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Potential Model Estimates of f_V and f_{PS} .

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Abstract

We use a refinement of the Van Royen - Weisskopf formula due to Poggio and Schnitzer to successfully calculate Γ_{ee} of all known vector mesons within a standard QCD based potential model. This encourages us to predict the decay constants of heavy pseudoscalars. Interesting consequences for D and F lifetimes occur.

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Following the classic paper of Van Royen and Weisskopf /1/ it has become familiar to express meson decay constants,

$$\langle 0 | \sum_{\text{colour}} \bar{d} \gamma_5 \gamma_\mu u | \pi(P) \rangle = f_\pi P_\mu, \quad \langle 0 | \sum_{\text{colour}} \bar{u} \gamma_\mu u | \rho(P, \epsilon) \rangle = f_\rho \epsilon_\mu M_\rho. \quad (1)$$

in terms of the (nonrelativistic) wave function at the origin. Using the Bethe-Salpeter amplitude in nonrelativistic approximation, $M=2m$,

$$\chi(q, M) = \frac{2\pi}{i} \sqrt{2M} \delta(q_0) \phi(\vec{q}) \frac{\not{\epsilon}/2 + m}{2m} \frac{1}{\sqrt{2}} \left\{ \begin{array}{l} \not{\epsilon} \\ \gamma_5 \end{array} \right\} \frac{\not{\epsilon}/2 - m}{2m} \{ \bar{q} q \} \quad \begin{array}{l} \text{vectors} \\ \text{pseudoscalars} \end{array}, \quad (2)$$

to describe the meson-quark-antiquark vertex, one finds

$$\begin{aligned} \langle 0 | \sum_{\text{colour}} \bar{d} \gamma_5 \gamma_\mu u | \pi(P) \rangle &= i \sqrt{3} \text{Tr} \int \frac{d^4 q}{(2\pi)^4} \gamma_5 \gamma_\mu \chi_\pi = \sqrt{3} 2 \sqrt{M_\pi} \phi_\pi(r=0) \hat{P}_\mu, \\ \langle 0 | \sum_{\text{colour}} Q(\bar{u} \gamma_\mu u - \bar{d} \gamma_\mu d) / 2 | \rho(P, \epsilon) \rangle &= i \sqrt{3} \text{Tr} \int \frac{d^4 q}{(2\pi)^4} \gamma_\mu Q \chi_\rho = \sqrt{3} 2 \sqrt{M_\rho} \phi_\rho(r=0) Q_\rho \epsilon_\mu. \end{aligned} \quad (3)$$

Here $\phi(\vec{q})$ is the Schroedinger wave function in momentum space, normalized to 1, and $\phi(r=0) = (2\pi)^{-3} \int d^3 q \phi(\vec{q})$ is the spatial wave function at the origin. Q is the quark charge matrix or its eigenvalues respectively: $Q(\rho^0, \omega, \phi, J/\psi) = (1/\sqrt{2}, 1/3\sqrt{2}, -1/3, 2/3)$. It follows that (we define the f 's including the colour factor)

$$f_\pi = \sqrt{3} 2 \phi_\pi(r=0) / \sqrt{M_\pi}, \quad f_\rho = \sqrt{3} 2 \phi_\rho(r=0) / \sqrt{M_\rho}. \quad (4)$$

Since experimentally $f_K = 1.25 f_\pi$ and $f_\rho = 1.5 f_\pi$ the f 's are much more similar than the wave functions at the origin. This fact is known as Van Royen - Weisskopf paradox, because in an $SU(6)$ symmetric quark model one expects only the wave functions to be equal, not the decay constants ($m_s > m_u = m_d$ is only a slight symmetry breaking).

When one wants to estimate the decay constants of pseudoscalars which carry new additive quantum numbers $c, b, t, \dots$ such as $D, B, \dots$ mesons, this puzzle persists. Should one continue with an approximate $SU(n)$ symmetry for the f 's? Or should one - because the wave functions of these heavy pseudoscalars are all very similar, governed by almost the same reduced mass $\mu \leq m_q$ - trust the wave function symmetry more? In the latter case the f 's would have to decrease asymptotically, in the former case nothing would prevent them from increasing further. In this note we try to shed some light on this question.

