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ON THE SUPPRESSION OF MONOPOLE PRODUCTION
IN THE VERY EARLY UNIVERSE

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A B S T R A C T

We investigate the effects of thermal fluctuations near the phase transition corresponding to the breakdown of a grand unified gauge group G to $G' \times U(1)$ in the very early Universe, especially as regards the production of superheavy monopoles. Using $SU(5)$ as a generic example of a grand unified theory, we find that this will be strongly suppressed for a reasonable range of parameters appearing in the Higgs potential. Allowing for a dynamical Higgs mechanism, leading to a considerable ratio of Higgs particle to leptoquark boson masses, the number of produced monopoles will be negligible.

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It has recently been argued, within the combined framework of grand unified theories of elementary particle interactions and standard hot Big Bang cosmology, that large numbers of 't Hooft-Polyakov monopoles would be created as the Universe cools down from the initial singularity and undergoes a phase transition corresponding to the breakdown of the grand unified gauge group G to $G' \times U(1) \subset G$, an expectation in apparent contradiction with observation^{1),2)}. Briefly, what was pointed out in Refs 1) and 2) is the following. As a result of the topological conservation of magnetic charge, these superheavy monopoles are stable and can only disappear through pair annihilation, a process amenable to a classical calculation. Now it is possible to deduce some stringent bounds on the relative monopole density $r(T)$, defined to be

$$r(T) \equiv \frac{n(T)}{T^3} = \frac{2 \zeta(3)}{\pi^2} \cdot \frac{n(T)}{n_\gamma(T)}, \quad (1)$$

where $n(T)$ and $n_\gamma(T)$ are the number densities of monopoles and photons, respectively, as a function of temperature, and the zeta function $\zeta(3)$ has the numerical value 1.202; here and elsewhere we set the Boltzmann constant $k = 1$. A good reference temperature at which to bound $r(T)$ turns out to be around 1 MeV, roughly corresponding to the time of primordial helium synthesis. By requiring that the possible presence of monopoles does not disturb the successful calculation of the helium abundance, one obtains the bound²⁾

$$r(T \simeq 1 \text{ MeV}) \lesssim 10^{-19}. \quad (2)$$

However, it is found^{1),2)} that the conventional annihilation mechanism can only reduce an arbitrarily large initial relative density of monopoles down to $r(T \simeq 1 \text{ MeV}) \sim 10^{-10}$. Initial values of less than this remain unchanged as the monopole density is too low for the annihilation mechanism to be effective in such a regime. Thus, monopole production must be suppressed near the phase transition. It has already been suggested³⁾ that the supercooling effect expected of a strongly first order transition could achieve such a suppression. However, the theoretical analysis of such a situation is extremely difficult; it is also far from obvious what the effects of the associated release of a large amount of latent heat in such transition would be as regards the number of produced monopoles.

It is the purpose of this note to present an analysis of monopole production, assuming that the relevant phase transition in the very early Universe is second order, or weakly first order. We are led to the conclusion that monopole production can be strongly suppressed, as a result of thermal fluctuations below but still close to the critical temperature T_c . The physical mechanism through which this is achieved will be described below, and numerical results obtained using the standard SU(5) grand unification scheme^{4),5)} as an illustrative example. We will also entertain the speculation that the symmetry breaking arises dynamically, in which case we will argue that monopole suppression is further enhanced due to the large mass of the physical (composite) Higgs fields relative to the masses of the gauge bosons.

We suppose that the free energy density can be expressed as a fourth-order polynomial in the relevant Higgs fields, with no cubic term: the discrete symmetry $\Phi \leftrightarrow -\Phi$ may be imposed to achieve this. In analogy with the phenomenological theory of superconductivity, we shall assume that this is also the case if the Higgs field is composite. This leads to a picture of a second order (or weakly first order) phase transition, at a finite critical temperature T_c , from the disordered (normal) to the ordered phase of lower energy of the Higgs field Φ , with the usual result

$$\langle \Phi(T) \rangle \sim v_0 (1 - T^2/T_c^2)^{1/2}, \quad T \leq T_c, \quad (3)$$

