes on the simple form

$$A = \frac{1}{4} \int d^4x \left\{ \frac{3}{2} \frac{1}{f(x)} (\partial_\mu f)^2 - \Lambda f^2(x) \right\}. \quad (9)$$

The analogue of the conventional treatment would now be to separate $f(x)$ as a constant term (the Newton constant) plus a quantum part $\varphi(x)$, i.e.

$$f(x) = \frac{1}{\ell^2} + \varphi(x) \quad (10)$$

and to expand in $\ell \varphi$. This procedure leads to a series of the form $\sum_u \ell^{2n} \varphi^n (\partial_\mu \varphi)^2$ and each term in ℓ is clearly accompanied by a non-renormalizable interaction. Such an unpleasant situation can be easily improved. It is indeed enough to choose a more appropriate field variable, namely

$$f(x) = h^2(x) \quad (11)$$

to obtain for the action the same form as Eq. (1)

$$A = \frac{1}{4} \int d^4x \left\{ 6 (\partial_\mu h)^2 - \Lambda h^4 \right\} \quad (12)$$

that is a perfectly respectable theory!

A more direct way of obtaining Eq. (12) is to express the Lagrangian in terms of the "vierbein" fields V_μ^a ($a = 1 \dots 4$), for which the ansatz reads

$$V_\mu^a(x) = h(x) \delta_{\mu a}. \quad (13)$$

It is tempting to suggest that such a feature is not accidental and that also in the case of a general $g_{\mu\nu}(x)$ the mathematical structure of the theory can be improved by an appropriate choice of the dynamical variables. In this spirit we shall be guided to use a formulation in which the vierbeine V_μ^a are the fundamental dynamical variables. They carry (according to our definition!) the dimensionality -1 of genuine vector fields. For these variables the Newtonian expansion takes on the form (pointed out by H.T. Nieh⁴).

$$V_\mu^a(x) = \frac{1}{\ell} \delta_{\mu a} + V_\mu^{a'}(x). \quad (14)$$

4. The action (3') in terms of the vierbeine reads

$$A_E = \int d^4x L = \int d^4x \left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\} \partial_{\mu_1} V_{\nu_1}^{a_1} \partial_{\mu_2} V_{\nu_2}^{a_2} \quad (15)$$

where

$$\begin{aligned} \frac{1}{\sqrt{g}} \left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\} = & + \frac{1}{8} \left\{ \delta_{a_1 a_2} (g^{\mu_1 \nu_2} g^{\mu_2 \nu_1} - g^{\mu_1 \mu_2} g^{\nu_1 \nu_2}) \right. \\ & - g^{\mu_1 \mu_2} V_{a_1}^{\nu_1} V_{a_2}^{\nu_2} + g^{\nu_1 \nu_2} V_{a_2}^{\mu_1} V_{a_1}^{\mu_2} \left. \right\} + \\ & + \frac{1}{4} \left\{ g^{\nu_1 \mu_2} V_{a_2}^{\mu_1} V_{a_1}^{\nu_2} - g^{\mu_1 \nu_2} V_{a_1}^{\nu_1} V_{a_2}^{\mu_2} + \right. \\ & \left. + g^{\mu_1 \mu_2} V_{a_1}^{\nu_1} V_{a_2}^{\nu_2} - g^{\mu_1 \nu_1} V_{a_2}^{\nu_2} V_{a_1}^{\mu_2} \right\}. \end{aligned} \quad (16)$$

A first important advantage of using the vierbeine emerges if one notices

that the fundamental coefficients $\left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\}$ (which determine the

dynamical and quantum properties of the theory) have zero dimensionality and are invariant under the local Weyl transformation *)

$$V_{\mu}^a(x) \rightarrow C(x) V_{\mu}^a(x), \quad V^{\mu a}(x) \rightarrow C^{-1}(x) V^{\mu a}(x). \quad (17)$$

As a consequence, the form of $\left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\}$ is for conformally flat spaces

the same as the flat one, $\left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\}_0$, obtained by simply taking

$$g_{\mu\nu} \rightarrow \delta_{\mu\nu}, \quad V_{\mu}^a \rightarrow \delta_{\mu a}.$$

*) For operators which do not contain field derivatives, the requirement of zero dimensionality and of Weyl invariance are of course equivalent.

