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STRONGLY FIRST ORDER PHASE TRANSITIONS IN GUTs

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ABSTRACT

In a GUT model where the symmetry breaking of the cold theory is of Coleman-Weinberg type, the hot theory shows a strong first order phase transition at high temperatures. The value of the order parameter at T_c is comparable with its value at zero temperature. If the symmetry breaking is driven by means of a negative mass squared term, then the transition is strongly or weakly first order, depending on the region of the coupling constants plane that one is exploiting. This state of affairs is demonstrated by taking the standard SU(5) as a prototype model.

A good measure of the size of a first order transition should be the ratio, δ , of the latent heat L to the jump in the specific heat, ΔC_p . A second order phase transition occurs with zero latent heat and a finite ΔC_p . It is therefore clear that δ is a measure by how much a first order transition differs from a second order one. When δ is small, the transition very much resembles a second order one and is called weakly first order. When δ is large the transition is strongly first order.

In Ref. 1) a criterion was introduced whereby the strength of the transition was measured in terms of the supercooling involved. Although in a strongly first order transition we could have a large amount of latent heat as well as strong supercooling, this might not always be the case. It is pointed out in Ref. 2) that it is possible to have small supercooling with a big jump in the order parameter, i.e., a big jump in the vacuum energy density. In such a case δ would be large and the transition strongly first order. We would like to show that in any GUT model of the $SU(N)$ or $O(N)$ type in the Coleman-E. Weinberg (CW) mode, the jump in the phase transition at high temperature is big, resulting in a strongly first order transition. Recently Linde³⁾ has advocated that the phase transitions in GUTs are almost always strongly first order.

Consider any GUT model of the type of an $SU(N)$ or $O(N)$ gauge theory (with gauge coupling maintained asymptotically free) in the CW mode. The effective potential of the theory in the tree approximation is given by

$$U_t(\phi) = U(0) + \frac{1}{4!} \lambda_{ijkl}(\kappa) \phi^i \phi^j \phi^k \phi^l \quad (1)$$

We assume that cubic terms are absent. $\lambda_{ijkl}(\kappa)$ are the renormalized quartic couplings at some renormalization scale κ . For a general value of κ , the only stationary point of $U(\phi)$ is $\phi_c = 0$. A non-trivial minimum arises if and only if⁴⁾ there exists a scale κ_0 , so that

$$\min_{|\vec{n}|^2=1} \{ \lambda_{ijkl}(\kappa_0) \phi^i \phi^j \phi^k \phi^l \} = 0 \quad (2)$$

If we assume that the minimum of (2) occurs in a direction $\vec{n}_0$, then the minimum of the potential occurs close to the line $\phi^i = n_0^i \phi$ and the magnitude of the minimum, ϕ_c , has to be determined by the one-loop terms which give⁵⁾

$$U(\phi) = U(0) + A_0 \phi^4 \left[\ln \left(\frac{\phi}{\kappa_0} \right) - \frac{25}{12} \right] \quad (3)$$

Note that

$$\left. \frac{\partial^4 U}{\partial \phi^4} \right|_{\phi = \kappa_0} = 0$$

in order to retain the definition of the scalar couplings

$$\lambda_{ijkl}(\kappa) = \left. \frac{\partial^4 U}{\partial \phi_i \partial \phi_j \partial \phi_k \partial \phi_l} \right|_{\phi = \kappa}$$

to one-loop order. The minimum of the potential occurs⁵⁾ at

$$\phi_c = \kappa_0 \exp \left(\frac{11}{6} \right) \quad (4)$$

Thus the scale of the order parameter, ϕ_c , at the symmetry breaking point is set by the renormalization scale κ_0 .