for the temperature dependence of the vacuum expectation value of the Higgs field (order parameter) near the critical point. In keeping with our assumption that the transition be second order, we will assume that the Higgs couplings are typically of the order of or larger than the gauge couplings: this requirement will be made more explicit later when we specialize our considerations to the case of SU(5). Under these circumstances, the critical temperature T_c is found to be of the order of the zero temperature vacuum expectation value v_0 of the Higgs field: this in turn is of the order of 10^{15} GeV for the case at hand. The correlation length $\xi(T)$ diverges at T_c ,

$$\xi(T) = m_H^{-1} (1 - T^2/T_c^2)^{-1/2}, \quad (4)$$

where m_H is the physical Higgs scalar mass at zero temperature. In later estimates, this will be taken to be the mass of the lightest Higgs scalar so the

calculated relative monopole density arising from the phase transition will actually be an upper bound. The divergence in Eq. (4) expresses the intuitive notion that field fluctuations occur on all length scales at the critical point.

THERMAL FLUCTUATIONS AND MONOPOLE FORMATION

Monopole production occurs immediately below T_c , and because of the expansion of the Universe, the relevant framework in which to study this is that of dynamical critical phenomena. Two questions arise: is thermal equilibrium maintained as the Universe undergoes the phase transition, and if so, what are the effects of thermal fluctuations on monopole formation?

For T very close to T_c , thermal fluctuations can create domains of normal vacuum which may shrink or break-up into domains with non-trivial winding number (i.e., non-zero magnetic charge), or recombine into larger domains adding their charges. Thus, thermal fluctuations very close to T_c provide a mechanism for the production and annihilation of proto-monopoles (-antimonopoles), by which we mean extended domains of normal vacuum with some charge. Note that since the field configurations in the system change continuously, the net charge is conserved, apart from boundary effects (fluctuations can change the boundary conditions).

These processes take place typically at a rate determined by the slowest time constant of the thermodynamic system. One expects this rate to be of the order of the lifetime τ of the longest wavelength ($\lambda = \xi$) fluctuations in the system, which is the rate at which the perturbed system relaxes back to equilibrium. This rate is determined by the intrinsic parameters of the system and has in principle nothing to do with the expansion rate of the Universe.

To estimate the relaxation time we will follow the lead of the theory of dynamical critical phenomena and consider a simple time-dependent extension of the Landau-Ginzburg theory, assuming that the rate at which the order parameter returns to its value $\langle \Phi \rangle$ is determined by the slope of the free energy density $F(\Phi)$ in function space^(6),7). One then finds that the relaxation time is typically given by

$$\tau = \tau_0 \left(1 - T^2/T_c^2 \right)^{-1}, \quad (5)$$

with τ_0 of the order of T_c^{-1} , or of m_H^{-1} . Note, again, the divergence of τ at the critical point, in agreement with our intuition that critical fluctuations occur on all length and time scales. The behaviour shown in Eq. (5) also obtains in the framework of dynamical renormalization group calculations in simple ϕ^4 type models, with small corrections to the critical exponent.

One requirement for thermal equilibrium of the basic system (and a fortiori of the monopoles) is of course that the relaxation (reaction) rate $1/\tau$ be larger than the expansion rate $\mathcal{H} = |\dot{T}/T|$ of the Universe, namely

$$\mathcal{H} \tau \ll 1 \quad . \quad (6)$$

In the framework of the usual Big Bang cosmology, and in the radiation dominated era, the expansion rate is related to the temperature:

$$\mathcal{H} = \frac{T^2}{M_P} \left(\frac{4\pi^3 g}{45} \right)^{1/2} , \quad (7)$$

where $M_P = G^{-1/2} = 1.2 \times 10^{19}$ GeV is the Planck mass and g is the effective number of particle states, a number of order 10^2 in the SU(5) grand unification scheme. Thus we require that, at some temperature T_* below T_c ,

$$O(20) \frac{T_*^2}{M_P \tau_0^{-1}} \left(1 - T_*^2/T_c^2 \right)^{-1} \ll 1 \quad . \quad (8)$$

