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A MODEL OF GLUERING EXCITATIONS IN QCD

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ABSTRACT

We present a theory of non-Abelian charged strings interacting with external gauge fields. The theory is applied to an approximative description of gluering excitations (i.e., closed, toroidal flux tubes of gluons) in the (Copenhagen) QCD vacuum. Using the usual QCD scale parameter Λ as an input, we suggest the existence of two spinless excitations in the form of gluerings with masses of the order of 1.7 and 2.3 GeV. The linear dimension of the corresponding objects are 0.3 and 0.5 Fermi, respectively. We also estimate the width of the "glueball" gluering excitations to be of the order of the excitation energy.

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1. INTRODUCTION

A basic object entering into considerations of QCD dynamics is the Wilson loop¹⁾

$$W(C) = \text{Tr} P \exp \left(i e \oint_C A_\mu dx^\mu \right) \quad (1)$$

where the integral is around a closed contour C . For an open path (and untraced) (1) describes the creation of an "electric" flux tube along the path. By closing such a path we obtain a representation of a closed ring of flux or a gluering.

Such gluerings have very recently been of considerable interest in the context of string dynamics in QCD²⁾ and a connection to the dual string model³⁾ has been suggested. Here we will assume that the existence of gluering bound states of gluons can be derived from QCD and that these states effectively can be described by closed dual strings with a universal tension given by the Regge slope α' . Since the gluons carry a charge we will endow the string with a (non-Abelian) charge as well. The gluerings we consider in this note are endowed with the following special properties: (i) The (non-Abelian) electric current $J^a(\sigma)$ travels along the ring with zero resistance; (ii) The Wilson loop $W(C)$, evaluated once around the ring on any given time slice is conserved. Furthermore, $W(C)$ is restricted to a discrete set of values associated with the central elements of the appropriate gauge group. (iii) The gluerings have zero total colour charge. In the next section we present a dynamical model for gluerings with properties (i), (ii) and (iii). Static solutions for gluerings in an external colour magnetic field are found in Sec. 3. In Sec. 4. we apply the results to find the energies of gluering excitations in the Copenhagen QCD vacuum. An estimate of their decay is also given. We suggest that such gluering excitations may be candidates for "glueballs". We refer the reader to Ref. 4) for a discussion of the phenomenology of gluonium, and to Ref. 5) for a study of toroidal glueball structures in the context of the MIT bag model.

2. DYNAMICAL MODEL FOR GLUERINGS

As indicated in the introduction we shall assume that the gluering can effectively be described by a closed dual string. Let $x^\mu(\sigma)$ correspond to the co-ordinate of the string, where $\sigma = (\sigma^0, \sigma^1)$ are the parameters of the string. The free action for the dual string is³⁾

$$\mathcal{S}_0 = \int d^2\tau L_0$$

$$L_0 = -\frac{1}{2\pi\alpha'} \sqrt{-\det M} ; M_{ab} = \partial_a z^\mu(\tau) \partial_b z_\mu(\tau) \quad (2)$$

where $\partial_a \equiv \partial/\partial\sigma^a$. To the dual string we will attach additional degrees of freedom to which we can couple a non-Abelian gauge field

$$A_\mu(x) \equiv A_\mu^\alpha(x) T(\alpha) \quad (3)$$

Here $A_\mu^\alpha(x)$ are the Yang-Mills potentials. The $T(\alpha)$'s correspond to the Hermitian generators of the gauge group which we take here to be $SU(N)$. They are subject to the normalization condition

$$\text{Tr } T(\alpha) T(\beta) = \delta_{\alpha\beta} \cdot \frac{1}{2} \quad (4)$$

To account for the Yang-Mills fields we add the following two terms to the action (2)

$$\mathcal{S}_F = -\frac{1}{2} \int d^4x \text{Tr } F_{\mu\nu} F^{\mu\nu} \quad (5)$$

and

$$\mathcal{S}_I = \int d^2\tau L_I \quad (6)$$

where (5) represents the free Yang-Mills action ($F_{\mu\nu} \equiv \partial_\mu A_\nu - \partial_\nu A_\mu - ie[A_\mu, A_\nu]$) and (6) is the coupling to the string. For L_I we take

$$L_I = ie^{ab} \text{Tr } K_a(\tau) g^\dagger(\tau) D_b g(\tau) \quad (7)$$

Here we define two additional dynamical variables on the string : (a) K_a which has values in the Lie-algebra associated with the gauge group, i.e.,

