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TOWARDS THE EARLY STAGES OF THE UNIVERSE

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A. - GENERALITIES ON COUPLING PARAMETERS

In 1953 a fundamental paper ¹⁾ questioned the notion of coupling parameters, used to expand in power series physical quantities in quantum field theories. This was somewhat revolutionary at a time when the fine structure constant in QED in the limit of vanishing energy momentum (from the $2P_{3/2} - 2P_{1/2}$ separation in deuterium or from the Thomson limit, etc.) seemed to be the only possible definition and was moreover consistent with the classical definition of the electron charge via the correspondence principle.

In the summary, at the beginning of this 1953 paper one can read indeed the following :

"...A coupling parameter, however, can only be specified in terms of a chosen development of a function $S(xy...x; c_1...)$ of physical significance ^{*)}. However, the terms of the actual correspondence development (in terms of e^2) $S = S_2 + S_4 + \dots$ have no physical meaning. Therefore the coefficient e^2 in S_2 has only a mathematical significance. It requires that the functions of $xy...; S_4, S_6, \dots, S_n$ have all been specified. As this specification involves the c_i 's ^{**)} , we must expect that a group of infinitesimal operations $P_i = (\partial/\partial c_i)_{c=0}$ exists ^{f1)}, satisfying

$$P_i S = h_i e (\kappa, \mu, e) \frac{\partial}{\partial e} S(\kappa, \mu, e; 0, 0, \dots) \quad (I)$$

admitting thus a renormalization of e ." ^{***)}

The above equation is the general form of the renormalization group equations for QED. Its restriction to the trivial one-parameter group for over-all scale transformations of the subtraction point μ

$$\mu' = \mu \exp(-c) \quad (1)$$

^{*)} Note : κ stands for the electron mass.

^{**)} Note : c_i are parameters originating in finite renormalizations.

^{***)} S , appearing on the left is of course the $S(\kappa, \mu, e; c_1, c_2, \dots)$.

is obvious. It gives the usual renormalization group equation for observable physical quantities

$$-\mu \frac{\partial}{\partial \mu} S_0(e, \kappa, \mu) = \beta \frac{\partial}{\partial e} S_0(e, \kappa, \mu) \quad (2)$$

[with $\beta = +h_e$ and $S_0(e, \kappa, \mu) \equiv S(e, \kappa, \mu; c=0)$] since (3)

$$\left. \frac{\partial}{\partial c} S_0(e, \kappa, \mu \exp(-c)) \right|_{c=0} \Big|_{e, \mu, \kappa \text{ fixed}} = -\mu \frac{\partial}{\partial \mu} S_0(e, \kappa, \mu) \Big|_{e, \kappa \text{ fixed}}$$

Therefore, Eq. (2) is, as announced, exactly equivalent to

$$\left. \frac{\partial}{\partial c} S(e, \kappa, \mu; c) \right|_{c=0} \Big|_{e, \kappa, \mu \text{ fixed}} = h_e \frac{\partial}{\partial e} S(e, \kappa, \mu; 0), \quad (4)$$

the equation (I) of the summary quoted above. Indeed, for the special subgroup considered (scale transformation of μ)

$$S(e, \kappa, \mu; c) = S_0(e, \kappa, \mu \exp(-c)) \quad (5)$$

This kind of equation ^{*)} has played various rôles since then in several topics of physics. For the first time, one had an equation with a derivative with respect to a coupling parameter, allowing thus to relate different orders in the perturbation expansion amongst themselves ^{f2)}. This feature has been explored and exploited to determine the asymptotic behaviour of all terms in perturbation theory. But the most exciting aspect is that it questions seriously the concept of coupling constant. Indeed, since physical quantities must satisfy relations like

$$S_0(e, \mu \exp(-c)) = S_0(e(c), \mu) \quad (6) \quad **)$$

*) Equation (I) extracted from the summary of Ref. 1). For multiplicatively renormalized quantities (Green functions, 1PI amplitudes, etc.) :

$$S(e, \kappa, \mu; c) = z^K(c) S_0(e, \kappa, \mu \exp(-c))$$

there is an additional term in I : $K(\partial z(c)/\partial c)|_{c=0}$ (the so-called anomalous dimensions γ). $z^K(c)$ stands for a product of various powers of z 's with $z(0)=1$.