In order of magnitude, $T_* \approx O(T_c) \approx O(m_H) \approx 10^{15}$ GeV, which means that provided T_* is not too close to T_c , i.e., $\epsilon_* \equiv 1 - T_*^2/T_c^2 \gg O(10^{-3})$, thermal equilibrium will be maintained through the phase transition. Note that a precise knowledge of τ_0^{-1} is unnecessary to reach this conclusion: it suffices that it be of the same order of magnitude as the other dimensional parameters of the Higgs system, and that these be much smaller than the Planck mass.

The second part of the criterion for the monopoles to be in thermal equilibrium is that the fluctuations are dense enough to ensure that the previously indicated mechanism for the creation and destruction of proto-monopoles is effective. This turns out to be the case for temperatures larger than the so-called Ginzburg temperature⁸⁾ T_G at which a typical fluctuation will no longer be energetic enough to induce the change from ordered to normal vacuum over a region of linear dimension of about a correlation length. So T_G is the "freeze-out" temperature below which the distinction between ordered and normal vacua becomes well established, and where fluctuations no longer provide an effective mechanism for keeping the monopoles in equilibrium. Below T_G the conventional annihilation mechanism considered by Zeldovich and Khlopov¹⁾ and by Preskill²⁾ is indeed the only way of making monopoles disappear, and is itself insufficient to maintain

equilibrium. Thus, according to our considerations, the initial density of monopoles and antimonopoles will be frozen out at a value proportional to $\exp(-m(T_G)/T_G)$, where $m(T_G)$ is the temperature dependent monopole mass.

The value of the Ginzburg temperature T_G may be estimated on the basis of the argument above by requiring⁹⁾ that the thermal energy T in a typical fluctuation be equal to the free energy required for a change from the ordered phase $\langle\Phi(T)\rangle \neq 0$ back to $\langle\Phi\rangle = 0$ on a scale of the correlation length $\xi(T)$. Thus,

$$T_G \simeq A \xi^3(T_G) |\Delta F(\langle\Phi(T_G)\rangle)|, \quad (9)$$

where ΔF is the free energy density difference between the normal and the ordered phases, and $A\xi^3$ is the correlation volume over which the longest wavelength fluctuations occur, with A a geometrical factor of order unity. Just what value of A should be used is not clear, and this uncertainty is magnified by the fact that A will appear in an exponent in the evaluation of the primordial density of monopoles: one can conceive that $A = 1$ for a cube of side ξ , $4\pi/3$ for a sphere of radius ξ , etc. This should be borne in mind for later estimates.

One may wonder how fluctuations with an energy $T_G = A\xi^3|\Delta F(T_G)|$ can affect monopole formation even when $T_G \ll m(T_G)$, which is of course the situation we are led to in obtaining a large suppression of the primordial monopole density. The essential point is that the probability p_ξ for any region of size $A\xi^3$ to be normal state is still large, approximately $p_\xi \approx 1/(1 + e) = 0.27$. This has important consequences, as may be seen from the following somewhat idealized picture. Suppose we divide space up in regions of volume $A\xi^3$, with a given probability of being, or not, in the normal state: this is similar to the percolation problem [see Ref. 10) for reviews], the study of which has established that when p_ξ is larger than a certain critical probability p_c called the percolation threshold, the (infinite) system contains at least one infinite cluster, here of interconnected regions of normal vacuum. The magnitude of p_c depends solely on the spatial dimensionality and the geometrical structure of the system under consideration: for the continuum in three dimensions one has¹⁰⁾ $p_c \approx 0.25$. Obviously the energy of large clusters can be much larger than $m(T)$, even if $m(T) > \xi^3|\Delta F(T)|$, and it is still possible to make monopoles. Combining this picture with the random dynamics of the fluctuations, which causes the clusters of normal vacuum to shrink, break up or swallow widely separated smaller domains,

etc., one concludes that proto-monopoles will be kept in equilibrium all the way down to $T = T_G$, after which the mechanism ceases to be effective because p_ξ drops sharply to values $p_\xi \ll p_c$.

