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PHASE TRANSITIONS AND MAGNETIC MONOPOLES IN $SO(10)$

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A B S T R A C T

In a unified gauge theory based on $SO(10)$, the combination of a strongly first order phase transition and a magnetic confinement mechanism can suppress the density of magnetic monopoles at the time of nucleosynthesis. However, this only occurs if $SO(10)$ breaks down to $SU(3)_C \otimes U(1)_{EM}$ via $SU(4)_C \otimes (SU(2)_L \times SU(2)_R)$. For the other symmetry breaking patterns of $SO(10)$ obtained with a minimal Higgs system, the potential conflict with the standard big bang cosmology is not naturally avoided.

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Unified gauge theories¹⁾ of strong, weak and electromagnetic interactions predict the existence of topologically stable, superheavy magnetic monopoles²⁾. In the framework of the hot big bang cosmology, such monopoles would have been produced in the very early history of the universe. Under the assumption that the transition during which the monopoles are created is second order (or weakly first order), their number density at the time of nucleosynthesis has been estimated to be unacceptably large^{3),4)}.

Two ways out of this difficulty have so far been proposed. In the first proposal^{4),5)}, it is suggested that the phase transition during which the monopoles are created is strongly first order. In this case, the transition proceeds through nucleation of bubbles, and appreciable supercooling occurs before the actual phase transition takes place. Monopole production can then be strongly suppressed if, immediately after the transition, the effective monopole mass is large compared with the temperature. This is discussed in detail for SU(5) in Ref. 5). The second proposal for resolving the conflict between the unified gauge theories and the hot big bang cosmology relies on a magnetic confinement mechanism⁶⁾.

In this letter we wish to show that these two proposals combine together in a natural way in the context of the unified SO(10)^{7)*)} gauge theory. By employing a minimal Higgs system (45, 16 and 10 or 54, 16 and 10 of Higgs fields), the SO(10) gauge group can break down to SU(3)_C ⊗ U(1)_{EM} in the following possible ways^{**)}:

- i) $SO(10) \rightarrow SU(5) \rightarrow (SU(3)_C \times SU(2)_L) \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$
- ii) $SO(10) \rightarrow (SU(3)_C \times SU(2)_L) \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$
- iii) $SO(10) \rightarrow SU(4)_C \otimes (SU(2)_L \times SU(2)_R) \rightarrow (SU(3)_C \times SU(2)_L) \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{EM}$

We shall argue that, for case iii), the monopole density at the time of nucleosynthesis can be naturally suppressed through the combination of a strongly first order phase transition and a magnetic confinement mechanism. In the other cases, however, the potential conflict with the standard hot big bang cosmology is not naturally avoided.

*) Throughout the paper, by SO(n) (n ≥ 3) we really mean its universal covering group Spin(n).

**) The symbols × and ⊗ mean direct product and local direct product respectively.

The minimal Higgs multiplet that can break $SO(10)$ down to $SU(4)_C \otimes (SU(2)_L \times SU(2)_R)$ is a 54-plet. However, in order to achieve the breaking, the presence of a cubic self-coupling of the 54-plet is unavoidable. This has the important consequence that in the early history of the universe the transition from $SO(10)$ down to $SU(4)_C \otimes (SU(2)_L \times SU(2)_R)$ proceeds through intermediate stages that are not part of the usual symmetry breaking pattern (at zero temperature). For a wide and natural choice of the parameters of the theory, the final transition to the $SU(4)_C \otimes (SU(2)_L \times SU(2)_R)$ phase is expected to be strongly first order, and monopole production would be suppressed. The next step of the breaking leads to the $(SU(3)_C \times SU(2)_L) \otimes U(1)_Y$ phase and proceeds via a second order phase transition. New, lighter monopoles are copiously produced. Fortunately, however, they get confined when $SU(2)_L \otimes U(1)_Y$ breaks down to $U(1)_{EM}$ and rapidly annihilate⁶⁾. The monopoles produced during the strongly first order phase transition eventually evolve into $SU(3)_C \otimes U(1)_{EM}$ monopoles, but their density is strongly suppressed.

