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SINGLET PARTON DENSITIES BEYOND LEADING ORDER

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A B S T R A C T

We present a short summary of the results for the evolution of two-loop singlet parton densities for both spacelike and timelike processes. There is a discrepancy between our results and those already available for deep inelastic scattering.

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We report on the results of the calculation of two-loop corrections to the Q^2 evolution of deep inelastic structure functions and fragmentation functions for the singlet sector, in the framework of QCD. Our technique is based on the factorization of mass singularities and it has been described in detail in Ref. 1). We remind the reader that the essential ingredients are :

- i) the choice of a lightlike gauge $n_\mu A_\mu = 0$, $n^2 = 0$;
- ii) the decomposition of the perturbative expansion in generalized ladders ;
- iii) the use of dimensional regularization for collinear singularities in $n = 4 + \epsilon$ ($\epsilon > 0$) dimensions, and of
- iv) renormalization group invariance in the minimal subtraction scheme.

The factorization programme in the singlet sector is almost identical to the non-singlet case ¹⁾. In the axial gauge mass singularities (poles in ϵ) are generated by the propagation of nearly on-shell partons (quarks and gluons) between neighbouring two-particle irreducible kernels and are factorized by means of suitable projectors acting on quark and gluon lines. The form of the quark projector P_F inserted between two kernels A, B which are connected by two quarks with momenta k, has been already introduced for the non-singlet case ¹⁾ and reads :

$$A P_F B = (A^{\alpha\alpha'} \cancel{\not{k}}_{\alpha\alpha'}) [P.P.] (\cancel{\not{k}}^{\beta\beta'} B_{\beta\beta'}) \quad (1)$$

where $\alpha, \beta, \dots$ are spinor indices and $[P.P.]$ extracts the pole part in ϵ of the projected kernel B.

For the singlet case, two kernels may be connected by a gluon line and one needs a corresponding projector P_G ^{*)} :

$$A P_G B = (A_{\mu\mu'} d^{\mu\mu'}(k) \frac{1}{2(1+\epsilon/2)}) [P.P.] (-g^{\nu\nu'} B_{\nu\nu'}) \quad (2)$$

where μ, ν are Lorentz indices and $d_{\mu\nu}(k)$ is the numerator of the gluon propagator in the lightlike gauge :

$$d_{\mu\nu}(k) = -g_{\mu\nu} + \frac{k_\mu \eta_\nu + k_\nu \eta_\mu}{\eta \cdot k} \quad (3)$$

*) In the timelike region, the average over gluon helicities $1/2(1+\epsilon/2)$ is on the right-hand side of the $[P.P.]$ projector.

The final result of the factorization procedure in the case, for example, of the cross-section $\sigma^{\ell H}$ of a lepton-hadron hard process is the following :

$$\sigma^{\ell H} = \sum_i C_i^{\ell} \Gamma_{ik} V_k^{(B,H)} = \sum_i C_i^{\ell} V_i^{(H)} \quad (4)$$

where :

- i) the indices i, k run over quarks and gluons ;
- ii) C_i denotes the "short-distance cross-section" for the lepton- i^{th} -parton scattering ;
- iii) Γ_{ik} contains all mass singularities of the parton cross-section ;
- iv) the vector $V^{(B,H)} [V^{(H)}]$ denotes the bare [renormalized] parton densities :
 $V_1 = q + \bar{q}$; $V_2 = G$.

Equation (4) can be read, as usual, either as a convolution in the longitudinal momenta (x) space, or as an ordinary product in the moment space.

The short-distance cross-section ¹⁾ is a power series in the running coupling constant $\alpha(Q^2)$:

$$C_i^{\ell} = (C_i^{\ell})^{(0)} + \frac{\alpha}{2\pi} (C_i^{\ell})^{(1)} + \dots$$

and is defined as the finite part of the "physical" inclusive lepton-parton cross-section : $\ell + V_i \rightarrow \text{anything}$ in the spacelike processes (deep inelastic scattering), $\ell + V_i \rightarrow \text{anything}$ in the timelike ones (one-particle inclusive annihilation). The calculation is made in $n = 4 + \epsilon$ dimensions for $\mu^2 = Q^2$ (or $\mu^2 = 4\pi Q^2 e^{-\gamma}$ in the $\overline{\text{MS}}$ scheme) and, at order α , it reduces to take the full result of the cross-section except of the pole part in ϵ . The results for coefficient functions which agree with our prescription for both spacelike ²⁾ and timelike ³⁾ processes are available in the literature.