$$K_a(\tau) = K_a^\alpha(\tau) T(\alpha) \quad (8)$$

and (b) g which has values in the group $SU(N)/Z_N$, Z_N being the centre group of $SU(N)$. D_a in (7) is the covariant derivative along the tangent directions on the string

$$D_a \equiv \partial_a - ie \mathcal{A}_\mu(z) \partial_a z^\mu \quad (9)$$

Lagrangians of the form (7) along with their resulting equations of motion have been discussed in detail in Ref. 6). We review some of the results here^{*)}. The equation of motion which results from variations of the string co-ordinate is

$$\partial_a \left(\frac{\delta \mathcal{L}_0}{\delta \partial_a z^\mu} \right) = e \text{Tr} \mathcal{F}_{\mu\nu}(z) J^a(\tau) \partial_a z^\nu \quad (10)$$

where $J^a(\sigma)$ is defined in terms of the variables g and K_a as follows:

$$\begin{aligned} J^a(\tau) &= \epsilon^{ba} g(\tau) K_b(\tau) g^\dagger(\tau) \\ &= J_\alpha^a(\tau) T(\alpha) \end{aligned} \quad (11)$$

We can interpret $J^a(\sigma)$ as a non-Abelian current on the string. It is covariantly conserved, i.e.,

$$\partial_a J^a(\tau) - ie [\mathcal{A}_\mu(z), J^a(\tau)] \partial_a z^\mu = 0 \quad (12)$$

Equation (12) results from variations of the variable g in the action (6). It was also shown to follow as a consistency condition for the field equations. A second consistency condition was found for this system:

$$\text{Tr} F_{ab}(z) J^b(\tau) = 0 ; F_{ab}(z) \equiv \mathcal{F}_{\mu\nu}(z) \partial_a z^\mu \partial_b z^\nu \quad (13)$$

^{*)} For a non-Abelian Kaluza-Klein formulation see Ref. 7).

Equation (13) results from variations of K_a^α in (7). Here, however, K_a^α was not varied fully. To obtain (13) one only retains the degrees of freedom necessary for the implementation of the reparametrization symmetry. In that which follows we shall depart from the system of Ref. 6) by allowing for full variations of the variable K_a^α ^{*)}. We therefore obtain the additional equation of motion:

$$0 = \frac{\delta L_I}{\delta K_a^\alpha} = i \epsilon^{ab} \text{Tr} T(\alpha) g^\dagger D_b g \quad (14)$$

Since $T(\alpha)$ forms a basis for the Lie algebra, it follows that

$$g^\dagger D_b g = 0 \quad (15)$$

or

$$\partial_b g(\sigma) = i e A_\mu(z) \partial_b z^\mu g(\sigma) \quad (16)$$

The general solution to (16) is

$$g(\sigma) = f_C(\sigma, \bar{\sigma}) g(\bar{\sigma})$$

$$f_C(\sigma, \bar{\sigma}) = P \exp \left(i e \int_C d\sigma' A_\mu(z) \partial_\sigma z^\mu(\sigma') \right) \quad (17)$$

where C is a path from some fixed point $\bar{\sigma}$ in the parameter space to the point σ and P stands for the usual path ordering. We can now show that the properties (i) and (ii) result from (17):

(i) If g is to be a well-defined quantity in $SU(N)/Z_N$, then $f_C(\sigma, \bar{\sigma})$ should be invariant under arbitrary continuous deformations of the path C . This is possible, however, only if the following condition is satisfied everywhere on the string surface:

$$F_{01}(z) = 0 ; F_{01}(z) \equiv F_{\mu\nu}(z) \partial_0 z^\mu \partial_1 z^\nu \quad (18)$$

*) This idea was suggested by G. Trahern.

This is a stronger version of the condition (13). Physically, (18) implies that in the rest frame of any infinitesimal segment of the string the tangential component of the electric field vanishes^{*)}. Since the current $J^a(\sigma)$ is, in general, non-zero, the string has infinite conductivity.

(ii) Let $\sigma = \bar{\sigma}$ in (17). Then C corresponds to a closed path and $\text{Tr } f_C(\sigma, \sigma)$ is the Wilson loop operator (1). From (17), $f_C(\sigma, \sigma)$ is the identity element in $SU(N)/Z_N$. Consequently, it can take on the values

$$f_C(\sigma, \sigma) = \exp\left(\frac{2\pi n i}{N}\right) \mathbb{1} ; \quad n = 0, 1, 2, \dots, N-1 \quad (19)$$

($\mathbb{1}$ is the identity element in $SU(N)$). Condition (19) states that the non-Abelian magnetic flux passing through the string is both quantized and conserved.

In addition to (i) and (ii) it is apparently necessary to add a third condition.

