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FOUR-JET EVENTS IN e^+e^- COLLISIONS AND THE
NON-ABELIAN NATURE OF QCD

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ABSTRACT

Suitably defined four-jet cross-sections are calculated in standard QCD and in an Abelian vector gluon model. We discuss various distributions that might show a difference between the two models, but the differences are so small that current experiments will not be able to exclude one model in favour of the other.

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1. INTRODUCTION

The possibility of the production of three-jet events in e^+e^- collisions due to the bremsstrahlung of gluons was already recognized a few years ago¹⁾. Several groups at DESY have now reported evidence for three-jet events²⁾⁻⁵⁾. According to these groups, the underlying dynamics is consistent with the QCD predictions. Additionally, QCD predicts the presence of four-jet events and there have been reports on the possible observation of four-jet events⁶⁾.

In view of all this, it will be interesting to try to separate a sample of four-jet events from all the hadronic events produced in e^+e^- collisions. The interest lies in the fact that in QCD the production has to include diagrams that contain an essential feature of QCD, namely the three gluon coupling. The importance of the four-jet cross-section has already been indicated by the authors of Refs 7) and 8). They studied various distributions in the framework of perturbative QCD. Their lowest order quark-gluon production cross-sections have been folded with realistic hadron fragmentation functions so that a direct confrontation with experiment should be possible. It is the purpose of this paper to compare QCD directly with a model in which the gluons do not have self-couplings.

For this purpose we use an Abelian version of QCD with which QCD can be compared. In Ref. 8) a comparison of total cross-sections is made on the basis of the two models. It turns out that the total four-jet cross-section is somewhat different in the two models. This is due to the rather artificial constraints one has to impose on the Abelian model. We do not think that measuring the total cross-section for four-jet events can serve as a useful discrimination between an Abelian or a non-Abelian theory. If, on the other hand, one studies normalized distributions of, for instance, the invariant mass of a two-jet sub-system, it turns out (disappointingly) that there is no obvious difference between Abelian and non-Abelian QCD. Only if one can distinguish gluon jets from quark jets on an event-by-event basis and with 100% efficiency is there a possibility of observing differences.

The outline of this paper is as follows. In Section 2 we give details of the calculation of the matrix element squared. The actual calculation has been done using the computer program SCHOONSCHIP⁹⁾. We pay attention to the problems of gauge invariance. We also introduce a simplifying method for the calculation of distributions of final state particles. Finally, in Section 3 we compare several distributions and averages for the two models from which we conclude that the differences are negligible if one cannot distinguish gluon-jets from quark-jets which seems to be the case at presently available energies. It is therefore likely that scale-breaking effects in gluon fragmentation are a better place to look for effects from the three-gluon vertex¹⁰⁾.

2. DETAILS OF THE CALCULATION

We will assume that the strong interactions between quarks are mediated by massless vector particles (V). Under this assumption there are several possibilities for the $q\bar{q}V$ couplings. One choice is the one made in QCD, where the quarks are in a triplet representation of $SU(3)_{\text{colour}}$ and the vector particles are the gauge bosons of this group. This choice has the big advantage of leading to asymptotic freedom.

Another choice (also renormalizable) for the coupling is one in which the vector particles are coupled to the quarks in a flavour and colour independent way and in which they do not have any self-couplings. We will call this the Abelian model.

In the context of e^+e^- physics we have the following constraints on the Abelian model:

- i) the magnitude of the two-jet cross-section must agree with experiment. This requires that each quark flavour has a three-fold multiplicity. Otherwise, the value of R would not be reproduced;
- ii) the magnitude of the three-jet cross-section must also agree with experiment. We know that the observed three-jet cross-section agrees with the QCD prediction.