The parton density $V_i^{(H)}$ evolves with Q^2 as follows :

$$Q^2 \frac{d}{dQ^2} V_i^{(H)}(Q^2) = P_{ij} [\alpha(Q^2)] V_j^{(H)}(Q^2) \quad (5)$$

where

$$P_{ij}(\alpha) = \frac{\alpha}{2\pi} P_{ij}^{(0)} + \left(\frac{\alpha}{2\pi}\right)^2 P_{ij}^{(1)} \quad (6)$$

The matrix $P^{(0)}$, universal for all hard processes, is well known ^{4),*)}

$$P^{(0)} \equiv \begin{pmatrix} P_{FF}^{(0)} & P_{GF}^{(0)} \\ P_{FG}^{(0)} & P_{GG}^{(0)} \end{pmatrix} \quad (7)$$

$$P_{FF}^{(0)} = C_F \cdot [P_{FF}(x)]_+$$

$$P_{FF}(x) = \frac{1+x^2}{1-x}$$

$$P_{FG}^{(0)} = C_F \cdot P_{FG}(x)$$

$$P_{FG}(x) = \frac{1+(1-x)^2}{x}$$

$$P_{GF}^{(0)} = 2T_R N_F \cdot P_{GF}(x)$$

$$P_{GF}(x) = x^2 + (1-x)^2$$

$$P_{GG}^{(0)} = 2C_G \frac{1}{x} [x P_{GG}(x)]_+ - \delta(1-x) \left(\frac{2}{3} T_R N_F\right); P_{GG}(x) = \frac{1}{1-x} + \frac{1}{x} - 2 + x - x^2 \quad (8)$$

where $C_F = \frac{N^2-1}{2N} = \frac{4}{3}$; $C_G = N=3$; $T_R = \frac{1}{2}$ and $[f(x)]_+ = f(x) - \delta(1-x) \int_0^1 dy f(y)$

The probability matrix $P^{(1)}$ is universal for all spacelike processes (where it will be denoted by $P^{(1,S)}$) and for all timelike ones ($P^{(1,T)}$). The diagonal entries of $P^{(1)}$ can be written by separating the part which is proportional to $\delta(1-x)$:

$$\begin{aligned} P_{FF}^{(1,U)} &= \hat{P}_{FF}^{(1,U)} - \delta(1-x) \zeta_F^{(1)} \\ P_{GG}^{(1,U)} &= \hat{P}_{GG}^{(1,U)} - \delta(1-x) \zeta_G^{(1)} \\ P_{AB}^{(1,U)} &= \hat{P}_{AB}^{(1,U)} \quad A \neq B \end{aligned} \quad (9)$$

where $U = T$ or S .

Using arguments based on the dispersion relations similar to those given in Ref. 1), we relate the constants ζ_F , ζ_G to the timelike densities in our scheme as follows :

*) For timelike processes the matrix is transposed.

$$\sum_F^{(1)} = \int_0^1 dx x \left[\hat{P}_{FP}^{(1,T)} + \hat{P}_{FG}^{(1,T)} \right]$$

$$\sum_G^{(1)} = \int_0^1 dx x \left[\hat{P}_{GG}^{(1,T)} + \hat{P}_{GF}^{(1,T)} \right] \quad (10)$$

Our results for the two-loop probabilities $\hat{P}(x)$ are ^{*} :

$$\begin{aligned} \hat{P}_{FF}^{(1,S)} = C_F^2 \cdot & \left[-1+x + \left(\frac{1}{2} - \frac{3}{2}x\right) \ln x - \frac{1}{2}(1+x) \ln^2 x - \left(\frac{3}{2} \ln x + 2 \ln x \ln(1-x)\right) p_{FF}(x) \right. \\ & \left. + 2 p_{FF}(-x) \cdot S_2(x) \right] + \end{aligned}$$

$$+ C_F C_G \left[\frac{14}{3}(1-x) + \left(\frac{11}{6} \ln x + \frac{1}{2} \ln^2 x + \frac{67}{18} - \frac{\pi^2}{6}\right) p_{FF}(x) - p_{FF}(-x) S_2(x) \right]$$