Case (i): Let us discuss the following symmetry breaking pattern of $SO(10)$:

$$\begin{array}{c}
 SO(10) \\
 \downarrow M_s \gg 10^{15} \text{ GeV} \\
 SU(5) \\
 \downarrow M_x \sim 10^{15} \text{ GeV} \\
 (SU(3)_C \times SU(2)_L) \otimes U(1)_Y \\
 \downarrow M_W \sim 10^2 \text{ GeV} \\
 SU(3)_C \otimes U(1)_{EM}
 \end{array}$$

In the first step, $SO(10)$ breaks down to $SU(5)$ through the vacuum expectation value of a 16-plet of Higgs fields χ . The second breaking is achieved by employing a Higgs 45-plet ϕ .

The most general renormalizable Higgs potential involving χ and ϕ is given by [the discrete symmetry $\phi \rightarrow -\phi$ is also assumed; otherwise $SO(10)$ naturally breaks down to $SU(4) \otimes U(1)$ rather than to $(SU(3)_C \times SU(2)_L) \otimes U(1)_Y$]

$$V(\chi, \phi) = V_1(\chi) + V_2(\phi) + V_3(\chi, \phi),$$

where ^{*)}

^{*)} The couplings $\lambda, \kappa, a, b, \alpha$ and γ are assumed to be of order g^2 , where g is the $SO(10)$ gauge coupling. Also $\kappa, b, \gamma > 0$ [See Ref. 8)].

$$V_1(x) = -\mu_x^2 x^\dagger x + \frac{\lambda}{2} (x^\dagger x)^2 + \frac{\kappa}{2} \sum_{i=1}^{10} (x^\dagger \Gamma^i x)^2$$

$$V_2(\phi) = \frac{a}{16} (\text{Tr } \phi^2)^2 + \frac{b}{4} \text{Tr } \phi^4 + \frac{1}{4} \mu_\phi^2 \text{Tr } \phi^2 \quad (1)$$

$$V_3(x, \phi) = -\frac{\alpha}{4} (x^\dagger x) \text{Tr } \phi^2 + \gamma \sum_{i,j,k,\ell=1}^{10} x^\dagger \sigma^{ij} \sigma^{k\ell} x \phi_{ij} \phi_{k\ell}.$$

Here x is a 16 component $SO(10)$ spinor and ϕ is an antisymmetric 10×10 matrix. Also $\sigma^{ij} = (1/2i) [\Gamma^i, \Gamma^j]$, where the Γ^i ($i = 1, 2, \dots, 10$) are the generalized Dirac matrices.

The matrix ϕ can be put in the following standard form:

$$\phi = \begin{pmatrix} y_1 & & & & \\ & y_2 & & & \\ & & y_3 & & \\ & & & y_4 & \\ & & & & y_5 \end{pmatrix} \otimes \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (2)$$

The $SU(5)$ singlet s and the $SU(5)$ 24-plet $\hat{\phi}$ in ϕ are then obtained by writing

$$\begin{pmatrix} y_1 & & & & \\ & y_2 & & & \\ & & y_3 & & \\ & & & y_4 & \\ & & & & y_5 \end{pmatrix} = \frac{s}{5} 1_5 + \begin{pmatrix} \hat{y}_1 & & & & \\ & \hat{y}_2 & & & \\ & & \hat{y}_3 & & \\ & & & \hat{y}_4 & \\ & & & & \hat{y}_5 \end{pmatrix} \equiv \frac{s}{5} 1_5 + \hat{\phi}, \quad (3)$$

where

$$s = \sum_{i=1}^5 y_i \quad \text{and} \quad \hat{y}_i = y_i - \frac{s}{5} \quad (i=1, 2, 3, 4, 5).$$

At a critical temperature $T_{c1} \sim M_S/g$, $SO(10)$ breaks down to $SU(5)$ through a second order phase transition. However, no monopoles are produced. The χ field starts to develop a vacuum expectation value which soon reaches its zero temperature value of order μ_χ/g . The effective Higgs potential involving $\hat{\phi}$ and s then reads:

$$\begin{aligned}
 V_{\text{eff}}(\hat{\phi}, s) = & \frac{a}{4} (\text{Tr } \hat{\phi}^2 + \frac{s^2}{5})^2 \\
 & + \frac{b}{2} \left(\text{Tr } \hat{\phi}^4 + \frac{4s}{5} \text{Tr } \hat{\phi}^3 + \frac{6s^2}{25} \text{Tr } \hat{\phi}^2 + \frac{s^4}{125} \right) \\
 & - \frac{1}{2} (\mu_\phi^2 - \alpha |\langle \chi \rangle|^2) (\text{Tr } \hat{\phi}^2 + \frac{s^2}{5}) + \gamma |\langle \chi \rangle|^2 s^2.
 \end{aligned} \tag{4}$$