In order to obtain the $q\bar{q}g$ cross-section in QCD we have to calculate the QED process $e^+e^- \rightarrow \mu^+\mu^-\gamma$ (where only the muons radiate) and make the substitution¹¹⁾:

$$\alpha^2 \rightarrow \sum_F 4\alpha\alpha_s Q_F^2 \quad (1)$$

Here $\alpha_s = g_s^2/4\pi$ is the strong fine structure constant, Q_F is the electric charge of the quark with flavour F . If we introduce the Abelian analogue of α_s , α_A we find the $q\bar{q}V$ cross-section by the substitution:

$$\alpha^2 \rightarrow \sum_F 3\alpha\alpha_A Q_F^2 \quad (2)$$

Comparing expressions (1) and (2) we see that we must choose:

$$\alpha_A = \frac{4}{3}\alpha_s \quad (3)$$

In all numerical calculations we will use:

$$\alpha_s = \frac{12 \pi}{(33 - 2 N_f) \ln s/\Lambda^2} \quad (4)$$

with $\Lambda = 500$ MeV and N_f the number of active flavours which we take equal to five.

Since we are primarily interested here in possible differences between Abelian and non-Abelian gluons we will calculate cross-sections depending only on the final state kinematics. Any angular dependence of the final state with respect to the e^+e^- beams is averaged over. We do this effectively by calculating the decay of a heavy unpolarized photon of mass $\sqrt{s}$.

For a final state f containing n particles we find the expression

$$\begin{aligned} \sigma(e^+e^- \rightarrow f) = & - \frac{8\pi^2 \alpha^2}{3 s^2} \sum_{\text{pol. } f} \langle 0 | J_\mu | f \rangle \langle f | J^\mu | 0 \rangle. \\ & \cdot (2\pi)^4 \delta^4(P_f - p_+ - p_-) \frac{d^3 q_1}{(2\pi)^3 2q_{10}} \dots \frac{d^3 q_n}{(2\pi)^3 2q_{n0}} \end{aligned} \quad (5)$$

We will first use this expression to calculate the differential cross-section for $e^+e^- \rightarrow q\bar{q}g$ in QCD according to the diagrams of Figure 1. The current matrix element (for one flavour) is given by:

$$\begin{aligned} \langle q_i(p_1), \bar{q}_j(p_2), g_a(p_3) | J_\mu | 0 \rangle = \\ e g_s Q_F \frac{\lambda_{ij}^a}{2} \bar{u}(p_1) \left[\not{\epsilon}(p_3) \frac{1}{\not{p}_1 + \not{p}_3 + i\epsilon} \gamma_\mu - \gamma_\mu \frac{1}{\not{p}_2 + \not{p}_3 + i\epsilon} \not{\epsilon}(p_3) \right] v(p_2) \end{aligned} \quad (6)$$

If we introduce the variables:

$$\epsilon_i = \frac{2 p_i \cdot Q}{Q^2} \quad (7)$$

with $Q = p_1 + p_2 + p_3$ we can easily find:

$$\frac{d^2 \sigma}{d\epsilon_1 d\epsilon_2} = Q_F^2 \frac{8 \alpha^2 \alpha_s}{3 s} \frac{\epsilon_1^2 + \epsilon_2^2}{(1 - \epsilon_1)(1 - \epsilon_2)} \quad (8)$$

In the derivation we have used the fact that for the generators of SU(3) in the triplet representation we have:

$$\text{Tr} \left(\frac{\lambda^a}{2} \frac{\lambda^a}{2} \right) = 4 \quad (9)$$

To zeroth order in α_s we have:

$$\sigma_0 (e^+e^- \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{s} Q_F^2 \quad (10)$$

so that:

$$\frac{1}{\sigma_0} \frac{d^2\sigma}{d\epsilon_1 d\epsilon_2} = \frac{2\alpha_s}{3\pi} \frac{\epsilon_1^2 + \epsilon_2^2}{(1-\epsilon_1)(1-\epsilon_2)} \quad (11)$$