**) κ has from now on been set equal to zero.

a change in $\mu' = \mu e^{-c}$ corresponds to a change in $e(c)$. More concretely, if

$$\mu \rightarrow \mu'' = \mu \exp(c), \text{ then } e \rightarrow e(c) \quad (7)$$

with $e(0) = e$ and the equation^{f3)}

$$\left. \frac{\partial e(c)}{\partial c} \right|_{c=0} = \beta(e) = h_e(e) \quad (8)$$

is easily derived from the preceding and also the RG equation

$$\frac{\partial e(c)}{\partial c} = \beta(e(c)) \quad (9)$$

The Equation (7) allows us to write

$$c = \text{Log } \mu'' / \mu$$

$e(c)$, the solution of Eq. (9), is most often written $\bar{e}(t)$ in the current literature and called the "running" or "effective" coupling constant.

At this point one sees why the preceding examination of the real meaning of a coupling parameter in quantum field theory was relevant. It led to a coupling which, if fixed at $\mu'' = \mu$ is equal to e , yields, at $\mu'' = \mu''$: $\bar{e}(\log \mu'' / \mu)$ which is a solution of Eq. (9) and can be written implicitly as

$$\int_e^{\bar{e}(t)} \frac{dx}{\beta(x)} = t \quad (10)$$

One knows in perturbation theory what $\beta(x)$ is. This is usually done by computing the relevant 1PI amplitudes and retaining their dimensionless part $S^{(i)}(p^2/\mu^2, e)$ evaluated at $p^2/\mu^2 = e^{2c}$ and then using the definition (8) of β [for details, see Ref. 2]. In field theories with one single coupling g , β is of the general form :

$$\beta(g) = b_0 g^k + b_1 g^m + \dots \quad k < m \quad (11)$$

and the non-written terms being of order bigger than m in g . For example, in a $g\phi^4$ theory, $K=2$, $m=3$; in QED, $K=3$, $m=5$, etc. In the so-called gauge theories with a single gauge coupling β has the same form as in QED.

B. - SEVERAL COUPLINGS

The situation becomes more involved if more than two fields are in interaction and several couplings occur. This situation has been examined in the fourth section of the paper ¹⁾. More comprehensive formulations have been made since then and the reader can refer to these standard reviews ³⁾. In general if one has a coupling constant space with vectors $\vec{g}$:

$$\vec{g} = (g_1, g_2, \dots, g_n) \quad (12)$$

then the term we had in Eq. (2) $\beta(e) \partial/\partial e$ becomes

$$\sum_{i=1}^n \beta^{(i)} \frac{\partial}{\partial g_i} \quad (13)$$

There is a β function $\beta^{(i)}$ for each g_i which depends on all these g_i , i.e.,

$$\vec{\beta} = (\beta^{(1)}, \beta^{(2)}, \dots, \beta^{(n)}) \quad (14)$$

and (13) is written

$$\vec{\beta} \cdot \frac{\partial}{\partial \vec{g}} \equiv \sum_{i=1}^n \beta^{(i)} \frac{\partial}{\partial g_i}$$

Similarly, the "anomalous dimensions" that we encountered in multiplicatively renormalizable quantities like Green functions of fields, 1PI amplitudes, Green functions of an operator, also depend on all the g_i (see the footnote on p. 2) :

$$\gamma = \gamma(\vec{g})$$

However, even for very complicated situations with several couplings, the evolution of the effective couplings $\bar{g}_i(t)$ can be determined, at least by numerical methods. The only essential assumption that must be fulfilled is that perturbation theory makes sense for each of these constants.