Since $SU(5)$ breaks down to $(SU(3)_C \times SU(2)_L) \otimes U(1)_Y$ at a scale $M_X \ll M_S$, one requires $\mu_\phi^2 - \alpha |\langle \chi \rangle|^2 \sim M_X^2$. For $T_{c1} > T > T_{c2} \sim M_X/g$, the expectation value of $\hat{\phi}$ is zero. In addition, the singlet field s acquires a mass of order M_S^*). Since, at this stage, the expectation value of s is zero, there is no effective cubic coupling for $\hat{\phi}$. Consequently, the transition at T_{c2} from the $SU(5)$ phase to the $(SU(3)_C \times SU(2)_L) \otimes U(1)_Y$ phase, through the vacuum expectation value of $\hat{\phi}^{**}$, is essentially second order and monopoles are copiously produced. Therefore, if $SO(10)$ breaks down to $(SU(3)_C \times SU(2)_L) \otimes U(1)_Y$ via $SU(5)$, the potential conflict with the standard big bang cosmology is not avoided.

Case (ii): The $SO(10)$ gauge symmetry can break directly down to $H = (SU(3)_C \times SU(2)_L) \otimes U(1)_Y$ by employing a 16-plet and a 45-plet of Higgs fields, with the Higgs potential given in Eq. (1)⁸⁾. The transition to H then is essentially second order and monopole production is therefore not suppressed.

Case (iii): Let us finally consider the following symmetry breaking pattern of $SO(10)$:

*) Remember that $\gamma > 0$.

**) Note that s starts to develop an expectation value when $\langle \hat{\phi} \rangle$ becomes non-zero.

$$\begin{array}{c}
 SO(10) \\
 \downarrow M_x \sim 10^{16} \text{ GeV} \\
 SU(4)_c \otimes (SU(2)_L \times SU(2)_R) \equiv SO(6) \otimes SO(4) \\
 \downarrow M_c \sim 10^{14} \text{ GeV} \\
 (SU(3)_c \times SU(2)_L) \otimes U(1)_Y \\
 \downarrow M_W \sim 10^2 \text{ GeV} \\
 SU(3)_c \otimes U(1)_{EM} .
 \end{array}$$

The first step in the breaking is achieved by a 54-plet of Higgs fields. The second step requires a 16-plet of Higgs fields, while the last breaking is done with a 10-plet. We now concentrate on the breaking caused by the 54-plet.

The 54 dimensional Higgs field ϕ may be considered as a symmetric, traceless 10×10 matrix which transforms under $SO(10)$ as

$$\phi \longrightarrow U \phi \tilde{U} \quad , \quad U \in SO(10) .$$

The most general renormalizable Higgs potential for ϕ is^{*)}

$$\begin{aligned}
 V(\phi) = & \frac{a}{4} (\text{Tr } \phi^2)^2 + \frac{b}{2} \text{Tr } \phi^4 + \frac{c}{3} \text{Tr } \phi^3 \\
 & - \frac{1}{2} \mu^2 \text{Tr } \phi^2 .
 \end{aligned} \tag{5}$$

The matrix ϕ can be diagonalized by an $SO(10)$ gauge transformation:

$$\phi = \text{diagonal} (y_1, y_2, \dots, y_{10}) \quad , \tag{6}$$

$$\sum_{i=1}^{10} y_i = 0 \quad , \quad y_i \in \mathbb{R} . \tag{7}$$

The potential $V(\phi)$ now becomes

$$V(y_i) = \frac{a}{4} S_2^2 + \frac{b}{2} S_4 + \frac{c}{3} S_3 - \frac{1}{2} \mu^2 S_2 \quad , \tag{8}$$