We will now proceed with the calculation of the cross-section for $e^+e^- \rightarrow q\bar{q}gg$. In QCD there are eight contributing diagrams (Figure 2). For the Abelian model the last two diagrams are absent. Indicating the contribution of diagram (a) to $\langle q\bar{q}gg | J_\mu | 0 \rangle$ as $J_\mu^{(a)}$ etc., we have:

$$\begin{aligned} J_\mu^{(a)} &= e Q_F g_s^2 \frac{\lambda_{jk}^b}{2} \frac{\lambda_{ki}^a}{2} \bar{u}(p_3) \not{\epsilon}(p_1) \frac{1}{\not{q}_1} \not{\epsilon}(p_2) \frac{1}{\not{q}_2} \gamma_\mu v(p_4) \\ J_\mu^{(b)} &= e Q_F g_s^2 \frac{\lambda_{jk}^b}{2} \frac{\lambda_{ki}^a}{2} \bar{u}(p_3) \not{\epsilon}(p_1) \frac{1}{\not{q}_1} \gamma_\mu \frac{1}{\not{q}_3} \not{\epsilon}(p_2) v(p_4) \\ J_\mu^{(c)} &= e Q_F g_s^2 \frac{\lambda_{jk}^b}{2} \frac{\lambda_{ki}^a}{2} \bar{u}(p_3) \gamma_\mu \frac{1}{\not{q}_4} \not{\epsilon}(p_1) \frac{1}{\not{q}_3} \not{\epsilon}(p_2) v(p_4) \\ J_\mu^{(d)} &= e Q_F g_s^2 \frac{\lambda_{jk}^a}{2} \frac{\lambda_{ki}^b}{2} \bar{u}(p_3) \not{\epsilon}(p_2) \frac{1}{\not{q}_5} \not{\epsilon}(p_1) \frac{1}{\not{q}_2} \gamma_\mu v(p_4) \\ J_\mu^{(e)} &= e Q_F g_s^2 \frac{\lambda_{jk}^a}{2} \frac{\lambda_{ki}^b}{2} u(p_3) \not{\epsilon}(p_2) \frac{1}{\not{q}_6} \gamma_\mu \frac{1}{\not{q}_6} \not{\epsilon}(p_1) v(p_4) \end{aligned} \quad (12)$$

$$J_{\mu}^{(f)} = e Q_F g_s^2 \frac{\lambda_{jk}^a}{2} \frac{\lambda_{ki}^b}{2} \bar{u}(p_3) \gamma_{\mu} \frac{1}{\not{q}_4} \not{p}_2 \frac{1}{\not{q}_6} \not{p}_1 v(p_4)$$

$$J_{\mu}^{(g)} = ie Q_F g_s^2 \frac{\lambda_{ij}^c}{2} \bar{u}(p_3) \gamma^{\alpha} \frac{1}{\not{q}_2} \gamma_{\mu} v(p_4) f^{abc} \frac{1}{q_7^2}$$

$$V_{\alpha e \sigma}(q_7, p_1, p_2) \epsilon^e(p_1) \epsilon^{\sigma}(p_2) \quad (12) \text{ cont.}$$

$$J_{\mu}^{(h)} = ie Q_F g_s^2 \frac{\lambda_{ij}^e}{2} \bar{u}(p_3) \gamma_{\mu} \frac{1}{\not{q}_2} \gamma^{\alpha} v(p_4) f^{abc} \frac{1}{q_7^2}$$

$$V_{\alpha e \sigma}(q_7, p_1, p_2) \epsilon^e(p_1) \epsilon^{\sigma}(p_2)$$

where:

$$\begin{aligned} q_1 &= p_1 + p_3 \\ q_2 &= p_1 + p_2 + p_3 \\ q_3 &= -p_2 - p_4 \\ q_4 &= -p_1 - p_2 - p_4 \\ q_5 &= p_2 + p_3 \\ q_6 &= -p_1 - p_4 \\ q_7 &= -p_1 - p_2 \end{aligned} \quad (13)$$

and V is the three-gluon vertex given by:

$$V_{\alpha e \sigma}(q_7, p_1, p_2) = g_{e \sigma} (p_1 - p_2)_{\alpha} + g_{\sigma \alpha} (p_2 - q_7)_{\epsilon} + g_{\epsilon \alpha} (q_7 - p_1)_{\sigma} \quad (14)$$

It is precisely this coupling whose presence we would like to establish.