$$+ C_F T_R N_F \left[-\frac{16}{3} + \frac{40}{3}x + \left(10x + \frac{16}{3}x^2 + 2\right) \ln x - \frac{112}{9}x^2 + \frac{40}{9} \cdot \frac{1}{x} - \right. \\ \left. - 2(1+x) \ln^2 x - \left(\frac{10}{9} + \frac{2}{3} \ln x\right) p_{FF}(x) \right]$$

$$\begin{aligned} \hat{P}_{FG}^{(1,S)} = C_F^2 & \left[-\frac{5}{2} - \frac{7}{2}x + \left(2 + \frac{7}{2}x\right) \ln x + \left(-1 + \frac{1}{2}x\right) \ln^2 x - 2x \ln(1-x) + \right. \\ & \left. + (-3 \ln(1-x) - \ln^2(1-x)) p_{FG}(x) \right] + \end{aligned}$$

$$+ C_F C_G \left[\frac{28}{9} + \frac{65}{18}x + \frac{44}{9}x^2 + \left(-12 - 5x - \frac{8}{3}x^2\right) \ln x + (4+x) \ln^2 x + \right. \\ \left. + 2x \ln(1-x) + (-2 \ln x \ln(1-x) + \frac{1}{2} \ln^2 x + \frac{11}{3} \ln(1-x) + \ln^2(1-x) \right. \\ \left. - \frac{\pi^2}{6} + \frac{1}{2}\right) p_{FG}(x) + p_{FG}(-x) S_2(x) \right]$$

$$+ C_F T_R N_F \left[-\frac{4}{3}x - \left(\frac{20}{9} + \frac{4}{3} \ln(1-x)\right) p_{FG}(x) \right]$$

^{*}) These results have been obtained by using extensively the algebraic programme "Schoonschip" written by M. Veltman.

$$\begin{aligned}
 \hat{P}_{GF}^{(1,S)} = & C_F T_R N_F \left[4 - 9x + (-1+4x) \ln x + (-1+2x) \ln^2 x + 4 \ln(1-x) \right. \\
 & + (-4 \ln x \ln(1-x) + 4 \ln x + 2 \ln^2 x - 4 \ln(1-x) + 2 \ln^2(1-x) - \frac{2}{3} \pi^2 \\
 & \left. + 10 \right) \cdot p_{GF}(x) \Big] \\
 & + C_G T_R N_F \left[\frac{182}{9} + \frac{14}{9} x + \frac{40}{9} \frac{1}{x} + \left(\frac{136}{3} x - \frac{38}{3} \right) \ln x - 4 \ln(1-x) \right. \\
 & - (2+8x) \ln^2 x + (-\ln^2 x + \frac{44}{3} \ln x - 2 \ln^2(1-x) + 4 \ln(1-x) + \frac{\pi^2}{3} \\
 & \left. - \frac{218}{9} \right) p_{GF}(x) + 2 p_{GF}(-x) \cdot S_2(x) \Big]
 \end{aligned}
 \tag{11}$$

$$\begin{aligned}
 \hat{P}_{GG}^{(1,S)} = & C_F T_R N_F \left[-16 + 8x + \frac{20}{3} x^2 + \frac{4}{3} \frac{1}{x} + (-6-10x) \ln x + (-2-2x) \ln^2 x \right] \\
 & + C_G T_R N_F \left[2 - 2x + \frac{26}{9} x^2 - \frac{26}{9} \frac{1}{x} - \frac{4}{3} (1+x) \ln x - \frac{20}{9} p_{GG}(x) \right] \\
 & + C_G^2 \left[\frac{27}{2} (1-x) + \frac{67}{9} (x^2 - \frac{1}{x}) + \left(-\frac{25}{3} + \frac{11}{3} x - \frac{44}{3} x^2 \right) \ln x + \right. \\
 & \left. + 4(1+x) \ln^2 x + \left(\frac{67}{9} - 4 \ln x \ln(1-x) + \ln^2 x - \frac{\pi^2}{3} \right) p_{GG}(x) + 2 p_{GG}(-x) S_2(x) \right]
 \end{aligned}$$

Here and in the following, $p_{AB}(x)$ are defined as in Eqs. (8) and

$$S_2(x) \equiv \int_{\frac{x}{1+x}}^{\frac{1+x}{1+x}} \frac{dz}{z} \ln\left(\frac{1-z}{z}\right) \quad ; \quad S_1(x) \equiv \int_0^{1-x} \frac{dz}{z} \ln(1-z)$$