Working at the one-loop level, and considering the renormalization group equations for gauge coupling constants only a simplification arises in that the various equations decouple ⁴⁾ and one has equations of the type

$$\frac{d\bar{\alpha}_i}{dt} = -2 \frac{\beta_0^{(i)}}{4\pi} \bar{\alpha}_i^2 \quad ; \quad i = 1, 2, \dots, n. \quad (15)$$

with $\bar{\alpha}_i = \bar{g}_i^2(t)/4\pi$, and β_0 being pure numbers [chosen with this definition without geometrical factors $(4\pi)^k$].

C. - SIGN OF β_0

The sign of the one-loop term in the expansion of the β function is of decisive importance. Indeed the solution of (15) is

$$2t = -4\pi \int_{\alpha_i}^{\bar{\alpha}_i} \frac{dx}{\beta_0^i x^2} = \frac{4\pi}{\beta_0^i} \frac{1}{x} \Big|_{\alpha_i}^{\bar{\alpha}_i} = \frac{4\pi}{\beta_0^i} (\bar{\alpha}_i^{-1} - \alpha_i^{-1})$$

$$\bar{\alpha}_i(0) = \alpha_i$$

so that

$$\bar{\alpha}_i(t) = \frac{\alpha_i}{1 + (\beta_0^i/2\pi) \cdot \alpha_i t} \quad (16)^*)$$

When t increases, $\bar{\alpha}$ decreases logarithmically with the ratio of the energy M at which $\bar{\alpha}$ is computed and the energy μ at which α is defined so that (16) is also often written

$$\alpha_i(M) = \alpha_i(\mu) \left[1 + \frac{\beta_0^i}{2\pi} \alpha_i(\mu) \text{Log } M/\mu \right]^{-1} \quad (17)$$

*) Remember that $t = \text{log } M/\mu$.

This phenomenon ($\beta_0 > 0$) is proper exclusively to non-Abelian gauge theories in four dimensions. It has received the name of asymptotic freedom ⁵⁾ and occurs spectacularly in QCD, the SU(3) colour non-Abelian gauge theory of strong interactions. But, for instance in QED, it is well-known that $\beta_0 < 0$ and, when $\log M/\mu \rightarrow \infty$, $\alpha(M)$ has a ghost pole when

$$\left[1 + \frac{\beta_0}{2\pi} \alpha(\mu) \log M/\mu \right]$$

is zero. This, however, is not a genuine difficulty as for these values of $t = \log M/\mu$, the two-loop terms are more important than the simple one-loop approximation term. It simply indicates that the perturbation expansion of $\beta(\alpha)$ breaks down. This is a good example of how important is the examination of the second term in the expansion of β . For instance, in QCD, even if the precision obtained at present in the current experiments does not require the knowledge of this two-loop term, it is important to know its sign and magnitude. This in order to make sure that at present Q^2 leading logarithms are indeed dominant (which is not the case near the Landau pole in QED) and that the results of the leading log approximation are not altered by the next-to-leading ones. It can be readily shown that they are not, and this is very satisfactory from the purely theoretical point of view.

D. - BACK IN TIME FOR U(1), SU(2) AND SU(3) COUPLINGS

In the present view of the origin of the Universe in a big bang, at very early times the temperature was very high and all particles were very energetic. "Upward in energy" or "upward in temperature" can therefore be expressed in a fancy way as "backward in time". Indeed knowing the value of gauge couplings at present energies ^{*}) allows, through the machinery of the renormalization group developed in the previous sections, to calculate what their value will be at much higher energies like those of the order of the Planck mass $M_P = 1.2 \times 10^{19}$ GeV, provided, of course, that one has verified that perturbational techniques are still valid in this range of energy. The only things to know are the β_0^i for $i=1,2,3$ corresponding respectively to the U(1), SU(2) and SU(3) β^i functions. We have seen that in the one-loop approximation the equations decouple so that it becomes elementary to perform this calculation. One has ⁶⁾, neglecting possible mass terms

^{*}) The gauge group at low energies seems to be QCD SU(3) and SU(2)×U(1) for the weak and electromagnetic interactions.