The SU(5) model in the CW mode was analyzed in Ref. 6). In this model

$$U_t(\Phi) = \lambda_M(\kappa) \left[\text{Tr } \Phi^4 - \frac{7}{30} (\text{Tr } \Phi^2)^2 \right] + \lambda_C(\kappa) \left[\frac{13}{20} (\text{Tr } \Phi^2)^2 - \text{Tr } \Phi^4 \right]$$

where Φ is the 24 Higgs multiplet and

$$\begin{aligned} \lambda_M &= \frac{3}{50} (10a + 13b) \\ \lambda_C &= \frac{1}{25} (15a + 7b) \end{aligned} \quad (5)$$

in terms of the conventional⁷⁾ a and b quartic couplings. If we demand the SU(5) to break into SU(3) × SU(2) × U(1) phase, then the effective potential

must have a zero in the direction $\Phi \propto \text{diag}(1,1,1,-3/2,-3/2)$ at a scale κ_0 , so that $\lambda_C(\kappa_0) = 0$. The renormalization group equations governing the evolution of the coupling constants are given by

$$\frac{1}{g^2} \frac{dR_C}{dt} = \frac{1}{8\pi^2} \left[\frac{52}{5} R_C^2 + \frac{56}{15} R_M^2 - \frac{4}{5} R_C R_M - (30+b_0) R_C + 15 \right] \quad (6)$$

$$\frac{1}{g^2} \frac{dR_M}{dt} = \frac{1}{8\pi^2} \left[9 R_C^2 + \frac{22}{3} R_M^2 - 3 R_C R_M - (30+b_0) R_M + \frac{45}{2} \right]$$

where

$$R_C = \frac{\lambda_C}{g^2}, \quad R_M = \frac{\lambda_M}{g^2}$$

In deriving Eq. (6) we have used Eqs (4.2a) and (4.3b) of Ref. 6) as well as

$$\frac{dg^2}{dt} = -\frac{1}{8\pi^2} b_0 g^4 \quad (7)$$

where $b_0 = 97/6$ ⁸⁾. If we assume that $R_C(\kappa)$ and $R_M(\kappa)$ are given at some scale $\kappa = M$ then we can compute the scale κ_0 at which λ_C vanishes. For instance, if we set $\lambda_M = 0$ (for the sake of simplicity), then we can integrate analytically the remaining differential equations and we find that λ_C vanishes at⁵⁾

$$\kappa_0 = M \exp \left(-\frac{16\pi^2}{g^2(M)} F(R_C(M)) \right) \quad (8)$$

where

$$F(\xi) = -\frac{1}{2b_0} \left[\exp \left(-2b_0 \int_0^\xi \frac{dR}{\frac{104}{5} R^2 - (60+2b_0)R + 30} \right) - 1 \right] \quad (9)$$

How does a GUT behave at high temperatures? According to the analysis of Ginsparg⁹⁾, in order to understand the critical behaviour of such a theory at finite temperature, we have to study an effective gauge theory in three dimensions

obtained from the same gauge theory but coupled only to its scalar sector. The fermion sector gets integrated away. Moreover at the tree approximation the scalar sector acquires a temperature dependent mass term.

The critical behaviour of such 3D theory can be analysed within the ϵ -expansion. In a sense, a 4D theory whose properties are well understood is used as a basis for an ϵ -expansion towards the 3D theory of interest. Consequently, given a GUT, we have to study a $(4-\epsilon)$ D theory with an effective potential of the form⁹⁾

$$U(\phi) = U(0) + \frac{m^2}{2} \phi^2 + \frac{A(\epsilon)}{\epsilon} \phi^{4-\epsilon} \left[\left(\frac{\phi}{\hat{\kappa}_0} \right)^\epsilon a(\epsilon) - 1 \right] \quad (10)$$

where

$$a(\epsilon) = (1-\epsilon) \left(1 - \frac{\epsilon}{2}\right) \left(1 - \frac{\epsilon}{3}\right) \left(1 - \frac{\epsilon}{4}\right)$$

This choice of $a(\epsilon)$ ensures that

$$\left. \frac{\partial U}{\partial \phi^4} \right|_{\phi=\hat{\kappa}_0} = 0$$

The above potential has a minimum at a value ϕ_c of ϕ which is of order $\hat{\kappa}_0$. It is straightforward to show that the jump in the order parameter at the phase transition is given by⁹⁾

$$\phi_c = \left[\frac{(1 - \frac{\epsilon}{2})}{a(\epsilon)} \right]^{1/\epsilon} \hat{\kappa}_0 \quad (11)$$