For timelike processes, we get :

$$\begin{aligned}
 \hat{P}_{FF}^{(1,T)} = & C_F^2 \left[-1+x + \left(-\frac{3}{2} + \frac{1}{2} x \right) \ln x + \frac{1}{2} (1+x) \ln^2 x + \left(\frac{3}{2} \ln x - 2 \ln^2 x + \right. \right. \\
 & \left. \left. + 2 \ln x \ln(1-x) \right) p_{FF}(x) + 2 p_{FF}(-x) S_2(x) \right]
 \end{aligned}$$

$$+ C_F C_G \left[\frac{14}{3}(1-x) + \left(\frac{11}{6} \ln x + \frac{1}{2} \ln^2 x - \frac{\pi^2}{6} + \frac{67}{18} \right) p_{FF}(x) - p_{FF}(-x) S_2(x) \right]$$

$$+ C_F T_R N_F \left[-\frac{52}{3} + \frac{28}{3}x + \frac{112}{9}x^2 - \frac{40}{9} \frac{1}{x} + (-10 - 18x - \frac{16}{3}x^2) \ln x + \right.$$

$$\left. + 2(1+x) \ln^2 x + \left(-\frac{2}{3} \ln x - \frac{10}{9} \right) p_{FF}(x) \right]$$

$$\hat{P}_{FG}^{(1,T)} = C_F^2 \left[-\frac{1}{2} + \frac{9}{2}x + \left(-8 + \frac{1}{2}x \right) \ln x + 2x \ln(1-x) + \left(1 - \frac{1}{2}x \right) \ln^2 x + \right.$$

$$\left. + \left(\ln^2(1-x) + 4 \ln x \ln(1-x) - 8 S_1(x) - \frac{4}{3} \pi^2 \right) p_{FG}(x) \right] +$$

$$+ C_F C_G \left[\frac{62}{9} - \frac{35}{18}x - \frac{44}{9}x^2 + \left(2 + 12x + \frac{8}{3}x^2 \right) \ln x - 2x \ln(1-x) + \right.$$

$$\left. + (-4-x) \ln^2 x + (-2 \ln x \ln(1-x) - 3 \ln x - \frac{3}{2} \ln^2 x - \ln^2(1-x) + \right.$$

$$\left. + 8 S_1(x) + \frac{7}{6} \pi^2 + \frac{17}{18} \right) p_{FG}(x) + p_{FG}(-x) S_2(x) \right]$$

$$\hat{P}_{GF}^{(1,T)} = (T_R N_F)^2 \left[-\frac{8}{3} - \left(\frac{16}{9} + \frac{8}{3} \ln x + \frac{8}{3} \ln(1-x) \right) p_{GF}(x) \right] +$$

$$+ C_F T_R N_F \left[-2 + 3x + (-7 + 8x) \ln x - 4 \ln(1-x) + (1-2x) \ln^2 x + \right.$$

$$\left. + (-4 \ln x \ln(1-x) - 2 \ln^2 x - 2 \ln(1-x) + 2 \ln x - 2 \ln^2(1-x) + \right.$$

$$\left. + 16 S_1(x) + 2 \pi^2 - 10 \right) p_{GF}(x) \right] +$$

$$+ C_G T_R N_F \left[-\frac{152}{9} + \frac{166}{9}x - \frac{40}{9} \frac{1}{x} + \left(-\frac{4}{3} - \frac{76}{3}x \right) \ln x + 4 \ln(1-x) + \right.$$

$$\left. + (2+8x) \ln^2 x + \left(8 \ln x \ln(1-x) - \ln^2 x - \frac{4}{3} \ln x + \frac{10}{3} \ln(1-x) + \right. \right.$$

$$\left. + 2 \ln^2(1-x) - 16 S_1(x) - \frac{7}{3} \pi^2 + \frac{178}{9} \right) p_{GF}(x) + 2 p_{GF}(-x) S_2(x) \right]$$

$$\hat{P}_{GG}^{(1,T)} = C_F T_R N_F \left[-4 + 12x - \frac{164}{9}x^2 + \frac{92}{9} \frac{1}{x} + \left(10 + 14x + \frac{16}{3}x^2 + \frac{16}{3} \frac{1}{x} \right) \ln x + \right.$$