$$\begin{aligned}\beta_0^1 &= -\frac{2}{3} \\ \beta_0^2 &= \frac{22}{3} + \beta_0^1 \\ \beta_0^3 &= 11 + \beta_0^1\end{aligned}\tag{18}$$

At first one remarks that g_1 , g_2 and g_3 evolve quite differently, g_1 increasing with energy whereas g_2 and g_3 decrease, the latter faster than g_2 . It is therefore to be expected that for some large value of M , these couplings, very different at $\mu \cong 10$ GeV, will be of the same order of magnitude, and this suggests the possibility of a unification of these three different interactions with quantization of electromagnetic charge. For instance, one sees that α_2 and α_3 will satisfy

$$\alpha_2(M) = \alpha_3(M)$$

for a value of M defined by

$$\alpha_2^{-1}(\mu) - \alpha_3^{-1}(\mu) = \frac{11}{6\pi} \text{Log } M/\mu\tag{19}$$

Equation (19) cannot be used at once to fix the value of M in terms of μ . If $\alpha_3(\mu)$ is a somewhat well-known quantity that can be derived from QCD this is not immediately true of $\alpha_2(\mu)$. At present moderately low energies [~ 10 GeV], the only exact symmetries are the $SU(3)$ colour for strong interaction and the $U(1)$ for electromagnetic phenomena. However, it is strongly believed that around 100 GeV, weak and electromagnetic interactions get unified with an $SU(2) \times U(1)$ symmetry. A popular model for this symmetry is the Weinberg-Salam ⁷⁾ model (WS) and it is so far supported by all experimental data on this topic, so that it does not take special courage to say that the WS model is the "right" one. Thus one can express α_2 in terms of $\alpha_{em} = e^2/4\pi$ and the weak mixing angle θ_W . One has

$$\alpha_2^{-1} = \sin^2 \theta_w / \alpha_{em} \quad (20)$$

$\sin^2 \theta_w$ is of course also μ dependent and it has been measured in neutral current experiments at an energy of about $\mu \ll 100$ GeV.

Its value is not yet firmly established and an average of all available results gives ⁸⁾

$$\sin^2 \theta_w (100 \text{ GeV}) = 0.23 \pm 0.01. \quad (21)$$

On the other hand, knowing $\alpha_{em}^{-1}(0) = 137.0$ one can, using the β function for QED with sharp thresholds of the θ function type, calculate the value of $\alpha_{em}^{-1}(100 \text{ GeV})$. One starts at $Q^2 = 4m_e^2$ and one considers the contributions of the different steps to α_{em}^{-1} (Table I).

Taking $\alpha_{em}^{-1}(2m_e) \cong 137$, one has

$$\alpha_{em}^{-1}(2m_w) \cong 137.0 - 8.9 = 128.1$$

This estimation is very crude but agrees rather well with more refined analyses done recently ⁹⁾. We know therefore the value $\alpha_2^{-1}(2m_w)$. If we choose $\sin^2 \theta_w = 0.2$, then Eq. (19) gives $M = 7.25 \times 10^{15}$ GeV. For $\sin^2 \theta_w = 0.21$, $M = 9.2 \times 10^{16}$ GeV, and for $\sin^2 \theta_w = 0.22$, $M = 8.2 \times 10^{17}$ GeV. The sensitivity of M to the value of $\sin^2 \theta_w$ is enormous. More refined estimations taking into account vector boson thresholds and two-loop β functions reduce appreciably the above crude evaluation and give $M \cong 4.8 \times 10^{14}$ GeV, but for $\sin^2 \theta_w = 0.19$ ⁹⁾. The exact value of M will be of interest later. But now, more interesting is the common value α_{GUM} taken both by $\alpha_2(M)$ and $\alpha_3(M)$ which is

$$\alpha_{GUM} = 0.024 = 1/41.7$$

By considering the order of magnitude of the energy at which the two coupling constants become equal, one sees that 10^{15} GeV is an extremely high temperature which has only been reached in the far past of the Universe, that is to say a very short time after its birth ; at least if we believe in the scenario of a big bang.