The frozen-out monopole density can now be calculated as being given by a Boltzmann factor at the Ginzburg temperature:

$$r(T_G) = \left(\frac{m(T_G)}{2\pi T_G} \right)^{3/2} \exp(-m(T_G)/T_G). \quad (10)$$

Before turning to the consideration of the explicit SU(5) grand unification scheme, a remark is in order on the crucial ratio $m(T_G)/T_G$. The ratio $m(T)/\xi^3(T)|\Delta F(T)|$ is actually T independent, so that one expects that the ratio $m(T_G)/T_G$ can be expressed in terms of the fundamental constants of the theory. The free energy density difference $\Delta F(T_G)$ is proportional to $m_H^2(T_G)m_X^2(T_G)/g^2$, where $m_{H,X}^2(T_G) = m_{H,X}^2 \epsilon_G$ are the temperature dependent Higgs and leptoquark boson masses, respectively, g is the grand unified gauge coupling constant, $g^2 = 4\pi\alpha_{GU}$, and where the ϵ factor has been defined in the text following Eq. (8). With $\xi(T_G) = m_H^{-1}(T_G)$ and the temperature dependent monopole mass $m(T_G) \sim m_X(T_G)/\alpha_{GU}$ one deduces that

$$m(T_G)/T_G = C m_H/m_X, \quad (11)$$

where C is a model dependent numerical constant and m_H and m_X are zero temperature masses. In particular, the result, Eq. (11), is independent of T_G . It remains now to discuss the evaluation of C , and to consider what values can be taken by the ratio m_H/m_X . We will find that C can be considerable, and that m_H/m_X can take large enough values, especially in a dynamical symmetry breaking scheme, to completely suppress the density of primordial superheavy monopoles.

MONOPOLES IN SU(5) GRAND UNIFICATION

For the evaluation of C , we will require an expression for the monopole mass in terms of the parameters appearing in the potential for the (perhaps composite) Higgs field. A brief discussion of the properties of monopoles in SU(5) is in order here.

In a suitable gauge, magnetic charge is quantized by demanding that

$$e^{ig\hbar} |\psi\rangle = |\psi\rangle, \quad (12)$$

where gh is the magnetic charge matrix in the Cartan subalgebra of $SU(5)$, and ψ is some faithful representation¹¹⁾. The solution is conveniently expressed as follows,

$$gh = 2\pi (n_1, n_2, n_3, n_4, -(n_1+n_2+n_3+n_4)), \quad (13)$$

where $(., ., ., ., .)$ denotes a diagonal 5×5 matrix. Below the Ginzburg (freeze-out) temperature $T_G \lesssim T_C$ the theory has an exact residual symmetry group $H \simeq SU(3) \times SU(2) \times U(1)$, due to a vacuum expectation value $\langle\Phi\rangle = v(1,1,1,-3/2,-3/2)$ of the Higgs field in the adjoint representation of $SU(5)$. The magnetic charge is labelled by the elements of

$$\pi_2(G/H) \simeq \pi_1(H) = \pi_1(U(1)) = \mathbb{Z}, \quad (14)$$

expressing the fact that only the component of gh along the $\langle\Phi\rangle$ direction is topologically conserved. The monopole mass is proportional to $\text{Tr}(gh\langle\Phi\rangle)$, and one may consider the monopoles with the smallest non-zero mass as fundamental. The multiply-charged monopoles can be considered as bound states of the fundamental ones¹²⁾, but since they will have higher masses they will not be produced as abundantly and so we need not worry about them and will restrict our considerations to the lightest ones. There are, of course, many solutions of the form given in Eq. (13) with the same minimal $U(1)$ charge: these differ by components of gh which are not conserved. Such configurations will lower their energy by losing charge in the non-conserved directions (within the Cartan subalgebra), until gh reaches a minimum gh' (in the sense that $\text{Tr}[(gh')^2]$ is minimal), gh' satisfies Eq. (12), and its $U(1)$ charge is the same, i.e., $\text{Tr}[gh'\langle\Phi\rangle] = \text{Tr}[gh\langle\Phi\rangle]$. If H is non-Abelian, then gh' is in general not unique, but all solutions gh' are gauge equivalent under H . In $SU(5)$, we may define for example the fundamental monopole as the one with $n_3 = -n_4 = 1$, and $n_1 = n_2 = n_5 = 0$, or gauge equivalently the ones obtained by permuting n_1, n_2, n_3 and/or n_4, n_5 : in fact, these are just members of a continuous class of equivalent classical solutions. On the quantum level, one expects the monopoles to be gauge