Taking $SU(5)$ as the prototype, the renormalization group equations governing the trajectories in the $(4-\epsilon)$ D theory are given by

$$\begin{aligned} \frac{d\lambda_c}{dt} &= \epsilon \lambda_c + \frac{1}{8\pi^2} \left[\frac{52}{5} \lambda_c^2 + \frac{56}{15} \lambda_M^2 - \frac{4}{5} \lambda_c \lambda_M - 30 \lambda_c g^2 + 15 g^4 \right] \\ \frac{d\lambda_M}{dt} &= \epsilon \lambda_M + \frac{1}{8\pi^2} \left[9 \lambda_c^2 + \frac{22}{3} \lambda_M^2 - 3 \lambda_c \lambda_M - 30 \lambda_M g^2 + \frac{45}{2} g^4 \right] \\ \frac{dg^2}{dt} &= \epsilon g^2 - \frac{1}{8\pi^2} \hat{b}_0 g^4 \end{aligned} \quad (12)$$

The $\hat{b}_0$ in (12) is different from the b_0 in (7), because the fermion contribution to the β_g function of the initial 4D theory has been eliminated in the 3D theory. In fact, for the $SU(5)$ ⁸⁾ $\hat{b}_0 = 105/6 > b_0$.

Now in terms of the R_C and R_M variables we have the following set of equations

$$\begin{aligned} \frac{1}{g^2} \frac{dR_C}{dt} &= \frac{1}{8\pi^2} \left[\frac{52}{5} R_C^2 + \frac{56}{15} R_M^2 - \frac{4}{5} R_C R_M - (30 + \hat{b}_0) R_C + 15 \right] \\ \frac{1}{g^2} \frac{dR_M}{dt} &= \frac{1}{8\pi^2} \left[9 R_C^2 + \frac{22}{3} R_M^2 - 3 R_C R_M - (30 + \hat{b}_0) R_M + \frac{45}{2} \right] \end{aligned} \quad (13)$$

These are the same as the set of equations (6) except that b_0 is replaced by $\hat{b}_0$. The form of Eq. (13) is in fact generic. For the general GUT model we have (in matrix notation)

$$\frac{1}{g^2} \frac{dR}{dt} = \frac{1}{8\pi^2} [A R^2 + (B - \hat{b}_0) R + C]$$

with

$$R_{ijkl} = \frac{\lambda_{ijkl}}{g^2}$$

We are now ready to compare $\phi_c(T_c)$ with its value, $\phi_c(T=0)$, at $T=0$. It follows from (4) and (11) that

$$\frac{\phi_c(T_c)}{\phi_c(T=0)} \sim \frac{\left[\frac{(1-\frac{\epsilon}{2})}{a(\epsilon)} \right]^{1/\epsilon}}{\exp(11/6)} \cdot \frac{\hat{\kappa}_0}{\kappa_0} \quad (14)$$

Since the differential Eqs (6) and (13) governing the flows are identical in both cases apart from the " b_0 " term, it is likely that the scales κ_0 and $\hat{\kappa}_0$ are comparable to within 30%. Indeed, within the $SU(5)$ model, it can be checked that values of $F(R_C(M))$, [see Eq. (9)] (for reasonable values of $R_C(M)$), computed with a b_0 and a $\hat{b}_0$ are different by a few parts in ten thousand. If this situation holds for the generic case, then we can give an estimate of $\phi_c(T_c)/\phi_c(T=0)$.

For $\epsilon \rightarrow 0$

$$\left[\frac{(1-\frac{\epsilon}{2})}{a(\epsilon)} \right]^{\frac{1}{\epsilon}} \rightarrow \exp(-19/12)$$

and in this limit we have

$$\frac{\phi_c(T_c)}{\phi_c(T=0)} \sim \exp(-1/4) \times 1.3$$

Consequently

$$\phi_c(T_c) \sim \phi_c(T=0) \quad (15)$$

Of course, instead of $\epsilon \rightarrow 0$ we must consider $\epsilon \rightarrow 1$, because the physics of interest occurs at $\epsilon = 1$. Unfortunately we cannot take this limit. However, for small finite values of ϵ it can be checked that the relation (15) holds.