We are now ready to discuss the improved form of perturbation theory. The first step is to use the conformal invariant separation

$$V_\mu^a(x) = h(x) \delta_{\mu a} + \bar{V}_\mu^a(x) \quad (18)$$

where

$$\bar{V}_\mu^{\mu} = 0 \quad (19)$$

and to construct an iteration procedure in which $\bar{V}_\mu^a/h$ is taken to be small. The Lagrangian L can be recast in the form

$$L = L_0 + L_1 \quad (20)$$

where

$$L_0 = \left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\}_0 \partial_{\mu_1} V_{\nu_1}^{a_1} \partial_{\mu_2} V_{\nu_2}^{a_2} =$$

$$= - \left[\frac{1}{4} (\partial_\mu V_\nu^a)^2 - \frac{1}{2} (\partial_\mu V_\mu^a - \frac{1}{2} \partial_a V_\mu^\mu)^2 - \frac{1}{8} (\partial_\nu V_\mu^\mu)^2 \right]^{(20)_0}$$

and

$$L_1 = \left\langle \left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\} - \left\{ \begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right\}_0 \right\rangle \partial_{\mu_1} V_{\nu_1}^{a_1} \partial_{\mu_2} V_{\nu_2}^{a_2} \equiv$$

$$\equiv \Delta \left(\begin{matrix} \mu_1 & \nu_1 & a_1 \\ \mu_2 & \nu_2 & a_2 \end{matrix} \right) \partial_{\mu_1} V_{\nu_1}^{a_1} \partial_{\mu_2} V_{\nu_2}^{a_2} \quad (20)_1$$

Since our aim is to examine the qualitative nature of the small distance divergences we shall proceed in a "naïve" fashion and adopt a covariant approach in the so-called "harmonic" gauge, disregarding the problems connected with the unitarity of the theory ⁵⁾.

In this framework the propagators are:

$$\langle \bar{V}_\mu^a(x) \bar{V}_\nu^b(y) \rangle_0 = \frac{1}{4\eta^2(x-y)^2} (\delta_{\mu\nu} \delta_{ab} + \delta_{\mu b} \delta_{\nu a} - \frac{1}{2} \delta_{\mu a} \delta_{\nu b}) \quad (21)$$

$$\langle \bar{V}_\mu^a(x) h(y) \rangle_0 = 0$$

As a consequence of Eq. (14) we shall write for the scalar part

$$h(x) = \frac{1}{\ell} + h'(x) \quad (22)$$

$$\langle h'(x) h'(y) \rangle_0 = \left(-\frac{1}{2}\right) \frac{1}{4\pi^2(x-y)^2} \equiv -G(x-y) \quad (23)$$

The quantity $\Delta \begin{pmatrix} u_1 & \dots \\ u_2 & \dots \end{pmatrix}$ depends only on $\bar{V}_\mu^a/h$ and can be expanded in powers of this ratio. The first terms of the "interaction" part L_1 can be sketched in the form

$$L_1 = \left(\frac{\partial h}{\partial x}\right)^2 \bar{V}/h + \frac{\partial h}{\partial x} \frac{\partial \bar{V}}{\partial x} \bar{V}/h + \left(\frac{\partial h}{\partial x}\right)^2 \left(\frac{\bar{V}}{h}\right)^2 + \dots \quad (24)$$

It is important to stress the fact that L_1 is of dimensionality -4 (as appropriate for a renormalizable theory) since the operator $\bar{V}/h$ in which we expand has zero dimensionality.

Before discussing in detail how this affects the small distance behaviour of this perturbation series it is fruitful to compare our expansion with the standard perturbation one.

The standard approach uses the expansion (14) in $2V_\mu^a$. Each higher order will thus acquire one more operator V_μ^a which worsens the small distance behaviour (say, of the two-point function) by adding an extra factor $1/(x_1 - x_2)^2$.

In our expansion based on Eq. (18) the same effect will exist. Each new operator $\bar{V}$ increases the small distance singularity. However, one should expect (on the basis of dimensionality arguments) that the $1/h$ operator accompanying each $\bar{V}$ will give a factor compensating the extra singularity created by $\bar{V}$. The problem is thus to investigate the effect in each perturbation diagram of the operator $1/h$.

5. We are now faced with the fundamental problem of computing propagators containing inverse powers of $h(x)$, in particular we shall discuss in detail the simple example of the quantity $\langle 1/h(x) \cdot 1/h(y) \rangle_0$.