invariant, and it is therefore useful to give an H invariant definition of the charge. This can be done by decomposing $\exp(igh) = \alpha\beta\gamma$ so that $\alpha \in U(1)$, $\beta \in SU(3)$ and $\gamma \in SU(2)$. For charges satisfying Eq. (12), one easily finds that $\alpha \in Z_\infty$, $\beta \in Z_3$ and $\gamma \in Z_2$ so that α , β and γ are indeed H invariant. Thus, a monopole with n units of $U(1)$ charge carries $2n \bmod 3$ units of Z_3 and $n \bmod 2$ units of Z_2 , i.e., the fundamental monopole may be characterized by $[1,2,1]$ and its antiparticle by $[-1,1,1]$. It is worth mentioning that these are in fact like Dirac monopoles in a theory with gauge group H , which have become non-singular as a result of the embedding in the larger group G .

In the Bogomolny-Prasad-Sommerfield (BPS) limit (i.e., $\lim m_H/m_X \rightarrow 0$), an exact solution for the fundamental monopole can be constructed easily¹³⁾ as a simple embedding of the PS solution¹⁴⁾. Its mass can be calculated to be

$$m(T) = \lim_{r \rightarrow \infty} 4\pi r^2 T_r [B \langle \Phi(r) \rangle] = \frac{5}{4} \cdot \frac{g v(T)}{\alpha_{GU}}, \quad (15)$$

where $B = h/4\pi r^2$ is the (matrix) radial component of the magnetic field and $v(T) = v_0 (1 - T^2/T_C^2)^{\frac{1}{2}}$ characterizes the vacuum expectation value of the Higgs field. However, we will be considering situations in which the Higgs particle mass is at least comparable to the gauge boson mass, or even much greater, and thus far from the BPS limit. It has been shown¹⁵⁾ that in a comparable situation, the monopole mass can be parametrized by the expression, Eq. (15), multiplied by a factor $f(m_H/m_X)$. Naturally this factor is normalized to $f(0) = 1$ and slowly increases to a value in excess of 2 for $m_H/m_X = 10$. Thus we shall take for the monopole mass the expression $m(T) = (5f/4) g v(T)/\alpha_{GU}$.

We now turn to the determination of the remaining parameters entering in the Ginzburg criterion, Eq. (9). We follow the notation of Ref. 5) in writing the usual potential for the adjoint Higgs field Φ in $SU(5)$:

$$V(\Phi) = -\frac{1}{2} \mu^2 \text{Tr} \Phi^2 + \frac{a}{4} (\text{Tr} \Phi^2)^2 + \frac{b}{2} \text{Tr} \Phi^4, \quad (16)$$

the effect of finite temperature fluctuations in the Higgs field being to shift μ^2 to $\mu^2 - \lambda T^2$, where λ is a function of the coupling constants, calculable in perturbation theory, in terms of which one defines the critical temperature T_C ¹⁶⁾. Then, provided $b > 0$, $a > -7b/15$, the lowest vacuum will be the asymmetric one corresponding to the desired pattern of symmetry breaking,

$SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$, with a Higgs vacuum expectation value matrix $\Phi(T) = v(T) (1, 1, 1, -3/2, -3/2)$, where

$$N(T) = \left(\frac{2\mu^2}{15a + 7b} \right)^{1/2} \left(1 - T^2/T_c^2 \right)^{1/2}. \quad (17)$$