Equation (15) shows that the jump in the order parameter at the transition temperature is comparable with its value at zero temperature. In all GUTs the order parameter at zero temperature is always very big. We can therefore conclude that the phase transition at high temperatures in any GUT model in the CW mode is almost always strongly first order.

We would like now to consider the explicit finite temperature calculations in the SU(5) model carried out in Ref. 1). The results of the calculations can be summarized as follows: the equilibrium equation

$$\left. \frac{dU}{d\phi} \right|_{\phi=\phi_c} = 0$$

where U is the effective potential at finite temperature T , yields

$$A\phi_c^2 - BT\phi_c + CT^2 - D = 0 \quad (16)$$

where

$$\begin{aligned} A &= \frac{15}{4} (15a+7b) \\ B &= \frac{1}{4\pi} \left[\frac{7}{2} (15a+7b)(10b)^{1/2} + 4(10b)^{3/2} + \frac{3}{2} (15a+7b)^{3/2} + \frac{1125\sqrt{2}}{8} g^3 \right] \\ C &= \frac{1}{12} \left[\frac{225}{2} g^2 + 13(15a+7b) + 50b \right] \\ D &= \frac{15}{2} \mu^2 \end{aligned} \quad (17)$$

where a and b are the conventional self couplings of the 24 Higgs multiplet, and μ^2 sets up the scale at which the transition takes place. In fact

$$\mu^2 = \frac{1}{2} (15a + 7b) \phi_c^2(T=0) \quad (18)$$

The scale μ^2 has been put in because we drive the symmetry breaking in terms of a negative mass squared term of the form $-\mu^2/2 \text{Tr } \phi^2$.

It is important to comment on the validity of (16). This equation arises from one-loop considerations and is valid whenever the limit of metastability of the disordered phase lies outside the critical region [see Ref. 1) for details]. This means that we must have sufficient supercooling so that the limit of metastability lies below the Ginzburg temperature. Recall that for temperatures too close to the critical temperature, T_c , standard perturbation expansion breaks down in spite of the smallness of the various couplings. Hence higher order terms become important and calculations derived from one-loop considerations are not in general reliable.

Let T_1 be the limit of metastability of the disordered phase $\phi_c = 0$. At $T = T_1 = \sqrt{D/C}$ we have

$$\phi_c(T_1) = 0$$

or

$$\phi_c(T_1) = \sqrt{\frac{D}{C}} \frac{B}{A} \quad (19)$$

Hence at $T = T_1$ we have a jump $\phi_c(T_1) \sim \sqrt{D/C} B/A$. The limit of metastability of the ordered phase is also given by

$$T_2 = T_1 \left[1 - \frac{B^2}{4AC} \right]^{-1/2} \quad (20)$$

The amount of supercooling that occurs during the transition can be measured in terms of the quantity $\Delta = (T_2 - T_1)/T_1$. In the region where (16) is valid, Δ has a simple expression

$$\Delta = \left[1 - \frac{B^2}{4AC} \right]^{-1/2} - 1 \quad (21)$$

It can be checked that within the validity of the calculations of Ref. 1) Δ takes values in the region $2 \times 10^{-1} - 5 \times 10^{-2}$. Now for $\Delta \sim (10^{-1})$ (21) implies

$$B^2 \sim AC \quad (22)$$

Moreover from Eqs (17)-(19) we obtain

$$\frac{\phi_c(T_1)}{\phi_c(T=0)} = \frac{B}{\sqrt{AC}}$$

As a consequence of (22) we deduce that

$$\phi_c(T_1) \sim O(1) \phi_c(T=0) \quad (23)$$

which implies that the jump from the supercooled symmetric phase to the asymmetric one is large, and the transition tends to be strongly first order. This occurs as we approach the boundaries for the stability region of the $SU(3) \times SU(2) \times U(1)$ phase. On the other hand for $\Delta \sim O(10^{-2})$ we get

$$\phi_c(T_1) \sim O(0.3) \phi_c(T=0) \quad (24)$$

and the transition tends to be weakly first order. This latter case occurs as, roughly speaking, we move away but parallel to the line $15a + 7b = 0$.