E. - THE GRAND UNIFICATION

Up to now we have not considered the constant $\alpha_1(\mu)$ and its evolution. Enlarging the frame of our assumptions (WS model) α_1 gets related to α_{em} and α_2 if we assume that the $SU(3)$, $SU(2)$ and $U(1)$ groups are subgroups of a larger group G ¹⁰⁾

$$\alpha_{em}^{-1}(\mu) = \alpha_2^{-1}(\mu) + C^2 \alpha_1^{-1}(\mu) \quad (22)$$

where C is a parameter depending on the grand group chosen for unification [in fact $C = (T_3 - Q)/T_0$ where $\vec{T}$ and T_0 are respectively the $SU(2)$ and $U(1)$ generators and Q the charge]. Having $\sin^2 \theta_W(\mu)$, $\alpha_{em}(\mu)$ and $\alpha_3(\mu)$ as input, one can eliminate $\log M/\mu$ of Eq. (19) with the help of

$$\alpha_1^{-1}(\mu) - \alpha_2^{-1}(\mu) = \frac{11}{3\pi} \log M/\mu \quad (23)$$

and writing $\alpha_1^{-1} = (\cos^2 \theta / \alpha_{em} C^2)$, using (22), and have an equation for C^2 . Taking as before $\alpha_{em}^{-1}(100) = 127$, $\alpha_3(100) = 0.139$ and $\sin^2 \theta_W(100) = 0.20$, C^2 turns out to be equal to 1.65 which is very close to 5/3. But if $\sin^2 \theta_W(100) = 0.21$, then $C^2 = 1.52$ and for $\sin^2 \theta_W(100) = 0.22$, $C^2 = 1.42$. So either G is $SU(5)$ and the present measurements of $\sin^2 \theta_W(100)$ suffer from some systematic error, or the measurements can be trusted at face value and one has to deal with another group which, perhaps, contains $SU(5)$ as a subgroup.

A lot has been written on $SU(5)$ chosen as the grand group ¹¹⁾ and very interesting results have come out which suggest that it has some rôle to play in the unification. But on the other hand there are also some conflicting results which could be better solved, though not destroying its nice features, in larger groups and especially by proceeding to the grand unification by steps like, for instance ¹²⁾:

$$\begin{array}{ccc} SU(3) \times SU(2) \times U(1) & \xleftarrow{m} & SU(4) \times SU(2) \times U(1) \\ & & \uparrow M_x \\ & & O(10) \end{array} \quad (24)$$

or 13)

$$\begin{array}{ccc}
 SU(3) \times SU(2)_L \times U(1) & \xleftarrow{M_R} & SU(3) \times SU(2)_L \times SU(2)_R \times U(1) \\
 & & \uparrow M_x \\
 & & E_6
 \end{array} \quad (25)$$

In these particular cases, it is possible to accommodate a theoretical value of $\sin^2 \theta_W$ (Fig. 2) which, through renormalization group equations, decreases from $3/8$ at M_x (the grand unification mass) down to a $\sin^2 \theta(100)$ which agrees with present experimental data (Table II).

One sees that the procedure of unification is perhaps not as simple as going directly from $SU(3) \times SU(2) \times U(1)$ to $SU(5)$. But whatever this procedure is, it will have to pass all the tests shown above. Namely going from the simple grand group G values of $\sin^2 \theta_W$ down to the observed $\sin^2 \theta_W(100)$ through the renormalization group equations. Moreover the β^i functions for the couplings of $SU(3)$, $SU(2)$ and $U(1)$ must be such that they provide an evolution which brings the three coupling parameters to a common value α_{GUM} , the unique coupling parameter of the simple group G . And this at a value of $M \gtrsim (2-4) \times 10^{14}$ GeV. This is necessary if one wishes that the proton lifetime does not become smaller than the present experimental lower bound of $\sim 2 \cdot 10^{30}$ years¹⁴⁾. This is because once grand unification arises, quarks and leptons are to be put in the same multiplets and there will be interactions changing quarks into leptons via the baryon number violating forces gauged by the bosons of mass M_x . Therefore the proton becomes instable and one can show that its lifetime τ_p is proportional to M_x^4 . A theoretical estimation of τ_p is :

$$\tau_p \sim K \cdot 10^3 M_x^4 / m_p^5 \quad (26)$$

K being of the order of unity.