On the basis of Eq. (22) we expand $h^{-1}(x)$ as a power series in $\ell h'(x)$.
We obtain (using Eq. (23))

$$\left\langle \frac{1}{h(x)} \frac{1}{h(y)} \right\rangle_0 = \ell^2 \sum_m m! (-\ell^2 G(x-y))^m \quad (25)$$

The asymptotic series on the right-hand side of Eq. (25) can be summed by introducing the integral representation

$$m! = \int_0^\infty e^{-\alpha} \alpha^m d\alpha$$

and we are led to the well known result ^{*}) of non polynomial field theories ⁶⁾

$$\left\langle \frac{1}{h(x)} \frac{1}{h(y)} \right\rangle_0 = I(G) = \ell^2 \int_0^\infty \frac{e^{-\alpha} d\alpha}{1 + \alpha \ell^2 G(x-y)}. \quad (26)$$

The function $I(G)$ can be expressed in terms of a known transcendental function, the exponential integral. Its asymptotic properties are the following ⁷⁾ as $G(x-y) \rightarrow \infty$ (i.e. for $(x-y) \rightarrow 0$)

$$I(G) \sim \frac{1}{G} \log(\ell^2 G) - \frac{\gamma}{G} \left(1 + O(1/G)\right) \quad (27)$$

On the other hand, for $G \rightarrow 0$, $I(G)$ tends to the constant value ℓ^2 .

We thus see that at small distances the vacuum expectation value $\langle 1/h(x) 1/h(y) \rangle_0$ has, apart from a logarithmic term, the power behaviour one could predict on the basis of a naïve dimensional argument by ascribing to $1/h$ dimensionality opposite to $h(x)$.

This trend is confirmed by the computation of more complicated propagators. We have for instance

$$\left\langle \frac{1}{h^m(x)} \frac{1}{h^m(y)} \right\rangle_0 = \frac{\ell^2}{[(m-1)!]^2} \left(\frac{d}{dG} \right)^{m-1} I(G), \quad (28)$$

^{*}) An elegant fashion to derive the same result (26) (which avoids asymptotic expansions) is to exploit the representation $h^{-1}(x) = \int_0^\infty e^{-\alpha} h(x) d\alpha$ and to use well known identities for exponential functions of operators.

whose asymptotic form $\sim (x-y)^{2n} \log \left\{ \frac{(x-y)^2}{\ell^2} \right\}$ again agrees with dimensional estimates. The same conclusions could be drawn for expectation values of operators involving derivatives of $h(x)$ *).

The overall outcome of our discussion is the following: If we consider the expectation value

$$\langle 0 | O(x) O(y) | 0 \rangle$$

of any operator $O(x)$ of dimensionality d , built of $\bar{V}_\mu^a$ and of positive or negative powers of $h(x)$, the small distance behaviour is dictated by dimensional arguments with a leading term

$$\langle 0 | O(x) O(y) | 0 \rangle \sim [(x-y)^2]^d \log \left\{ \frac{(x-y)^2}{\ell^2} \right\} \quad (29)$$

Recalling now that the term L_1 , Eq. (20)₁, has (as it should!) dimensionality -4, we can conclude that at least in the case of two-point functions the small distance nature of the integrals is essentially the same as for renormalizable theories.

If we consider for example the first corrections to the graviton propagator, these involve the vacuum expectation value of the operator $\delta A / \delta V_\mu^a(x)$ and its behaviour in the ultraviolet limit is

$$\langle 0 | \frac{\delta A}{\delta V_\mu^a(x)} \frac{\delta A}{\delta V_\nu^b(y)} | 0 \rangle \sim \frac{1}{[(x-y)^2]^3} \log \left\{ \frac{(x-y)^2}{\ell^2} \right\} \quad (30)$$

This is completely analogous apart from logarithms to what happens in the $\lambda\phi^4$ for the vacuum expectation values (7) related to the propagator of the scalar field.

*) The previous technique can be applied to these more complicated cases. One introduces the generalized operator

$$H(x | z^{(1)} z^{(2)} \dots) = h(x) + z_\lambda^{(1)} \frac{\partial h}{\partial x_\lambda} + z_\lambda^{(2)} \frac{\partial h}{\partial y_\lambda} + \dots \quad (a)$$

and quantities of the form $\partial_\mu h \partial_\nu h \dots / h^n$ can be reduced to derivatives (with respect to $z_\lambda^{(i)}$ at $z_\lambda^{(i)} = 0$) of H^N whose expectation values can be computed with the same method as for $1/h^n$.

6. Our procedure can be easily extended to discuss the small distance behaviour of theories where gravity is coupled to matter fields. Again, the action will be dimensionless and the expansion (19) can be applied. A main point is the definition of the "best" dynamical variables to be used in order to describe the matter fields.