We now indicate an important property of the sum of expressions (12). If we replace $\epsilon_p(p_1)$ everywhere by $p_{1\rho}$ we get zero, provided we take for $\epsilon_{\sigma}(p_2)$ a physical polarization vector for which $p_2^{\sigma} \epsilon_{\sigma}(p_2) = 0$. We do not have this gauge invariance property for gluon 1 if the polarization vector for gluon 2 is not transverse. If we write:

$$\langle q\bar{q}gg | J_\mu | 0 \rangle = A_{\mu e\sigma}(p_1, p_2, p_3, p_4) \epsilon^\mu(p_1) \epsilon^\sigma(p_2) \quad (15)$$

we can show using (12) that

$$\begin{aligned} A_{\mu e\sigma} p_1^\mu &\sim p_{2\sigma} \\ A_{\mu e\sigma} p_2^\sigma &\sim p_{1\mu} \end{aligned} \quad (16)$$

These residual terms find their origin in the vertex (14). If we take instead of (14) the following form for the vertex,

$$V'_{\alpha e\sigma}(q, p_1, p_2) = g_{e\sigma}(p_1 - p_2)_\alpha + 2 p_{2e} g_{\sigma\alpha} - 2 p_{1\sigma} g_{e\alpha} \quad (17)$$

the amplitude for physical polarizations remains the same whereas we now have

$$A'_{\mu e\sigma} p_1^\mu = A'_{\mu e\sigma} p_2^\sigma = 0 \quad (18)$$

independent of what is contracted with the other gluon index. We may now, thanks to this property, calculate cross-sections by making the polarization sum:

$$\sum_{pol} \epsilon_\mu \epsilon_\nu \Rightarrow -g_{\mu\nu} \quad (19)$$

for both gluons. This modification simplifies the calculation of the cross-section considerably. By this method we avoid using ghosts as in Ref. 8). It is to be noted that in the Abelian model we do not have this complication due to the absence of diagrams 2(g) and 2(h). Also in the Abelian model $\lambda^a/2$ is to be replaced everywhere with the unit matrix. We would like to point out further that it is important to be sure of gauge invariance in a theory where all particles are massless, since incomplete gauge cancellations will not show up in a spurious high energy behaviour of cross-sections calculated from tree graphs. The reason is the lack of a parameter with dimension which causes such cross-sections to scale dimensionally.

In the further calculation of cross-sections we need the following identities for SU(3):

$$\begin{aligned}
\frac{1}{16} \text{Tr}(\lambda^a \lambda^a \lambda^b \lambda^b) &= \frac{16}{3} \\
\frac{1}{16} \text{Tr}(\lambda^a \lambda^b \lambda^a \lambda^b) &= -\frac{2}{3} \\
\frac{i}{8} \text{Tr}(\lambda^a \lambda^b \lambda^c) f^{abc} &= -6 \\
\frac{(i)^2}{4} \text{Tr}(\lambda^a \lambda^b) f^{acd} f^{bcd} &= -12 \\
\frac{1}{4} \text{Tr}(\lambda^a \lambda^b) \frac{1}{4} \text{Tr}(\lambda^a \lambda^b) &= 2
\end{aligned} \tag{20}$$

The contributions due to the diagrams of Figure 3 can be added separately to the cross-section since there is no interference with the diagrams of Figure 2.