With a value of $M_x \cong (2-4) \times 10^{14}$ GeV one gets $\tau_p = 10^{31 \pm 2}$ years.

We will soon end our travel back to the very early time of the Universe. But we shall, before concluding, have a look on the evolution of the masses.

F. - FERMION MASSES

Masses in perturbative quantum field theory can be considered on the same footing as the coupling parameters ¹⁵⁾. One is led to consider "effective" or "running" masses $\bar{m}_i(t)$ which evolve according to an equation of the type

$$\partial/\partial t \text{Log } \bar{m}(t) = \gamma_m(\bar{g}(t)) \quad (27)$$

where γ_m is a mass anomalous dimension similar to the γ encountered for multiplicatively renormalizable quantities. Its definition comes from the fact that masses can get renormalized by finite renormalization z_m such that $m_R = z_m m_{R'}$ (R and R' are two different renormalization prescriptions). In our rescaling of $\mu \rightarrow \mu' = \mu e^{-c}$, z_m is a function of the parameter c and

$$\gamma_m = \partial/\partial c \log z_m(c) \big|_{c=0}, \quad z_m(0) = 1 \quad (28)$$

When one has several interactions like in our cases, fermion masses receive contributions from the $SU(3)$, $SU(2)$ and $U(1)$ types of interaction. They have been computed and one can write ¹⁶⁾

$$\gamma_{m_f} \hat{=} \gamma_{m_f}^{(3)} + \gamma_{m_f}^{(2)} + \gamma_{m_f}^{(1)} \quad (29)$$

with, at the one-loop approximation and neglecting mass corrections

$$\gamma_{m_f}^{(3)} = \begin{cases} -\frac{2}{\pi} \alpha_3 & \text{for quarks} \\ 0 & \text{for leptons} \end{cases}$$

$$\gamma_{m_f}^{(1)} = \begin{cases} -\frac{1}{16\pi} \alpha_1 & \text{for } u, c, t \text{ quarks} \\ \frac{7}{80\pi} \alpha_1 & \text{for } d, s, b \text{ quarks}^*) \\ -\frac{33}{80\pi} \alpha_1 & \text{for } e, \mu, \tau \text{ leptons} \end{cases} \quad (30)$$

*) $\gamma_{m_f}^{(1)} = 5\alpha_1(80\pi)^{-1}$ for neutrinos.

Using the values of the one-loop β functions for α_1 , α_2 and α_3 , one has the following solutions ¹⁶⁾ for the various $m_{if}(\mu)$

$$\frac{m_{d,s,b}(\mu)}{m_{e,\mu,\tau}(\mu)} = \frac{m_{d,s,b}(M_x)}{m_{e,\mu,\tau}(M_x)} \left[\frac{\alpha_3(\mu)}{\alpha_3(M_x)} \right]^{\frac{12}{33-2f}} \cdot \left[\frac{\alpha_1(\mu)}{\alpha_1(M_x)} \right]^{\frac{3}{2f}}$$

$$\frac{m_{u,c,t}(\mu)}{m_{d,s,b}(\mu)} = \frac{m_{u,c,t}(M_x)}{m_{d,s,b}(M_x)} \left[\frac{\alpha_1(\mu)}{\alpha_1(M_x)} \right]^{-9/20f} \quad (31)$$

If the grand group G gives simple relations between quarks and fermion masses, then one can use them as an input in the ratios (31) at M_x and get their values at present energies, i.e., those values which are experimentally determined nowadays. For instance, if $m_b(M_x) = m_\tau(M_x)$ one gets $m_b(10 \text{ GeV}) = 5.2 \text{ GeV}$ ^{*}). Depending from the models of symmetry breaking and the grand group chosen, zeroth order mass ratios (i.e., at M_x) can be chosen in different ways ^{**}). As an example, within the $O(10)$ model ¹²⁾ one can account for several quark to lepton mass ratios (m_t is predicted to be 13-14 GeV, for instance)