$$\left. + 2(1+x) \ln^2 x \right] +$$

$$+ C_G T_R N_F \left[2 - 2x + \frac{26}{9}(x^2 - \frac{1}{x}) - \frac{4}{3}(1+x) \ln x - \left(\frac{20}{9} + \frac{8}{3} \ln x \right) p_{GG}(x) \right] +$$

$$+ C_G^2 \left[\frac{27}{2} (1-x) + \frac{67}{9} \left(x^2 - \frac{1}{x} \right) + \left(\frac{11}{3} - \frac{25}{3} x - \frac{44}{3} \frac{1}{x} \right) \ln x - 4(1+x) \ln^2 x + \right. \\ \left. + \left(4 \ln x \ln(1-x) - 3 \ln^2 x + \frac{22}{3} \ln x - \frac{\pi^2}{3} + \frac{67}{9} \right) P_{GG}(x) + 2 P_{Gg}(-x) S_2(x) \right]$$

From Eq. (9) follows the energy momentum conservation law for timelike probabilities, which reads :

$$\int dx x \left[P_{FF}^{(1\pi)} + P_{FG}^{(1\pi)} \right] = 0 \\ \int dx x \left[P_{GG}^{(1\pi)} + P_{GF}^{(1\pi)} \right] = 0 \quad (13)$$

The validity of the same relation for spacelike probabilities is guaranteed by the identity of moments of our probabilities with the anomalous dimensions of Wilson operators in the minimal subtraction scheme ⁵⁾. This relation has already been discussed for non-singlet fermion operators : by using the explicit form of the gluon projector [Eq. (2)] and the same analyticity arguments of Ref. 1), one can extend the relation to the singlet case and get :

$$-\frac{1}{8} \gamma_{AB}^{(1)}(N) = \eta \cdot \int_0^1 P_{AB}^{(1)}(x) x^{N-1} dx \quad (14)$$

where $\gamma_{AB}^{(1)}(N)$ are the anomalous dimension of Wilson operators and

$$\eta = +1 \text{ if } A=B \quad ; \quad \eta = -1 \text{ if } A \neq B$$

Equations (13) for spacelike probabilities then follow from the vanishing of the anomalous dimensions of the energy momentum tensor operator. The validity of Eqs. (13) both for spacelike and timelike probabilities puts a severe constraint on the second moment of their difference :

$$\int dx x \left\{ \left[\hat{P}_{FF}^{(1,\pi)} + \hat{P}_{FG}^{(1,\pi)} \right] - \left[\hat{P}_{FF}^{(1,s)} + \hat{P}_{FG}^{(1,s)} \right] \right\} = 0 \\ \int dx x \left\{ \left[\hat{P}_{GG}^{(1,\pi)} + \hat{P}_{GF}^{(1,\pi)} \right] - \left[\hat{P}_{GG}^{(1,s)} + \hat{P}_{GF}^{(1,s)} \right] \right\} = 0 \quad (15)$$

which has to hold independently for each of the colour factors of Eqs. (11), (12). By explicit integration, one can verify that the above conditions are satisfied by the probabilities of Eqs. (11), (12).

Using Eqs. (9), (10), (15), one can rewrite the full result for the diagonal probabilities as follows

$$\begin{aligned} P_{FF}^{(1,0)}(x) &= \frac{1}{x} \left[x \hat{P}_{FF}^{(1,0)}(x) \right]_+ - \delta(1-x) \int_0^1 dy y \hat{P}_{FG}^{(1,0)}(y) \\ P_{GG}^{(1,0)}(x) &= \frac{1}{x} \left[x \hat{P}_{GG}^{(1,0)}(x) \right]_+ - \delta(1-x) \int_0^1 dy y \hat{P}_{GF}^{(1,0)}(y) \end{aligned} \quad (16)$$

Our results for the spacelike case differ from those already obtained with the operator product expansion formalism in the part of $\hat{P}_{GG}^{(1,S)}$ proportional to c_G^2 5).

Therefore, we have decided to perform, as a further independent test, the calculation of the two-loop contribution to the β functions. This can be obtained by collecting next-to-leading ultra-violet divergences inside proper vertices and self-energies. By analytic continuation in the number of dimensions n from the region $n < 4$ to the region $n > 4$, ultra-violet divergences are converted into mass singularities and are therefore calculable with our techniques.

