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NEUTRINO MASSES FROM QUARK-LEPTON TRANSITIONS

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A B S T R A C T

Quark-lepton transitions can lead to a small Dirac mass for the electron neutrino and can contribute to the electron mass. This mass must be less than 10^{-3} eV or else the calculated ratio $\Gamma(\pi \rightarrow e\nu)/\Gamma(\pi \rightarrow \mu\nu)$ will be in disagreement with experiment.

Quark-lepton symmetry calls for the existence of right-handed neutrinos, and these fields are a natural part of many grand unified models¹⁾. The observed neutrino masses are very small even on leptonic scales, or perhaps they are zero. Two possibilities must be considered for the coupling of the left-handed neutrino to the right-handed neutrino, i.e., the Dirac mass of the neutrino:

- i) The Dirac mass is exactly (or very nearly) zero. Perhaps this is guaranteed by an unknown symmetry;
- ii) The Dirac mass is not zero but is near the mass m_ℓ of the corresponding charged lepton. The light mass of the observed neutrino is due²⁾ to a large Majorana mass M for the right-handed neutrino. The observed neutrino has a small admixture of right-handed neutrinos and has a mass m_ℓ^2/M .

In models with quark-lepton transitions, there is a possible source for Dirac mass terms besides the usual Lagrangian Higgs couplings. Imagine that the quark-lepton transitions are mediated by boson exchange or some other means, but at low energy there is an effective four-Fermi interaction of the form:

$$\frac{1}{\Lambda^2} \bar{\nu}_L \Gamma_1 q \bar{q} \Gamma_2 \nu_R \quad (1)$$

where Γ_1 and Γ_2 are some gamma matrices, ν and q represent neutrino and quark fields, and the subscripts L and R mean left-handed and right-handed. Λ is a scale parameter characteristic of the interactions. If this can be rearranged or Fierz-transformed to the form:

$$\frac{1}{\Lambda^2} \bar{q} q \bar{\nu}_L \nu_R \quad (2)$$

then the vacuum expectation value of $\bar{q}q$ induced by quantum chromodynamics will give the neutrino a Dirac mass. Since the scale Λ is large, we expect this mass to be small. In case i) above, it would account for a small non-zero neutrino mass. In case ii) it would be suppressed by a factor m_ℓ/M . In either case, we would also expect the charged lepton ℓ corresponding to ν to have an interaction:

$$\frac{1}{\Lambda^2} \bar{q} q \bar{l}_L l_R \quad (3)$$

which would induce a small contribution to the Dirac mass of the charged lepton. I will call this mass m^d to distinguish it from the usual Higgs mass m . We then have for the two cases:

$$\begin{aligned} (i) \quad m_\nu &= m^d \\ m_\ell &= m + m^d \end{aligned} \quad (4)$$

$$\begin{aligned} (ii) \quad m_\nu &= m_\ell^2 / M \\ m_\ell &= m + m^d \end{aligned} \quad (5)$$

I will now argue that the mass m^d must be very small, or else the observed pattern of ordinary pion decay will be unacceptably altered. Consider first the coupling of the Higgs field to the electron-neutrino doublet in the standard³⁾ weak interaction theory. I will use a σ model notation where the four real components of the complex Higgs doublet explicitly form an $SU(2)_L \times SU(2)_R = O(4)$ four vector

$$(\phi_0, \vec{\phi}) \quad (6)$$

It is ϕ_0 which gets the vacuum expectation value V , breaking the symmetry in the standard way. Under an $SU(2)_L$ transformation $\mathcal{U}_L$ and an $SU(2)_R$ transformation $\mathcal{U}_R$, the combination

$$H = \phi_0 + i \vec{\tau} \cdot \vec{\phi} \quad (7)$$

transforms as follows:

$$H \rightarrow u_L H u_R^\dagger \quad (8)$$

The Higgs coupling to the lepton doublet,

$$\Psi = \begin{pmatrix} \nu \\ e \end{pmatrix} \quad (9)$$

can be written as

$$\bar{\Psi}_L H (a + b \tau_3) \Psi_R + h.c. \quad (10)$$

With a and b arbitrary, this is the most general $SU(2)_L \times U(1)_{\text{HYP}}$ invariant coupling. (If $b \neq 0$ it is not $SU(2)_R$ invariant, but it is $U(1)_{\text{HYP}}$ invariant because hypercharge can be constructed from $(\tau_3)_R$). In order to make $m_\nu = 0$, a and b are chosen so that Eq. (10) has the form

$$\frac{m}{2V} \bar{\Psi}_L H (1 - \tau_3) \Psi_R + h.c. \quad (11)$$

Written out explicitly this becomes:

$$\begin{aligned} & \frac{m}{V} \bar{e} \phi_0 e - i \frac{m}{V} \phi_3 \bar{e} \gamma_5 e \\ & + \frac{im}{\sqrt{2}V} \phi^+ \bar{\nu} (1 + \gamma_5) e - \frac{im}{\sqrt{2}V} \phi^- \bar{e} (1 - \gamma_5) \nu \end{aligned} \quad (12)$$

When ϕ_0 gets the vacuum expectation value V , the electron gets a Higgs mass m while the neutrino remains massless³⁾.