G. - SYMMETRY BREAKING

Having gone back in time close to the Planck time, $t_p = \hbar/KT_p \cong \cong 10^{-43}$ sec after the birth of the Universe, we have to come back now at our starting point, namely at energies of the order 10-100 GeV. During this return journey, we shall observe the most noteworthy features that we encounter. At the Planck temperature T_p , we suppose that grand unification has taken place, there is one single coupling (we neglect gravitation which has not entered the game yet). The particles are classified in generations, i.e., they have been put in well-defined representations of the grand group G [in $5+10$ for the $SU(5)$ case]. The coupling parameter increases as we decrease T by several orders of magnitude. In this region, aside the fermions, there are also scalar particles present, the so-called Higgs particles. We then attend to the breaking of the G symmetry which will

^{*}) $m_q(\mu)$ is always computed in such a way that $\mu_{th} = 2m_q(\mu_{th})$, μ_{th} being the approximate position of the $q\bar{q}$ threshold.

^{**}) Depending the way we choose to distribute the fermions in the various representations of G .

take place by same mechanism. As discussed in Ref. 17) for $SU(5)$, the breaking can arise through radiative corrections. The quartic Higgs couplings, which are assumed to be of order g^2 at $M_p = 1.2 \times 10^{19}$ GeV, decrease and when one of these couplings (more precisely one of the two adjoint Higgs couplings) has a zero, the symmetry breaks. This will occur rather rapidly in the $SU(5)$ model, when Q will be of the order $10^{-4} - 10^{-5}$ times the Planck mass, i.e., $\lambda_{adj}(Q_c) = 0$ when Q_c is around 10^{15} GeV. In $SU(5)$, according to which of the two adjoint couplings vanishes first, then the grand group will break into $SU(4) \times U(1)$ or into $SU(3) \times SU(2) \times U(1)$. The first way of breaking is undesired but nothing compelling forces $SU(5)$ to break into the other way. Concretely,

Meanwhile, the vector Higgs couplings $\left[\begin{smallmatrix} 5 \\ \bar{5} \end{smallmatrix} \right]$ in $SU(5)$ evolve much more slowly than the adjoint which have a large non-Abelian charge (see Fig. 3).

After passing $Q_c = 10^{15}$ GeV, nothing occurs in $SU(5)$, because these fundamental couplings evolve even more slowly below Q_c , in part because large masses have been generated for some particles which therefore do not contribute to the renormalization group equations below Q_c . $\lambda_{vect}(\mu)$ can be arranged to vanish for $\mu = 0(10^2)$ GeV¹⁸⁾. Therefore between the time corresponding to $T_c = 10^{15}$ GeV and that corresponding to $T_0 = 10^2$ GeV we have crossed a real desert. At T_0 , $\lambda_{vect}(T_0) = 0$ (see Fig. 4) and

$$SU(3) \times SU(2) \times U(1) \longrightarrow SU(3) \times U(1)$$

This is the second symmetry breaking generating the W and Z masses and exhibiting finally the symmetry which we left at the beginning of our trip, namely the $SU(3) \times U(1)$ characteristic of our current energies of a few GeV. Of course, as Eqs. (24) and (25) show, in other schemes, something may occur between 10^{15} and 10^2 GeV. While the first symmetry breaking (which is not necessarily due to radiative corrections) occurs somewhere in between the Planck mass and 10^{15} GeV, it is followed by a second step

of breaking, somewhere between 10^{15} GeV and 10^2 GeV ^{*)}. And then breaks finally to $SU(3) \times U(1)$ at $T_0 = O(10^2)$ GeV. This makes the return journey a little less boring than it was for $SU(5)$ above.

As a conclusion, one may say that we have discovered a strange world ($M_x < Q < M_p$) where all gauge particles are governed by a single coupling parameter ; where scalar particles are in great abundance and where no stable baryonic matter exists. Moreover, the interactions of the scalar particles with the fermions create forces which violate also a C and CP symmetry. They might very well be a source of the residual baryons which are the constituents of our present Universe ; this Universe where we have returned and where we live again now.