Let us first consider states where the wave function concept is successful, because the nonrelativistic approximation is justified. These states are $J/\psi, Y, \dots$. Here potential models /2/ explain $\Gamma(J/\psi \rightarrow e^+e^-) = 4\Gamma(Y \rightarrow e^+e^-)$ and predict $\Gamma(\bar{t}t \rightarrow e^+e^-) = \Gamma(J/\psi \rightarrow e^+e^-)$ for $M(\bar{t}t)$ below the Z^0 pole in terms of the wave function at the origin and different constituent masses. However, the zeroth order nonrelativistic picture is only applicable to states with a very large reduced mass $\mu \gg m_c/2$. In charmonium, for which $\mu = m_c/2$, large next order corrections are present. In particular, the observed constancy of $\Gamma_{ee}(V)/Q^2$ even below $\mu = m_c/2$, from ρ to J/ψ , cannot be explained in terms of the zeroth order nonrelativistic picture /2/. In fact, the naive potential models would imply a rise of $\Gamma_{ee}(V)/Q^2$ with falling reduced mass below $\mu = m_c/2$. For small masses, relativistic corrections become important. In a recent paper, Poggio and Schnitzer /3/ have shown that most of the next to leading order corrections in α_s and the quark velocity to the leptonic decay of a quarkonium state in a static linear confinement potential can be included in the Van Royen - Weisskopf formula by replacing $\psi(r=0)$ by $\psi(r=1/m)$. The first order formula reads

$$\frac{M_V^2}{16\pi\alpha^2 Q_V^2} \Gamma_{ee}(V) = |\phi(\tau=0)|^2 \left(1 - \frac{16}{3\pi} \alpha_s(M) - 2\frac{\lambda}{m^2} + \dots\right) \quad (5)$$

$$= |\phi(\tau=1/m)|^2 (1 - 0.36 \alpha_s(M) + \dots)$$

An intuitively appealing feature of this formula is the appearance of the Compton wavelength $1/m$. One might interpret this to mean that the quarks annihilate once they come to within their relativistic extension to each other. For a truly nonrelativistic system like hydrogen or positronium, of course, $1/m=0$ on the bound state scale $1/\alpha m$ and $\psi(1/m)=\psi(0)$. But for the quarkonium systems of interest clearly $\psi(1/m) \neq \psi(0)$. In charmonium $|\psi(1/m)|^2/|\psi(0)|^2 = 60\%$ to 70% depending on the choice of the quark mass. Of course the description in Equ.(5) of the decay in terms of $\psi(1/m)$ is not a zeroth order description in terms of a local field theory, i.e. we cannot interpret it in terms of Feynman diagrams. Using the wave function at $1/m$ instead of zero distance means in momentum space that the integral over the momentum wave function gets an extra factor damping the higher momenta

$$\phi(\tau=1/m) = (2\pi)^{-3} \int d^3p \phi(\vec{p}) \exp(i\vec{p} \cdot \hat{x}/m). \quad (6)$$

With this in mind we now try to apply the improved Van Royen - Weisskopf formula Equ.(5) to the vector mesons lighter than J/ψ . We find that a standard QCD based potential /4/ together with Equ.(5) is able to reproduce all leptonic vector meson decays in the wide mass range from ρ (<1 GeV) to beyond Y (>10 GeV), Table 1.

We shall take the successful description of the leptonic widths of vector mesons as an indication, that decay probabilities can be estimated via the nonrelativistic wave function at the Compton

wavelength, even in a regime, where the nonrelativistic approximation itself is open to question. This enables us to discuss the decay constants of light and heavy pseudoscalars, $\pi, K, D, B, \dots$. From a simple geometric consideration it is clear that in the unequal mass case $1/m$ has to be replaced by $1/2\mu = (1/m_1 + 1/m_2)/2$. Including pseudoscalars into our discussion, however, we have to worry about spin-spin forces. Our potential model includes most of the repulsive spin-spin forces in triplet states. This is so because the short distance part of the potential is tailored to give a good fit to the vector mesons $J/\psi, \psi, Y, Y'$ and the total spin triplet sector of the $\bar{c}c$ system. It can therefore - if at all - describe the ρ or the K^* , but not the π or the K . But we can profit from the fact that the spin-spin forces are proportional to the colour magnetic moments of the constituents, as suggested by QCD. If one of the constituents is very heavy, spin-spin effects therefore become very small, even though the wave function of the system is governed by a small reduced mass. Then the wave function of a spin triplet state is the same as that of a spin singlet state. We will therefore proceed in the following way. With our potential we calculate the wave functions of spin triplet states at $r=1/2\mu$. Those of the spin singlet states will now be smaller, depending on $(m_1 m_2)^{-1}$. They will be smaller because the attractive spin-spin force increases the average bound state momenta, but $\propto(1/2\mu)$ weights the higher momenta less, cf. Equ.(6). We do not attempt to calculate this difference, Δf , but from the experimentally measured decay constants of π and K we can try to parameterize them in a fashion corresponding to second order perturbation theory (perturbations of the wave function). The results are summarized in Table 2.