^{*)}We assume that $a \sim b \sim g^2$ and $c \sim g\mu$, where g is the $SO(10)$ gauge coupling.

where

$$s_k = \sum_{i=1}^{10} (y_i)^k \quad (9)$$

In order to discuss the possible symmetry breakings induced by the ϕ field, we shall need the following lemma^{*)}:

Lemma: Consider a real valued function

$$f = \frac{b}{2} s_4 + \frac{c}{3} s_3$$

of n real variables y_i , subject to the constraints

$$\begin{aligned} s_1 &= \sigma, \\ s_2 &= \rho^2, \end{aligned} \quad (10)$$

where $s_k = \sum_{i=1}^n (y_i)^k$ and σ and ρ are real constants. At any absolute extremum of f , the set of values of the y_i 's has at most two elements.

By applying this lemma to the potential $V(y_i)$ in Eq. (8), we arrive at the following result:

Theorem: At the absolute minimum of $V(y_i)$,

$$(y_1, y_2, \dots, y_{10}) = y \left[m(10-m)10 \right]^{-1/2} \underbrace{(10-m, \dots, 10-m)}_m, \underbrace{(-m, \dots, -m)}_{(10-m)}, \quad (11)$$

where $y \in \mathbb{R}$ and $m = 1, 2, 3, 4$ or 5 .

It is then clear that for $y \neq 0$, the possible subgroups of $SO(10)$ that leave invariant the vacuum expectation value of the 54-plet ϕ are $SO(m) \otimes SO(10-m)$ ($m = 1, 2, 3, 4, 5$). In order to discuss these five cases, let us define the quantities

$$\hat{s}_k(m) = y^{-k} s_k(m), \quad m = 1, 2, \dots, 5, \quad (12)$$

where $s_k(m)$ is evaluated for the y_i 's given in (11). Substitution in (8) gives

$$V(y, m) = \left[\frac{a}{4} + \frac{b}{2} \hat{s}_4(m) \right] y^4 - \frac{|c|}{3} \hat{s}_3(m) y^3 - \frac{1}{2} \mu^2 y^2, \quad y \geq 0, \quad (13)$$

^{*)} This is a generalization of a lemma given in Ref. 9).

with

$$\hat{S}_4(m) = [100m(10-m)]^{-1} [(10-m)^3 + m^3] , \quad (14)$$

$$\hat{S}_3(m) = [1000m(10-m)]^{-1/2} [(10-m)^2 - m^2] . \quad (15)$$

Note that at the absolute minimum of $V(y,m)$, the cubic term must be non-positive.

Now consider the case $b < 0$. Given any $y > 0$, the smallest value of $V(y,m)$ is achieved for $m = 1$. Thus, $SO(10)$ can only break down to $SO(9)$ which is uninteresting¹⁰⁾.

In the rest of the paper we shall take $b > 0$. In this case, the stability of the potential V requires that $\eta \equiv a/b > -1/5$. The $SO(10)$ symmetric phase is unstable with respect to a breakdown to $SO(m) \otimes SO(10-m)$ if

$$\xi \equiv -\mu^2 \frac{b}{c^2} < \frac{2}{9} \frac{\hat{S}_3^2(m)}{\eta + 2\hat{S}_4(m)} . \quad (16)$$

The minimal value of $V(y,m)$, for m fixed, is then given by

$$-\frac{3}{b(\mu^2)^2} V_{\min}^{(m)} = \frac{1}{\eta + 2\hat{S}_4(m)} \cdot \left[3 + 3z + \frac{1}{2}z^2 + (z^2 + 4z)\left(\frac{1}{4} + \frac{1}{2}\right)^{1/2} \right] , \quad (17)$$

where

$$z = -\frac{1}{\xi} \frac{\hat{S}_3^2(m)}{\eta + 2\hat{S}_4(m)} . \quad (18)$$

From this equation one can determine the absolute minimum of the potential for different values of ξ and η , and so plot the phase diagram of the $SO(10)$ system with one 54-plet of Higgs fields (see figure).

Note that in the absence of the cubic term in the potential V , the $SO(10)$ symmetry can only break down to $SO(5) \otimes SO(5)$ ^{10)*)}. Therefore, the presence of a non-zero cubic self-coupling of the 54-plet is necessary in order to break $SO(10)$ down to the physically relevant subgroup $SO(6) \otimes SO(4)$.