The theory also has quarks interacting through quantum chromodynamics. I will only talk about massless quarks because quark masses are irrelevant to this discussion. The u and d quarks of this sector have a $SU(2)_L \times SU(2)_R$ symmetry which is spontaneously broken down to $SU(2)_{\text{ISOSPIN}}$. The result is three massless

pions, $\vec{\pi}^{\text{QCD}}$, which couple to the axial currents $J_{5\mu}^a$ with strength f_π :

$$\langle 0 | J_{5\mu}^a | \pi_b^{\text{QCD}} \rangle = f_\pi q_\mu \delta_b^a \quad (13)$$

f_π is a measure of the spontaneous symmetry breaking. $J_{5\mu}^a$ is part of the weak current, so these QCD pions couple to the weak gauge bosons. If there were no Higgs fields, then the QCD pions would disappear from the spectrum giving masses $\sim gf_\pi$ to the $W^\pm$ and Z ⁴⁾ (g is the weak coupling constant). In a theory with both Higgs fields and QCD pions, the linear combinations

$$\vec{\phi}^{\text{absorbed}} = \frac{V\vec{\phi} + f_\pi \vec{\pi}^{\text{QCD}}}{\sqrt{V^2 + f_\pi^2}} \quad (14)$$

couple maximally to the weak current and they are absorbed as longitudinal components of $W^\pm$ and Z ⁵⁾. The states

$$\vec{\pi}^{\text{physical}} = \frac{V\vec{\pi}^{\text{QCD}} - f_\pi \vec{\phi}}{\sqrt{V^2 + f_\pi^2}} \quad (15)$$

have no couplings to the weak currents, so they remain massless states in the spectrum. (The introduction of quark masses gives the physical pions masses.) Since $V \gg f_\pi$, the absorbed states are mostly Higgs, while the physical pions are almost pure QCD pions. Since the physical pion does not couple to the weak current, the pion cannot decay into $e\nu$ through W boson exchange. The decay proceeds through the component of $\vec{\pi}^{\text{physical}}$ which is a Higgs particle, since the Higgs couple to leptons via the Yukawa coupling previously discussed. Using equations (12) and (15) we get:

$$A(\pi \rightarrow e\nu) = \frac{if_\pi}{\sqrt{V^2 + f_\pi^2}} \frac{m}{\sqrt{2}V} \bar{e}(1-\gamma_5)\nu \quad (16)$$

Since $V \gg f_\pi$,

$$A(\pi \rightarrow e \nu) = \frac{i f_\pi m}{\sqrt{2} V^2} \bar{e} (1 - \gamma_5) \nu \quad (17)$$

which agrees with the familiar form

$$A(\pi \rightarrow e \nu) = i G_w f_\pi q^\mu \bar{e} \gamma_\mu (1 - \gamma_5) \nu \quad (18)$$

because the Fermi constant G_w is $1/\sqrt{2}V^2$ and q on the spinors is m . Of course the familiar form comes from making the approximation that the physical pion is π^{QCD} , which couples to the weak current with strength f_π . Current algebra theorems guarantee that there are no differences in pion decay between the calculation presented here and the usual picture⁴⁾. The introduction of more quarks and leptons introduces a Cabibbo angle into Eq. (17) in the usual way.

This decay of the pion will be altered if there are quark-lepton transitions which can give the electron-neutrino a mass through the spontaneous breakdown of chiral symmetry associated with QCD. I will assume for simplicity that these interactions are $SU(2)_L \times SU(2)_R$ invariant, and I will describe their low energy interactions with a σ model. Let

$$(\sigma, \vec{\pi}^{\text{QCD}}) \quad (19)$$

be a $SU(2)_L \times SU(2)_R$ four vector, and let

$$\Sigma = \sigma + i \vec{T} \cdot \vec{\pi}^{\text{QCD}} \quad (20)$$

have the same transformation properties as H (Eq. (8)). The σ field develops the vacuum expectation value f_π which is a measure of the symmetry breaking. The coupling to leptons can be described⁶⁾ by

$$\frac{m^d}{f_\pi} \bar{\Psi}_L \leq \Psi_R + h.c. \quad (21)$$

which, written out, is

$$\frac{m^d}{f_\pi} \bar{\Psi} [\sigma + i \vec{\tau} \cdot \vec{\pi} \gamma_5] \Psi \quad (22)$$