We note that, although $\Delta f(\pi) = 341$ MeV and $\Delta f(K) = 124$ MeV are not just in the ratio $m_s/m_u = 3/2$, the decrease of f with the constituent masses fits in our picture. Although the spin effects for π, K are substantial, not to say overwhelming, their rapid decrease with the constituent masses allows to roughly estimate the decay constants for the D , $f_D \approx 150$ MeV, the F , $f_F \approx 230$ MeV and higher mass pseudoscalars of the type $\bar{q}Q$ (see Table 2). We find it unlikely that pseudoscalar decay constants should lie above the calculated decay constants (column 4 of Table 2), which might be considered as upper limits.

We close with a short discussion of lifetime estimates. The conventional cascade decay of a $\bar{q}Q$ meson is

$$\Gamma_{\text{cas}}(\bar{q}Q \rightarrow \alpha\ell) \approx N_f G_F^2 m_Q^5 / 192 \pi^3. \quad (7)$$

Here N_f is the number of effective weak doublets: leptons + 3 quarks. Only three heavy pseudoscalars below 10 GeV solely decay via the cascade mechanism Equ.(7) (on the least forbidden Cabibbo level): D^+, B_u^+, B_c^+ . All others containing a c or b quark can in principle annihilate /6/ as well: the D^0, B_d^0, B_s^0 through a charged current in t -channel exchange (here no colour enhancement factor is present, so that with our definition of the f 's, Equ.(4), a factor of $1/3$ has to be applied) and the F with a charged

current in the s-channel (here we have the conventional colour factors). Usually it is argued that pseudoscalar annihilations are suppressed by a helicity factor $(m_q/m_Q)^2$ which is of the order of 5% or smaller. But one should be careful: Colour and spin can be carried off by an arbitrary number of relatively soft gluons emitted before the weak interaction. When these gluons do not recombine with final state quarks immediately, there is no helicity suppression. Since an expansion in the number of gluons emitted before the weak interaction from a bound state whose radius is dominated by confinement is not expected to converge rapidly it seems to be unrealistic to claim the presence of this helicity suppression factor in the total rate. We therefore feel that a conservative estimate of the pseudoscalar annihilation rate can be obtained by using the lowest order formulae, but without the helicity factor,

$$\Gamma_{ann}(\bar{q}Q \rightarrow \text{all}) \lesssim \left\{ \begin{matrix} 1/3 \\ N_f \end{matrix} \right\} G_F^2 f_{PS}^2 m_Q^3 / 8\pi \quad \text{for} \left\{ \begin{matrix} t \\ s \end{matrix} \right\} \text{channel weak currents.} \quad (8)$$

This leads to the ratio

$$\Gamma_{ann}/\Gamma_{cas} \lesssim \left\{ \begin{matrix} 1/3 N_f \\ 1 \end{matrix} \right\} 24\pi^2 (f_{PS}/m_Q)^2 \quad \text{for} \left\{ \begin{matrix} t \\ s \end{matrix} \right\} \text{channel weak currents.} \quad (9)$$

Inserting the numbers of Table 2 for the f's we find

$$\Gamma_{ann}(D^0)/\Gamma_{cas}(D^0) \lesssim 13 (f_D/m_c)^2 \approx 0.15 \quad . \quad (10)$$