*) Remember that $b > 0$.

The temperature dependent effective potential $V_{\text{eff}}(\phi, T)$ can be evaluated by employing the methods given in Ref. 11). It will be assumed that $a \sim b \sim g^2$ and $c \sim g\mu$, where g is the $SO(10)$ gauge coupling. For $T \sim \mu/g$, the effective potential $V_{\text{eff}}(\phi, T)$ can be obtained from $V(\phi)$ by replacing μ^2 with $\mu_{\text{eff}}^2(T) = \mu^2 - \sigma T^2$, where

$$\sigma = \frac{1}{60} (75g^2 + 280a + 112b) . \quad (19)$$

From the figure one sees that, for $T > T_{c1} \approx \mu/\sqrt{\sigma} \sim 10^{16}$ GeV, the $SO(10)$ gauge symmetry is unbroken. At $T = T_{c1}$, $SO(10)$ breaks down to $SO(9)$. For a wide and natural choice of the parameters, $|\mu_{\text{eff}}^2(T_{c1})| \ll \mu^2$ and the phase transition is expected to be weakly first order. Further cooling causes the system to pass through two more intermediate phases, before it breaks down to $SO(6) \otimes SO(4)$ *). The precise nature of the intermediate phase transitions does not interest us here. Let T_c be the critical temperature for the transition to the $SO(6) \otimes SO(4)$ symmetric phase. There exists a wide and natural choice of values for the parameters ξ and η at zero temperature, such that

$$\mu_{\text{eff}}^2(T_c) = \mu^2 - \sigma T_c^2 \sim \mu^2$$

and consequently $T_c \sim 10^{15} - 10^{16}$ GeV. The transition to the $SO(6) \otimes SO(4)$ phase is then expected to be strongly first order. Thus, the actual phase transition can occur after appreciable supercooling.

Suppose now that, very early in the history of the universe, the temperature T exceeds $T_{c1} \sim 10^{16}$ GeV. The $SO(10)$ gauge symmetry is then fully restored and no monopoles are present. As the universe cools below T_{c1} , $SO(10)$ breaks down to $SO(9)$ through a weakly first order phase transition. However, no monopoles are produced. Under further cooling bubbles of the $SO(8) \otimes U(1)$, $SO(7) \otimes SO(3)$ and $SO(6) \otimes SO(4)$ phases start to nucleate, and stable monopoles can be produced. To simplify the discussion, let us assume that the transition to the intermediate $SO(8) \otimes U(1)$ and $SO(7) \otimes SO(3)$ phases is weakly first order. However, from the previous discussion the transition to the $SO(6) \otimes SO(4)$ is expected to be strongly first order. We will assume that there is negligible probability of converting an $SO(7) \otimes SO(3)$ monopole into an $SO(6) \otimes SO(4)$ monopole when a bubble wall of the latter phase crosses over it⁵⁾.

*) The parameters ξ and η are suitably chosen so that $SO(10)$ is broken down to $SO(6) \otimes SO(4)$ at zero temperature.

The topological magnetic charges of the $SO(6) \otimes SO(4)$ monopoles correspond to the elements of the second homotopy group $\pi_2(SO(10)/SO(6) \otimes SO(4))$ ¹²⁾. Since $SO(10)$ is simply connected,

$$\pi_2(SO(10)/SO(6) \otimes SO(4)) = \pi_1(SO(6) \otimes SO(4)) = \mathbb{Z}_2. \quad (20)$$

The last relation is a consequence of the non-trivial intersection of $SO(6)$ and $SO(4)$ ⁶⁾. The monopoles simultaneously carry $SO(6)$ ($\equiv SU(4)_{\text{colour}}$) and $SO(4)$ ($\equiv SU(2)_L \times SU(2)_R$) magnetic fields, and have mass $\approx M_X/\alpha_G \approx 4 \times 10^{17}$ GeV.

Since the transition to the $SO(6) \otimes SO(4)$ phase is strongly first order, one expects appreciable supercooling before the actual transition occurs. The latent heat will then warm the universe back up to $T_R \lesssim T_c^*$. Under these circumstances the production of the $SO(6) \otimes SO(4)$ monopoles can be strongly suppressed⁵⁾.