In keeping with our assumption that the phase transition is second order (or weakly first order), we assume that the Higgs couplings are typically larger, or at least of the order of the gauge coupling: specifically, radiative corrections to the potential $V(\Phi)$ in Eq. (16) which could change the nature of the transition (at least in a theory with explicit scalars), will be negligible upon requiring that $15a + 7b \gg 0(g^4)$: for a discussion of this point, see Ref. 17). Recall that with $\alpha_{GU} \approx 0.02$, $g^2 \approx 1/4$. For the case at hand, and to lowest order in perturbation theory, T_c turns out to be^{3), 16)}

$$T_c = \mu \left(\frac{130a + 94b + 75g^2}{60} \right)^{-1/2}. \quad (18)$$

The superheavy vector particles have masses $m_{X,Y}^2(T) = (25/8)g^2v^2(T)$. With the usual definition for the shifted Higgs field,

$$\Phi - \langle \Phi \rangle = \begin{bmatrix} H_8 + \frac{2}{\sqrt{30}} H_0 & \tilde{H}_X & \tilde{H}_Y \\ \hline \tilde{H}_X^\dagger & \sqrt{\frac{1}{2}} H_Z - \frac{3}{\sqrt{30}} H_0 & H_+ \\ \tilde{H}_Y^\dagger & H_- & -\sqrt{\frac{1}{2}} H_Z - \frac{3}{\sqrt{30}} H_0 \end{bmatrix} \quad (19)$$

then the massless H_X and H_Y are eaten by the X and Y leptoquark bosons, leaving twelve superheavy Higgs particles with masses given by

$$\begin{aligned} m_{H_8}^2(\tau) &= \frac{5}{2} b v^2(\tau), \quad m_{H_2}^2(\tau) = m_{H_{\pm}}^2(\tau) = 10 b v^2(\tau), \\ m_{H_0}^2(\tau) &= (15a + 7b) v^2(\tau) \end{aligned} \quad (20)$$

Finally, the difference in free energy density between the normal and asymmetric vacua is given by

$$|\Delta F(\langle \Phi(\tau) \rangle)| = V(0) - V(\langle \Phi(\tau) \rangle) = \frac{15}{16} (15a + 7b) v^4(\tau). \quad (21)$$

It will be convenient to use the ratio $\rho = a/b$ and the mass of the lightest Higgs particle m_H as independent parameters characterizing the Higgs system.

The constant C appearing in Eq. (11) can be calculated in two regimes of the parameter ρ . In the first, corresponding to $\rho \geq -3/10$ (for which the lightest scalar mass is given by $m_H^2 = (5/2)bv^2$), one finds:

$$C = \frac{4\pi f}{A} \cdot \frac{25\sqrt{2}}{42} (1 + 15\rho/7)^{-1}, \quad (22)$$

and also:

$$\epsilon_G \equiv 1 - T_G^2/T_C^2 \simeq \left[1 + 3.9A^2 (1 + 15\rho/7) \left(1 + \frac{65\rho}{47} + \frac{30}{47} \frac{m_X^2}{m_H^2} \right) \frac{m_X^2}{m_H^2} \right]^{-1} \quad (23)$$

where in the evaluation of (23) we have used $\alpha_{GU} = 0.02$: note that this expression for ϵ_G is only valid when perturbation theory is applicable so that, for example, m_X/m_H cannot be too small. We will return to this point momentarily.

The second regime corresponds to the interval $\rho_0 \leq \rho \leq -3/10$ where $\rho_0 \geq -7/15$, chosen to be consistent with the previously stated requirements of the desired pattern of symmetry breaking on the one hand, and of smallness of

the radiative corrections to the tree-level potential, on the other. Then the lightest Higgs particle has mass $m_H^2 = 7bv^2(1 + 15\rho/7)$, and one finds

$$C = \frac{4\pi f}{A} \cdot \frac{5\sqrt{2}}{3}, \quad (24)$$

and correspondingly,

$$\epsilon_G \simeq \left[1 + .5 A^2 (1 + 15\rho/7)^{-1} \left(1 + \frac{65\rho}{47} + \frac{84}{47} \left(1 + \frac{15\rho}{7} \right) \frac{m_X^2}{m_H^2} \right) \frac{m_X^2}{m_H^2} \right]^{-1}. \quad (25)$$