Now when σ develops the vacuum expectation value f_π , both the neutrino and electron get a mass m^d , so added to the previous mass terms:

$$\begin{aligned} m_{\nu_e} &= m^d \\ m_e &= m + m^d \end{aligned} \quad (23)$$

which corresponds to case i) discussed before.

The interaction (22) makes a contribution to $\Pi \rightarrow e\nu$. The additional amplitude is

$$\frac{V}{\sqrt{V^2 + f_\pi^2}} i \frac{m^d}{f_\pi} \sqrt{2} \bar{e} \gamma_5 \nu \quad (24)$$

and since $V \gg f_\pi$, this, when added to (17), gives

$$A(\pi \rightarrow e\nu) = \frac{if_\pi m_e}{\sqrt{2} V^2} \bar{e} (1 - \gamma_5) \nu + i \frac{m^d}{f_\pi} \sqrt{2} \bar{e} \gamma_5 \nu \quad (25)$$

I have replaced m by m_e since m^d is small. Now unless m^d/f_π is much less than $f_\pi m_e/V^2$, the observed pion decay will be disturbed. This means that m^d/m_e must be less than $(f_\pi/V)^2 \sim 10^{-7}$, so a stringent limit is imposed on the QCD induced mass.

More specifically, consider the effect of the term proportional to m^d on the ratio

$$R = \frac{\Gamma(\pi \rightarrow e \nu)}{\Gamma(\pi \rightarrow \mu \nu)} \quad (26)$$

There are two reasonable assumptions about the possibility of quark-lepton transitions inducing an extra term in $\pi \rightarrow \mu \nu$. First, it might be zero if the quark-lepton transitions are always between particles of the same generation. Secondly, it might just have the same form as Eq. (24), with the same m^d , but with muon replacing electron. Either assumption gives

$$R(m^d \neq 0) = \left(1 - \frac{2m^d}{m_e} \frac{V^2}{f_\pi^2}\right) R(m^d = 0) \quad (27)$$

where terms of order m^d/m_μ have been neglected and $R(m^d = 0)$ is the standard result for the theoretical calculation of R .

The standard theoretical calculation (including radiative corrections) agrees fairly well with experiment⁷⁾:

$$R(\text{experiment}) = (1.033 \pm .019) R(m^d = 0) \quad (28)$$

So, within three standard deviations,

$$\frac{2m^d}{m_e} \frac{V^2}{f_\pi^2} < .024 \quad (29)$$

or

$$m^d \lesssim (2 \times 10^{-9}) m_e \approx 10^{-3} \text{ eV} \quad (29)$$

cont.

This result was calculated for case i) above, where m^d is the mass of the electron neutrino and is also a contribution to the electron mass. The calculation for case ii) gives the same result for m^d , but the interpretation comes from Eq. (5).

We can ask what scale of interactions would be required to produce such a small m^d . Using Eq. (2), we see that

$$m^d = \frac{\langle \bar{q}q \rangle}{\Lambda^2} \quad (30)$$

Estimating $\langle \bar{q}q \rangle$ from the current algebra formula

$$f_\pi^2 m_\pi^2 = (m_u + m_d) \langle \bar{q}q \rangle \quad (31)$$

gives $\langle \bar{q}q \rangle \sim (250 \text{ MeV})^3$, so

$$\Lambda \gtrsim 125 \text{ TeV} \quad (32)$$

In a Pati-Salam⁸⁾ model, these quark-lepton transitions are built into the theory via boson exchange, and the scale Λ would be close to the mass of the boson exchanged. The best limit on the scale Λ in a Pati-Salam model comes from limits on $K_L \rightarrow \mu e$, and Λ comes out about twice as high as estimated here⁹⁾.

In conclusion, quark-lepton transitions can induce a small mass for the neutrinos and can contribute equally to the corresponding charged lepton mass. For the electron and the electron neutrino, this induced mass must be very small or else there will be unacceptable changes in pion decay. The scale of interactions which give rise to this mass must be greater than 125 TeV. If the neutrino gets a mass from quark-lepton transitions, then the decay modes of the pion should be measured with higher precision. The decay of the pion could be a rich source of information about physics at higher scales.

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