In the case of gauge theories (electroweak or of gluons) the choice is nearly obvious: the best variable is the gauge field itself. For scalar and spinor fields it is easy to get convinced that the appropriate choices are those discussed in Ref. 1) (section 2), namely

$$\hat{\phi} = g^{1/8} \phi, \quad \hat{\psi} = g^{3/16} \psi \quad (31)$$

The new fields $\hat{\phi}, \hat{\psi}$, whose dimensionality is -1 and $-3/2$ respectively, were shown in Ref. 1) to behave in the flat limit as the canonical scalar and spinor fields.

It is important to emphasize that our procedure works for all theories invariant under general co-ordinate transformations, even for theories (like for instance $\sqrt{g} \phi^n$ $n > 4$) which are non-renormalizable in the flat limit. Thus in agreement with the hopes expressed several times, the coupling to gravity works to adjust the small distance behaviour of theories which in the flat limit would be hopelessly non-renormalizable.

Before closing this section we wish to emphasize that our perturbation treatment makes explicit use of the invariance of the Lagrangian under the 15 parameter conformal group.

On the other hand, the Lagrangians are invariant under the much larger set of general co-ordinate transformations. This can possibly lead to extra constraints among different terms of the perturbative expansion which might improve the small distance behaviour. For instance it is well known that the form of the graviton propagator is gauge dependent so that strong cancellations should occur in order to reach a gauge independent result.

A final interesting point concerns the choice of the background field. In this paper we have followed the conventional approach based on a flat background. However, it is not hard to see that our summation procedure would work equally well in the presence of any conformally flat background field. It would lead essentially to the same leading term at small distances: the ultraviolet behaviour depends on the form of the Lagrangian and not on the nature of the vacuum.

7. The heuristic discussion of this paper seems to confirm the validity and usefulness of the point of view that the Einstein action is fully dilatational invariant. This is achieved by ascribing the appropriate dimensionality -1 to the "gravitational field" V_{μ}^a . The apparent "lack of scale invariance" of the Lagrangian (3) follows, in our opinion, from the unjustified assignment of zero dimensionality to V_{μ}^a . We do not thus see at this point any reason to abandon the original Einstein formulation which is, until now, in perfect agreement with experiment.

Our faith in the validity and internal consistency of the Lagrangian (3) does not exclude the possibility that the whole problem could be seen in a more modern light. In this respect we think that the new approach of "induced gravity" 8), which tries to derive the term R and to compute the Newton constant from other interactions, is both very promising and interesting.

The main suggestion of this work is that the "non renormalizability" of the general theory of gravitation is not an intrinsic feature of the theory but is rather a consequence of the use of the standard perturbative expansion of Eq. (5) which, by introducing the Newton constant, completely breaks the original invariance. We have indeed shown that using the expansion of Eq. (14), which preserves dilatation invariance, one is led to an enormously improved small distance behaviour of the Green functions.

We are of course aware that it will take much time and effort before one can hope to tackle the full problem of renormalizability. We believe that our preliminary results may encourage optimism about the final outcome of such a programme.

REFERENCES

- 1) V. de Alfaro, S. Fubini and G. Furlan - Il Nuovo Cimento 50A, 523 (1979),
Il Nuovo Cimento 57B, 227 (1980).
- 2) See for instance, L. Landau and E. Lifshitz - "The classical theory of fields",
ch. XI, Pergamon Press (Oxford).
- 3) H.T. Nieh - "Generalization of Einstein's theory of gravitation",
New York University at Stony Brook, preprint (1979).
- 4) A. Salam - "Impact of quantum gravity theory on particle physics" in
"Quantum Gravity", an Oxford Symposium, Clarendon Press (Oxford, 1975) and
papers quoted therein.
- 5) See for instance, the lectures by M.J.G. Veltman in "Methods in field theory",
Les Houches Summer School 1975, North Holland Publ.
- 6) For a more complete introduction to the subject see the contributions in
A. Salam - "Non-polynomial Lagrangians, renormalization and gravity",
Tracts in Mathematics and Natural Sciences. Vol. 1 (1971).
- 7) "Higher transcendental functions", Vol II, Bateman manuscript project,
A. Erdélyi ed., McGraw Hill (New York).
- 8) S.L. Adler - Phys. Rev. Letters 44, 1567 (1980).
See also, S.L. Adler - "A formula for the induced gravitational constant",
Institute for Advanced Study, preprint (1980);
B. Hasslacher and E. Mottola - "Gauge field model of induced classical gravity",
Institute for Advanced Study, preprint (1980);
A. Zee - "Spontaneously generated gravity", University of Pennsylvania,
preprint (1980).