Recall, finally, the expression for the primordial density of monopoles:

$$r(T_G) = \left(\frac{C}{2\pi} \cdot \frac{m_H}{m_X} \right)^{3/2} \exp \left(-C m_H/m_X \right). \quad (26)$$

A definite prediction for the value of $r(T_G)$ is difficult because of the absence of precise values for the parameters A , ρ and m_H/m_X . As long as perturbation theory is applicable, it is possible to check explicitly, using Eq. (23) or (25), that thermal equilibrium is in fact maintained through the phase transition, in spite of the expansion of the Universe: then the ratio m_H/m_X cannot be too large. To estimate the limit on m_H/m_X , consistent with the perturbative regime, we may require that the one-loop corrections to the potential due to the scalar particles¹⁸⁾, V_r , be small as compared to the tree level result,

$$V_r = \frac{1}{64\pi^2} \text{Tr } m_H^4 < V(\langle \Phi \rangle). \quad (27)$$

Again, the bound thus obtained depends on the parameter ρ . Corresponding to the previously mentioned regimes for this parameter, one obtains (m_H is again the lightest Higgs mass):

$$\begin{aligned} \rho \geq -3/10 : \quad \left(\frac{m_H}{m_X} \right)^2 &< \frac{168\pi \alpha_{GU}^{-1} (1 + 15\rho/7)}{700 + 98(1 + 15\rho/7)^2}, \\ \rho_0 \leq \rho \leq -3/10 : \quad \left(\frac{m_H}{m_X} \right)^2 &< \frac{168\pi \alpha_{GU}^{-1} (1 + 15\rho/7)^2}{250 + 35(1 + 15\rho/7)^2}. \end{aligned}$$

It is now easy to establish that there exists a range of plausible values for A , ρ and m_H/m_X for which the bound on r in Eq. (2) and the condition for equilibrium $\epsilon_G \gg 0(10^{-3})$ are amply satisfied, and perturbation theory should be valid. For example, with $A = 1$, $\rho = -3/10$ and $m_H/m_X = 2.5$ (i.e., $f \approx 1.2$ for the factor in the monopole mass) one obtains:

$$C \approx 36, \quad 1 - T_G^2/T_C^2 \approx .87 \gg 0(10^{-3}) \quad (28)$$

so that

$$r(T_G) \approx 10^{-38} \ll 10^{-19}. \quad (29)$$

It thus seems likely that monopole extinction very close to the phase transition can be understood as arising from the effects of thermal fluctuations on the Higgs system. In general, this will require that A be closer to unity than to, say, $0(10)$, and that $m_H/m_X \gtrsim 1$. Note that, for a given value of ρ , a smaller value of A and a larger value of m_H/m_X (assumed to be consistent with perturbation theory calculations) lead to a larger suppression of the number of produced monopoles and to a larger value of ϵ_G at the same time, a nicely consistent situation. Of course, there are also plausible choices of parameters that lead to small values of C and m_H/m_X : thus our considerations may be viewed as implying that constraints on the Higgs system parameters arise from the non-observation of primordial superheavy monopoles.

A DYNAMICAL HIGGS MECHANISM ?

Finally, we consider the interesting possibility that the Higgs mechanism at the grand unification scale is implemented in a dynamical way, a suggestion which has received a great deal of attention recently¹⁹⁾. Within such a framework, the Higgs masses are expected to be considerably larger than the gauge boson masses, with the ratio m_H/m_X easily reaching values in excess of ten, by an argument we now outline¹⁹⁾. Note that under these circumstances, the critical temperature T_C is no longer known in perturbation theory and the potential $V(\Phi)$ in Eq. (16) must be viewed, as is the case in the theory of superconductivity, as a phenomenological description of the self-interaction of the composite Higgs fields near the phase transition. On the basis of the remarks following Eq. (8), we suppose that the conditions for thermal equilibrium are still fulfilled: then the independence of $r(T_G)$ in Eq. (26) on the precise value of T_C (presumably still of the order of 10^{15} GeV) encourages us to believe that this expression is still valid for the primordial density of monopoles.