The final state phase space for four particles is given by the usual

$$(2\pi)^4 \delta^4(p_1 + p_2 + p_3 + p_4 - Q) \frac{d^3 p_1}{(2\pi)^3 2p_{10}} \frac{d^3 p_2}{(2\pi)^3 2p_{20}} \frac{d^3 p_3}{(2\pi)^3 2p_{30}} \frac{d^3 p_4}{(2\pi)^3 2p_{40}} \tag{21}$$

We can reduce this formula by introducing the invariant mass of p_1 and p_2 which we call M_{12} , the polar angles Ω_1^* of p_1 in the c.m. frame of $p_1 + p_2$ and the energies E_3 and E_4 of p_3 and p_4 in the over-all c.m. system. With these variables (21) takes the form

$$\frac{2}{(4\pi)^6} d\Omega_1^* dE_3 dE_4 dM_{12}^2 \tag{22}$$

We close this section with the remark that the Abelian model as constructed here is probably quite artificial with respect to the couplings and hence with respect to the numerical values of the four-jet cross-sections. We feel, however, that it is important to establish a signature for the coupling (13) and that we therefore have to compare QCD with a theory where this coupling is absent.

3. RESULTS AND DISCUSSION

Using the formalism described in Section 2 we can calculate four-jet differential cross-sections. It is, of course, not possible to calculate total four-jet cross-sections due to infra-red divergences and mass singularities. We can, however, impose cuts on certain variables which allow us to define cross-sections which are finite. The four four-momenta of the quarks and gluons are labelled as follows: p_1, p_2 gluons, p_3 quark and p_4 antiquark. For the diagram of Figure 3 p_1 and p_2 also correspond to a quark and antiquark, respectively. Using these four-momenta we define the quantities:

$$x_{ij} = 2 \frac{p_i \cdot p_j}{s} \quad (i < j) \quad (23)$$

In the case of infra-red or collinear divergences at least one of the six variables x_{ij} goes to zero. If we impose a cut on all x_{ij} simultaneously, we find a finite cross-section. We will use a parameter ξ to describe these cuts by:

$$x_{ij} \geq \xi^2 \quad (24)$$

In this way we can define a finite four-jet cross-section as a function of ξ , where in the case $\xi \rightarrow 0$ the cross-section diverges. It is also clear that in the limit $\xi \rightarrow 0$ the $q\bar{q}gg$ system will present itself as three, or even two jets.

If we use σ_0 as defined in Eq. (8) as a reference cross-section, we can calculate the ratios $R_4 = \sigma_{4\text{-jet}}(\xi)/\sigma_0$ and $R_3 = \sigma_{3\text{-jet}}(\xi)/\sigma_0$. In Figure 4 we plot R_4 as a function of ξ both for QCD and the Abelian gluon model. We also give R_3 which by construction is identical for the two models. The calculation has been done at $\sqrt{s} = 30$ GeV, $\Lambda = 500$ MeV and five active flavours. The only thing that will change if we vary s is a slow logarithmic variation with s due to the dependence of α_s on s . With the parameters used here $\alpha_s = 0.20$. Due to its definition, R_3 is independent of the number of flavours. The value of R_4 is not independent of the number of flavours due to the diagrams of Figure 3.

From the graphs in Figure 4 it is clear that for small values of ξ the three-jet and the four-jet cross-sections may become larger than the two-jet cross-section. This is, of course, an artefact of the use of lowest order perturbation theory. If we use $\xi = 0.25$ and $\sqrt{s} = 30$ GeV we may be confident that the lowest order

results make sense. In the sequel all calculations will be done with these values for ξ and $\sqrt{s}$. This means that all invariant masses of two-jet systems are larger than 7.5 GeV.

We can now calculate the normalized distributions of the variables $\sqrt{x_{ij}}$. If we assume, however, that it is not possible to distinguish quark and gluon jets, the ordering described above expression (23) cannot be made experimentally. The only thing that can be done then is to order jets according to energy. If we follow this prescription, $\sqrt{x_{12}s}$, for instance, is the invariant mass of the two-jet system formed by the two most energetic jets etc. We have calculated the normalized distributions of the variables $\sqrt{x_{ij}}$ in this way, both for the Abelian and for the non-Abelian model. It turned out that the distributions were not distinguishable within the accuracy of the Monte Carlo calculation which was such that the errors on the total cross-sections were about 0.5%. If we distributed the four jets according to phase space, the distributions looked different. In Figure 5 we show $1/\sigma_{4j} d\sigma_{4j}/d\sqrt{x_{12}}$ as a function of $\sqrt{x_{12}}$ for QCD, the Abelian model and phase space.