It turns out that the proper vertex is related to the first moment of our spacelike probabilities. The particular diagrams, however, enter with quite different weights to the final sum as it is illustrated in Fig. 1. Figures 1b and 1c show the two different contractions of colour indices of the forward four-point amplitude of Fig. 1a which are performed in the calculation of spacelike probabilities and of proper vertex contribution to the β function, respectively. The kinematics of diagrams entering in the calculation of spacelike probabilities corresponds to that of a proper vertex with zero momentum transfer. From Eqs. (10), we can relate self-energies with finite incoming momenta to the second moment of our timelike probabilities. However, the zero momentum gluon self-energy, $\zeta_G(0)$, is not easily accessible in the lightlike gauge and would require an independent calculation. We can get rid of this extra contribution, by comparing the coefficients of ultra-violet poles of gluon-fermion and gluon-gluon vertices which are expected to be equal on the basis of the gauge invariance of the theory.

Denoting by $\tilde{P}_{AB}(x)$ the densities derived with the contraction of colour indices as in Fig. 1c, we get the relation :

$$\begin{aligned} \beta_1 - \frac{1}{2} \zeta_G(0) &= \\ &= \int_0^1 dx [\tilde{P}_{FF}(x) + \tilde{P}_{FG}(x)] - \zeta_F^{(1)} = \int_0^1 dx [\tilde{P}_{GG}(x) + \tilde{P}_{GF}(x)] - \zeta_G^{(1)} \quad (17) \\ &= C_F T_R N_F \left[-\frac{1}{2\epsilon} \right] + C_G T_R N_F \left[\frac{1}{\epsilon} \left(-\frac{86}{27} + \frac{2}{3} Z_2 \right) \right] + \\ &\quad + C_G^2 \left[\frac{1}{\epsilon} \left(\frac{943}{108} + \frac{1}{2} Z_3 - \frac{11}{6} Z_2 \right) \right] \quad Z_2 = \frac{\pi^2}{6} \quad Z_3 = 1.2021 \end{aligned}$$

We have checked that the relation (17) connecting all diagrams entering the calculation of the probabilities, holds.

Properties of singlet probabilities like analytic continuation and the Gribov-Lipatov relation between spacelike and timelike regions will be exhaustively discussed in a more complete paper where also the details of our calculation will be included. However, as a last point, we want to comment on the rather peculiar relation which, at leading log level ⁶⁾, allows one to compute the gluon gluon probability in terms of the other three, once the substitution $C_F = C_G = 2T_R$ has been operated

$$\Delta^{(0)}(x) = \hat{P}_{FF}^{(0)}(x) + \hat{P}_{FG}^{(0)}(x) - \hat{P}_{GF}^{(0)}(x) - \hat{P}_{GG}^{(0)}(x) = 0 \quad (18)$$

The validity of this equation may be related ⁷⁾ to the existence of a supersymmetric theory where gluon and Majorana fermions both transform according to the adjoint representation ⁸⁾.

In the dimensional regularization scheme, the above relation is broken at two-loop level :

$$\begin{aligned} \Delta^{(1,S)}(x) &\equiv \hat{P}_{FF}^{(1,S)}(x) + \hat{P}_{FG}^{(1,S)}(x) - \hat{P}_{GF}^{(1,S)}(x) - \hat{P}_{GG}^{(1,S)}(x) = \\ &= \frac{5}{6} - \frac{23}{3}x + 7x^2 - (2x + 4x^2 - 1) \ln x + \frac{2}{3} \frac{1}{x} \\ \Delta^{(1,T)}(x) &= \frac{13}{6} + \frac{5}{3}x - x^2 + (2x + 4x^2 - 1) \ln x - \frac{2}{3} \frac{1}{x} \end{aligned} \quad (19)$$

where $\Delta^{(1,T)}(x)$ represents the corresponding quantity in the timelike region. The particular simplicity of $\Delta^{(1)}(x)$ with respect to the complexity of terms which appear in the full result of Eqs. (11), (12) is rather suggestive, and might indicate that in other regularization schemes more appropriate to supersymmetry ⁹⁾ the relation of Eq. (18) could be exact.

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FIGURE CAPTION

Two different colour contractions [(b) and (c)] of the forward four-point amplitude (a) : the matrices T^a_{ik} are the generator of the group in the appropriate representation (octet or triplet).