The author wishes to thank John Ellis for the numerous and enlightening discussions.

^{*)} In (24), $M_x = O(M_p)$ and $M_R > 3M_L$, with $M_L = m_W$. In (25), $M_x = O(10^{15})$ GeV and 10^9 GeV $< m < M_x$.

Q^2 range	Contributions to α_{em}^{-1}
$4m_e^2 - 4m_\mu^2$	-1.132
$4m_\mu^2 - 0.5 \text{ GeV}^2$	-0.513
$0.5 \text{ GeV}^2 - 12 \text{ GeV}^2$	-1.349
$12 \text{ GeV}^2 - 10^2 \text{ GeV}^2$	-1.425
$10^2 \text{ GeV}^2 - 10^3 \text{ GeV}^2$	-1.629
$10^3 \text{ GeV}^2 - 4m_W^2$	-2.835
Total for $4m_e^2 - 4m_W^2$	-8.883

TABLE I - The contributions to the finite renormalization of α_{em}^{-1} between $4m_e^2$ and $4m_W^2$ (with step functions).

M_R/M_L	$\sin^2 \theta_W$	$\alpha_s (10 \text{ GeV})$
10	0.274	0.093
10^2	0.266	0.102
10^3	0.259	0.113
10^4	0.251	0.128
10^5	0.244	0.144
10^6	0.236	0.169
10^7	0.229	0.199
10^8	0.221	0.251
10^9	0.214	0.328
10^{10}	0.206	0.487
10^{11}	0.199	0.869
10^{12}	0.191	
10^{13}	0.184	

TABLE II - Plots of M_R/M_L versus $\sin^2 \theta_W$ and of $\sin^2 \theta_W$ versus $\alpha_s (10 \text{ GeV})$. (M_x is taken to be $1.2 \times 10^{19} \text{ GeV}$ and $M_L = 80 \text{ GeV}$).

SPECIAL FOOTNOTES

- f1) The fact that one restricts oneself to continuous groups results from the fact that non-parametrized renormalization prescriptions $R, R', R'', R'''\dots$ do not form a group ¹⁹⁾. This is due to the absence of a rule for multiplying $R \rightarrow R'$ and $R'' \rightarrow R'''$, which, however, can be obtained for special subsets of prescriptions which can be explicitly parametrized.

- f2) A well-known application has been made in the calculations of the muon anomalous magnetic moment for the coefficients of the powers of $\log(m_\mu^2/m_e^2)$. See Ref. 20).

- f3) Gell-Mann and Low ²¹⁾ have been the first to give explicitly the first two terms of $h_e(\psi)$ in their notation) in an expansion in e^2 , in the limit of vanishing electron mass. They have shown that this limit exists in all orders of perturbation expansion.

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FIGURE CAPTIONS

Figure 1 Qualitative picture of the evolution of the $SU(3)$, $SU(2)$ and $U(1)$ couplings in a one-step grand unified theory such as $SU(5)$.

Figure 2 M_x and m (in GeV) and the ratio m_b/m_τ (at 10 GeV) for $f=6$ and 8, plotted versus $\sin^2\theta_W$ for the subgroup $SU(4) \times SU(2) \times U(1)$ of $O(10)$. $\Lambda = 0.2$ GeV.

Figure 3 The two adjoint (λ_M and λ_C) and the fundamental (λ_D) couplings. Their average slope between $Q = 2 \cdot 10^{14}$ GeV and $Q = 2 \cdot 10^{19}$ GeV are plotted. These are solutions of a system of coupled renormalization group equations. λ_M and λ_D have been normalized at the same value for $Q = 2 \cdot 10^{14}$ in order to show the difference in average slope [SU(5) case].

Figure 4 Qualitative picture of the Q evolution of the effective Higgs couplings of fundamental and adjoint representations in the $SU(5)$ model.

fig 1

fig 2

fig 3

fig 4