It is instructive to check this picture by doing the calculation in a different manner. The $SU(5)$ model with a negative mass squared term is equivalent at the critical temperature to a 3D theory with an effective potential which has vanishing mass term. The temperature dependent mass term of the scalar sector of the 3D theory is cancelled by $-(\mu^2/2) \text{Tr } \Phi_0^2$ (where Φ_0 is a field in three dimensions) induced by the corresponding $-(\mu^2/2) \text{Tr } \Phi^2$ of the original 4D theory. More explicitly, the scalar sector of the 3D theory has a tree approximation mass term given by

$$\left(\frac{T}{g_0^2} [13(15a+7b)+50b+\frac{225}{2}g^2] - \mu^2 \right) \int_{\vec{p}} \Phi_0^i(\vec{p}) \Phi_0^i(-\vec{p})$$

So, at

$$T \sim \frac{g_0 \mu^2}{\left[\frac{225}{2} g^2 + 13(15a+7b) + 50b \right]}$$

we have an effective 3D theory with vanishing mass term.

Consider now the effective potential of the corresponding $(4-\varepsilon)D$ theory. This is given by

$$U(\phi) = U(0) + \frac{A(\varepsilon)}{\varepsilon} \phi^{4-\varepsilon} \left[\left(\frac{\phi}{\kappa_0} \right)^\varepsilon a(\varepsilon) - 1 \right]$$

with a non-trivial minimum at

$$\phi_c = \left[\frac{(1 - \frac{\varepsilon}{4})}{a(\varepsilon)} \right]^{1/\varepsilon} \kappa_0 \quad (25)$$

κ_0 must be the scale at which λ_C vanishes because we demand that the breaking of the $SU(5)$ be in the $SU(3) \times SU(2) \times U(1)$ phase. Moreover we have from (5) and (18) that

$$\phi_c(T=0) = \frac{\sqrt{2} \mu}{\sqrt{25} \lambda_C(\mu)} \quad (26)$$

Therefore in order to compare the jump in the order parameter at the transition point, we have the task of comparing μ with κ_0 . In Ref. 1) the analysis was carried out in a region of the coupling constants' plane where $R_C(\mu) = \lambda_C(\mu)/g^2(\mu)$ roughly takes values between 0.16 and 0.04. The value $R_C = 0.04$ corresponds to the region close to the line $15a + 7b = 0$, i.e., $\lambda_C = 0$.

Let us consider the two ends of the allowed region for R_C separately. With $R_C(\mu) = 0.04$ as an input we can calculate the scale κ_0 at which $R_C(\kappa_0) = 0$. Again if we set $\lambda_M = 0$ we get from (9) $F(0.04) = 1.28 \times 10^{-3}$. Equation (8) now yields $\kappa_0 = 0.44\mu$. Consequently from (25) we deduce, in the $\varepsilon \rightarrow 0$ limit, that

$$\phi_c(T_c) = \exp(11/6) \times 0.44\mu \sim 2.78\mu \quad (27)$$

whereas Eq. (26) yields

$$\phi_c(T=0) = 2.83 \mu \quad (28)$$

Consequently $\phi_c(T_c) \sim \phi_c(T=0)$ and the transition is expected to be strongly first order. So as we approach the boundary $15a + 7b = 0$ for the stability region of the $SU(3) \times SU(2) \times U(1)$ phase, there is a tendency for the phase transition to become strongly first order.

In the other end of the region where $R_C(\mu) = 0.16$, we have $F(0.16) = 6.7 \times 10^{-3}$ and

$$\phi_c(T_c) \sim 0.09 \mu$$

Comparing this with $\phi_c(T=0)$ calculated for this case we finally get

$$\phi_c(T_c) \sim 0(0.06) \phi_c(T=0)$$

It is clear that the transition tends to become weakly first order in agreement with the results of Ref. 1).

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