To see how a dynamical symmetry breaking mechanism naturally leads to a large value of m_H/m_X , a number of plausible results pertaining to flavour symmetries in QCD are used. The basic idea is to introduce a new species of "techniquarks" Q' characterized by a new quantum number called "technicolour": the group associated with these new degrees of freedom is gauged, leading to a new strong interaction among the techniquarks. Typically this technicolour group is taken to be $SU(N'_C)$ where N'_C is greater than or equal to four to allow for unification with the known $SU(3) \times SU(2) \times U(1)$ at some high energy: the weak and electromagnetic interactions of techniquarks are the same as those of the ordinary quarks, with the usual classification in terms of left-handed flavour doublets and right-handed singlets. At the energy scale Λ' where the technicolour coupling reaches a value of order unity, $\bar{Q}'Q'$ condensates form leading to a spontaneous breakdown of chiral symmetry, and a number of massless technipions appear with decay constant $F_{\pi'}$. These mix with the flavour gauge fields which thus acquire a mass: it has been shown that such a mechanism, applied to the standard $SU(2) \times U(1)$ model of electroweak interactions, implies the well-known mass formula $m_Z \cos \theta_W = m_W$ ¹⁹⁾. The remaining physical Higgs meson(s) are technihadrons, with masses of the order of the scale Λ' . The estimation of m_H/m_X now proceeds in several steps. First, the gauge field masses are given in terms of the technipion decay constants, as follows¹⁹⁾

$$m_X = g \sqrt{n'_f} F_{\pi'}/2 \quad (30)$$

where g is now the electroweak coupling and n'_f the number of flavour doublets. We may now appeal to the $1/N'_C$ expansion [see, e.g., Ref. 20) for a review] to establish that $F_{\pi'}$ scales like $\sqrt{N'_C}$; denoting by $m_{\sigma'}$ ($= m_H$) the mass of a typical O^+ technihadron we may form the dimensionless ratio $\sqrt{N'_C} m_{\sigma'}/F_{\pi'}$, which is independent of N'_C , n'_f and Λ' . Thus we have the estimate

$$\sqrt{N'_C} \frac{m_{\sigma'}}{F_{\pi'}} \simeq \sqrt{3} \frac{m_{\sigma}}{F_{\pi}} \quad , \quad (31)$$

where the right-hand side of Eq. (31) refers to the usual $SU(3)$ of QCD; m_{σ} is not too well pinned down, but may be taken to be around 900 MeV, so that

$$\sqrt{N'_C} \frac{m_{\sigma'}}{F_{\pi'}} \simeq 17 \quad . \quad (32)$$

Combining Eqs (30) and (32) leads to the estimate

$$\frac{m_H}{m_X} \simeq \frac{34}{g \sqrt{N'_c n'_f}} \quad (33)$$

To pin down values of N'_c and n'_f would require a realistic explicit model, the construction of which is beset by many difficulties. Let us, however, take $N'_c = n'_f = 4$ for the purposes of a crude estimate; with $g \simeq 1/2$, this leads to

$$m_H/m_X \simeq 17 \quad (34)$$

With $C \simeq O(10)$ for example, this is easily seen to lead to the complete extinction of primordial superheavy monopoles.

To summarize: we have presented a general mechanism for the suppression of the number of superheavy monopoles produced in the very early Universe at the grand unified phase transition, assumed to be second order (or weakly first order). Although we have specialized our considerations to the minimal grand unified theory of $SU(5)$, we believe our conclusions to be more general. In particular, similar results are expected to follow from theories with more complicated Higgs sectors, i.e., more free parameters. We have also suggested that if the Higgs mechanism is of dynamical origin, monopole extinction is expected to be essentially complete.

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