(iii) We require that the only physically observable gluing states are colour singlets, i.e.

$$Q = \int_0^{2\pi} d\sigma^1 J^0(\sigma) = 0 \quad (20)$$

Equation (20) does not follow from the equations of motion of the preceding system, but rather must be imposed as an external constraint.

3. STRINGS IN A UNIFORM BACKGROUND FIELD

We now exhibit static solutions of the non-Abelian string in a uniform external field. As a first example we consider the case where the gauge group is $SU(2)$. For the external field we take

$$F_{ij}(x) = \epsilon_{ij3} H T(3) ; \quad F_{0i}(x) = 0 \quad (21)$$

Here H is a constant and we can write $T(\alpha) = \sigma_\alpha/2$, σ_α being the usual Pauli matrices. We look for solutions where the string is a circle of radius R and lies in the 1-2 plane

^{*)} This follows after making the gauge choice $x^0 = \sigma^0$.

$$\begin{aligned}
 z^0 &= r^0 \\
 z^1 &= R \cos \sigma^1 \\
 z^2 &= R \sin \sigma^1 \\
 z^3 &= 0
 \end{aligned}
 \tag{22}$$

For the current we write

$$J^a(\sigma) = j^a T(z) \tag{23}$$

where j^a 's are constants.

Let us now substitute (21), (22) and (23) into the equations of motion. Clearly (18) is satisfied. Equation (12) holds, provided we choose $\mathcal{A}_\mu$ purely along the 3-direction in internal space, i.e.,

$$\mathcal{A}_i(x) = -\frac{1}{2} \epsilon_{ij3} x_j H T(z) ; \mathcal{A}_0(x) = 0 \tag{24}$$

In order to check Eq. (10) we need to compute the matrix M [cf., (2)]

$$M = \begin{pmatrix} -1 & 0 \\ 0 & R^2 \end{pmatrix} \tag{25}$$

After substitution into (10) we find that we must impose the following condition

$$\frac{1}{\pi \alpha'} = e H j^1 R \tag{26}$$

Thus, the current density is determined by the string size and the magnetic field present.

Let us now examine the flux quantization condition (19) for the case $SU(2)$. Since $\mathcal{A}_\mu(x)$ points along the third direction in internal space for all x , the path ordering may be removed in $f_C(\sigma, \sigma)$. Using (17) we then find

$$f_C(\sigma, \sigma) = \exp\left(i e \pi R^2 H \frac{\sqrt{3}}{2}\right) \tag{27}$$

If the flux is to be non-trivial we have the single possibility that $f_C = -1 = \exp \pi i \sigma_3$. Consequently,

$$R^2 = \frac{2}{|eH|} \quad (28)$$

To determine the mass of the gluering let us compute the canonical momentum density

$$p^\mu(\tau) = \frac{\delta(\mathcal{L}_0 + \mathcal{L}_I)}{\delta \partial_0 z^\mu} = -\frac{1}{2\pi\alpha'} \sqrt{-\det M} M_{0a}^{-1} \partial_a z^\mu + e \text{Tr} J^0 \mathcal{A}^\mu(z) \quad (29)$$

For the total momentum we find

$$P^\mu = \int_0^{2\pi} d\tau p^\mu(\tau) = \frac{R}{\alpha'} \delta_{\mu 0} \quad (30)$$

It follows that the gluering mass is given by

$$m^2 = \frac{2}{\alpha'^2 |eH|} \quad (31)$$

The generalization of the above to $SU(N)$ is straightforward. In the case of $SU(3)$ we choose the following external field

$$F_{ij}(x) = \epsilon_{ij3} HT(8) \quad ; \quad F_{0i}(x) = 0 \quad (32)$$

where $T(\alpha) = \lambda_\alpha/2$, λ_α being the usual Gell-Mann matrices. Corresponding Yang-Mills potentials are

$$A_i(x) = -\frac{1}{2} \epsilon_{ij3} x_j HT(8) \quad ; \quad A_0(x) = 0 \quad (33)$$