Annihilation contributes at most 15% to D^0 decays! It is therefore quite unlikely that a difference in D meson lifetimes /7/, $\tau(D^0) < \tau(D^+)$, is caused by the D^0 annihilation channel /8/. If such a difference is actually present, in our opinion it must be due to (e.g. interference) suppressions in D^+ decays /9/ and consequently leptonic branching ratios should be almost "normal", i.e. of the order of 15%, in D^0 and enhanced, i.e. $>15\%$ in D^+ . For the F meson

$$\Gamma_{ann}(F)/\Gamma_{cas}(F) \lesssim 240 (f_F/m_c)^2 \approx 5 \quad . \quad (11)$$

One more comment is in order. The inequality in Equ.(11) might be saturated by nature less than that in Equ.(10) because of two reasons: i) The argument that the helicity factor is absent because of gluon radiation off the initial state is weaker here than in the D^0 case, since we need at least two gluons to carry off zero net colour (the weak current is colour singlet!), ii) in the

limit where a large number of gluons is emitted there is an additional colour factor of $1/3$, which one might be tempted to multiply to Equ.(11) /10/ (for two gluons, however, this factor is $2/3$). It is nevertheless possible that the F meson might annihilate an appreciable fraction of the time. Consequently its branching ratio into pions might be substantial. For B mesons

$$\Gamma_{\text{ann}}(B)/\Gamma_{\text{cas}}(B) \leq 9(f_B/m_b)^2 \approx 0.6\% (1.2\%) \text{ for } B_d^0(B_s^0) \quad , \quad (12)$$

so that annihilations are negligible, while B_c^+ cascades down to B_s^0 weakly (Cabibbo allowed). Thus the B_c^+ meson decaying by this mechanism has a much shorter lifetime than the other B mesons.

We conclude that the potential model description using the (nonrelativistic) wave function at the Compton wavelength, i.e. with the corrections as given by Poggio and Schnitzer, works astonishingly well for the leptonic decay of all known vector mesons, even the light ones. Thus encouraged we have applied the same picture to the weak decays of light and heavy pseudoscalars (which all have a very small reduced mass) obtaining overall consistent and interesting results, which are amenable to experimental tests.

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TABLE CAPTIONS.

Table 1. Γ_{ee} of all vector ground states in a standard QCD based potential model /4/. Γ_{ee} is calculated using the Van Royen - Weisskopf formula with $\psi(1/m)$, Equ.(5). Experimental numbers are from Ref. /11/.

Table 2. Wave functions at $r=1/2\mu$ and the resulting decay constants for pseudoscalar $\bar{q}Q$ mesons. The wave functions are calculated for vector states and spin effects are scaled with $(m_1 m_2)^{-1}$ using the experimental π and K decay constants as input. Asymptotic values for $|\psi(1/2\mu)|^2$ are (1.5, 2.65, 16) fm^{-3} for $\mu=(0.38, 0.50, 1.5)$ GeV.

Table 1.

state	$m \approx M_V/2$ [GeV]	$ H(0) ^2$ [fm ⁻³]	$ H(1/m) ^2$ [fm ⁻³]	f [MeV]	$\Gamma_{ee}^*(V)/Q^2$ [keV]	$\Gamma_{ee}^*(V)/Q^2(\text{exp})$ [keV]
ρ^0						
ω	0.38	1.2	0.33	203	12	13.3 ± 1.6
ϕ	0.50	1.4	0.6	238	12	13.8 ± 3.1
J/ψ	1.5	7.6	5.4	409	12	11.4 ± 1.1
Υ	4.9	55	45	676	11	10.8 ± 1.1
						10.8 ± 1.8

Table 2.

state	$\mu(m_q, m_Q)$ [GeV]	$ H(1/2\mu) ^2$ [fm ⁻³]	f [MeV]	$\Delta f = f - f_{ps} \sim \frac{1}{m_q m_Q}$ [MeV]	f_{ps} [MeV]
π	0.19(0.38, 0.38)	0.33	480	341	139 (input)
K	0.22(0.38, 0.50)	0.47	300	124	176 (input)
D	0.30(0.38, 1.50)	0.83	207	≈ 60	≈ 150
$B_{u,d}$	0.35(0.38, 4.90)	1.15	144	≈ 20	≈ 125
F	0.38(0.50, 1.50)	1.50	270	≈ 40	≈ 230
B_s	0.45(0.50, 4.90)	2.15	192	≈ 10	≈ 180
B_c	1.15(1.50, 4.90)	9.3	360	≈ 5	≈ 350