After the heat spike the universe cools again. At a critical temperature $T_c' \sim 10^{14}$ GeV, the $SO(6) \otimes SO(4)$ symmetry breaks down to $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$ through an essentially second order phase transition. The $SO(6) \otimes SO(4)$ monopoles then evolve into elementary $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$ monopoles which simultaneously carry $SU(3)_c$, $SU(2)_L$ and $U(1)_Y$ fields. After the breaking of $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{EM}$, these monopoles can evolve into elementary $SU(3)_c \otimes U(1)_{EM}$ monopoles. In addition, the breaking of $SO(6) \otimes SO(4)$ down to $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$ copiously produces new lighter (mass $\approx M_c/\alpha_G \sim 4 \times 10^{15}$ GeV) monopoles. Their topological quantum numbers correspond to the elements of $\pi_2(SO(6) \otimes SO(4)/(SU(3)_c \times SU(2)_L) \otimes U(1)_Y)$. Since $SO(6) \otimes SO(4)$ is not simply connected,

$$\begin{aligned} & \pi_2(SO(6) \otimes SO(4)/(SU(3)_c \times SU(2)_L) \otimes U(1)_Y) \\ &= \pi_1((SU(3)_c \times SU(2)_L) \otimes U(1)_Y)_{SO(6) \otimes SO(4)} = \mathbb{Z}. \end{aligned} \quad (21)$$

This means that the topological charges of the lighter monopoles are even multiples of the topological charge of the elementary $(SU(3)_c \times SU(2)_L) \otimes U(1)_Y$ monopole. Therefore, they do not carry $SU(2)_L$ magnetic fields. As explained in a previous paper⁶⁾, after the breaking of $SU(2)_L \otimes U(1)_Y$ down to $U(1)_{EM}$, all these monopoles get connected to their antimonopoles by a magnetic flux string of the Z^0 field and are rapidly annihilated.

^{*)} During supercooling, the temperature of the universe drops below 10^{14} GeV and the 16 Higgs field will start to develop a vacuum expectation value. However, this becomes zero again when the universe gets re-heated to T_R .

To summarize, it is seen that a combination of a strongly first order phase transition and a magnetic confinement mechanism suppresses the density of magnetic monopoles in $SO(10)$, at the time of nucleosynthesis. This happens only if $SO(10)$ breaks down to $SU(3)_C \otimes U(1)_{EM}$ via $SO(6) \otimes SO(4)$. This breaking pattern requires a 54-plet of Higgs fields with a non-zero cubic self-coupling. Consequently, for a wide and natural choice of the parameters of the theory, the transition to the $SO(6) \otimes SO(4)$ phase is strongly first order and the production of $SO(6) \otimes SO(4)$ monopoles can be suppressed. These monopoles eventually evolve into elementary $SU(3)_C \otimes U(1)_{EM}$ monopoles. Lighter monopoles, although copiously produced when $SO(6) \otimes SO(4)$ breaks down to $(SU(3)_C \times SU(2)_L) \otimes U(1)_Y$, undergo complete annihilation when $SU(2)_L \otimes U(1)_Y$ breaks down to $U(1)_{EM}$. Therefore, there need not be any conflict with the standard big bang cosmology.

Finally, we would like to stress once again that whereas present phenomenology is consistent with the various breaking schemes of $SO(10)$ discussed in the paper, cosmological considerations seem to select out one of them.

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FIGURE CAPTION

Phase diagram for the spontaneous breaking of the $SO(10)$ symmetry induced by the vacuum expectation value of a 54-plet of Higgs fields. The phases depend on two combinations of the coupling parameters in the Higgs potential

$$\eta = \frac{a}{b} \quad , \quad \xi = -\mu^2 \frac{b}{c^2} \quad .$$

The cross hatched region $\eta < -1/5$ is forbidden. The point $(\eta, \xi) = (-1/5, 1/9)$ is common to all phases, except the $SO(5) \otimes SO(5)$ phase. The transition line separating the $SO(6) \otimes SO(4)$ phase from the $SO(5) \otimes SO(5)$ phase touches the forbidden region at $\xi = 0$.