Equation (12) will be satisfied by the following choice for the string current

$$J^a(\tau) = \sum_{A=1}^3 j_A^a T(A) + j_8^a T(8) \quad (34)$$

where j_A^a and j_8^a are constants. Condition (26) is now replaced by

$$\frac{1}{\alpha' \pi} = e H j_8^1 R \quad (35)$$

The magnetic flux can now take on two non-trivial values corresponding to $f_c = \exp(2n\pi i/\sqrt{3} \lambda_8)$, $n = 1, 2$. Using (27) with σ_3 replaced by λ_8 , we find

$$R^2 = \frac{4}{\sqrt{3}} \frac{n}{|eH|} ; \quad n=1, 2 \quad (36)$$

Consequently

$$m^2 = \frac{4}{\sqrt{3}} \frac{n}{\alpha'^2 |eH|} ; \quad n=1, 2 \quad (37)$$

Since we want equations (36) and (37) to correspond to gluing states, condition (iii) requires that we set $j_a^0 = 0$. This implies that the strings have zero angular momentum. From (29), the angular momentum in the third direction is

$$M^{12} = \int_0^{2\pi} d\tau^1 (p^1(\tau) \tilde{z}^2(\tau) - p^2(\tau) \tilde{z}^1(\tau)) = -\frac{e\pi}{2} j_8^0 H R^2 \quad (38)$$

The result therefore follows by setting $j_8^0 = 0$.

4. GLUERING EXCITATIONS IN THE QCD VACUUM

We would now like to apply the above considerations to get an estimate for the excitation energy of a gluing in the QCD vacuum. Here we make use of the Copenhagen picture⁸⁾ of the QCD vacuum^{*)}. Flux tubes with an approximately constant chromomagnetic field H are randomly distributed in space forming a quantum liquid. The colour magnetic instability¹⁰⁾, which is responsible for the formation of the liquid, is a long range effect characterized by a "coherence length" $d_c = \sqrt{2\pi/|eH|}$. In comparing d_c with R [cf., Eq. (36)], we note that it is

*) For another application of the Copenhagen vacuum see Ref. 9).

possible to imagine a gluering embedded within an individual flux tube. To compute the mass and radius of the gluerings we can utilize the relationship

$$\sqrt{|eH|} = 1.85 \Lambda \quad (39)$$

which was found valid at the one-loop level of perturbation theory¹¹⁾. Using the values $\alpha' = 1 \text{ GeV}^{-2}$ and $\Lambda = 0.5 \text{ GeV}$, we find $m = 1.7 \text{ GeV}$ and $R = 0.3 \text{ Fermi}$ for $n = 1$ and $m = 2.3 \text{ GeV}$ and $R = 0.5 \text{ Fermi}$ for $n = 2$ ^{*}). It is perhaps interesting to note that the masses computed here are of the same order of magnitude as other phenomenological and theoretical estimates for glueball states^{4),13)}

In addition to computing the energy of the glueball states, it is possible to estimate their lifetimes. We find these states to be extremely unstable^{**)}. Consider two neighbouring flux tubes with opposing magnetic fields. If one should tunnel into the other the fields will cancel, yielding the usual perturbative vacuum in the overlapping region. Such a process requires an energy equal to BV where B is the MIT bag constant and V is the overlap volume. If a gluering was originally present in the volume V the tunnelling process will remove its magnetic flux, leading to the demise of the gluering. To compute the decay width we need an estimate for the volume of the gluering which requires giving the string a thickness. A liberal estimate for V would be $\frac{4}{3}\pi R^3$, R being the radius of the gluering. Then $BV \approx 10 \text{ MeV}$. Since the energy is negligible compared with the gluering energy, the decay width is extremely broad so it is likely that these glueball states are unobservable^{***)}. All the arguments presented here necessitate that the gluering be a "test" object, i.e., it does not influence the background field. We are not clear as to the validity of this assumption. Perhaps a more accurate description would require replacing the closed string with a two-dimensional torus (at a given time-slice). To be consistent with the usual interpretation a confined electric flux could be present within the torus. This, however, would require placing a magnetic, as well as an electric current, on the object.

*) Due to the relation (39) instanton corrections¹²⁾ will actually change the masses given here. Since we, however, are only interested in the order of magnitude estimates, we will not dwell on this question in the present note. If we use the relation¹¹⁾ $\langle E \rangle = -(0.42\Lambda)^4 = -B$, where B is the bag constant, the values of m and R will be somewhat larger than those given in the text.

**) As classical objects the circular gluerings are stable since they have a minimum circumference.

***) This argument was suggested to us by V.P. Nair. The tunnelling frequency, ω was estimated by Nielsen and Olesen⁸⁾ to be $\omega = 2\sqrt{3}|eH|$. This expression leads to a decay width of the order of the gluering excitation energy.

It is a deficiency of our model of gluing "glueballs" that we cannot predict the charge-parity quantum numbers^{*)} as well as decay modes. We believe, however, that our model can be improved with regard to decay modes by making use of a quark-pair production mechanism similar to the one used in the context of the chromoelectric flux-tube model¹⁵⁾.

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^{*)} See in this context Ref. 14).

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