We can see that the QCD and Abelian curves fall more or less on top of one another. The distributions for the other $\sqrt{x_{ij}}$ show much the same features and we will therefore not present them here.

It should be noted at this point that if there had been a marked difference between QCD and the Abelian model it would have been necessary to fold the distributions with the hadronic fragmentation functions in order to see how much of the effect would remain observable in a real experiment. Since we find, however, that there is hardly any difference at the quark and gluon level, this folding is unnecessary.

We will now look at another variable which characterizes the possible presence of four jets in the final state, the acoplanarity A . The acoplanarity is defined by varying a plane to determine the minimal value of the momenta components perpendicular to it¹²⁾

$$A = 4 \min \left[\frac{\sum_i |\vec{p}_{i\perp}|}{\sum_i |\vec{p}_i|} \right]^2 \quad (25)$$

In Figure 6 we give the A distribution again for QCD, the Abelian model and phase space. As we see, the curves for QCD and the Abelian model hardly differ, whereas the phase space curve can be easily distinguished.

In the Table we give averages for various kinematical quantities. The errors given are due to the finite statistics of the Monte Carlo method used. From this Table we see also that there is very little difference between the two models which we compare.

Table

	QCD	Abelian Model	Phase Space
$\sqrt{x_{12}}$	0.571 ± 0.001	0.576 ± 0.002	0.550 ± 0.002
$\sqrt{x_{13}}$	0.451 ± 0.002	0.451 ± 0.002	0.450 ± 0.001
$\sqrt{x_{14}}$	0.346 ± 0.002	0.345 ± 0.002	0.358 ± 0.001
$\sqrt{x_{23}}$	0.346 ± 0.002	0.345 ± 0.002	0.358 ± 0.001
$\sqrt{x_{24}}$	0.327 ± 0.001	0.324 ± 0.001	0.333 ± 0.002
$\sqrt{x_{34}}$	0.317 ± 0.001	0.315 ± 0.001	0.324 ± 0.001
ϵ_1	0.663 ± 0.001	0.667 ± 0.002	0.646 ± 0.001
ϵ_2	0.566 ± 0.002	0.569 ± 0.002	0.554 ± 0.001
ϵ_3	0.435 ± 0.002	0.432 ± 0.002	0.446 ± 0.001
ϵ_4	0.337 ± 0.001	0.333 ± 0.002	0.354 ± 0.001
θ_{12}	142.7 ± 0.3	142.7 ± 0.3	137.8 ± 0.3
θ_{13}	117.8 ± 0.3	117.5 ± 0.3	117.3 ± 0.4
θ_{14}	97.3 ± 0.3	97.1 ± 0.3	100.3 ± 0.3
θ_{23}	91.1 ± 0.3	90.7 ± 0.4	94.8 ± 0.3
θ_{24}	100.9 ± 0.3	100.0 ± 0.4	101.3 ± 0.4
θ_{34}	118.8 ± 0.4	119.3 ± 0.5	115.7 ± 0.4
Acoplanarity	0.164 ± 0.002	0.167 ± 0.002	0.183 ± 0.002
Thrust	0.758 ± 0.001	0.756 ± 0.001	0.745 ± 0.001

So far we have seen that, if it is not possible experimentally to distinguish a quark jet from a gluon jet, one cannot distinguish the two vector gluon models. We will now assume that it is possible to tell the difference between quark and gluon jets, event by event. In this case it is possible to distinguish final states due to the diagrams of Figure 3, from those of Figure 2. We will now use the numbers 1 and 2 for the gluons and 3 and 4 for quark and antiquark, respectively.

Looking again at the $\sqrt{x_{12}}$ distribution for the two models (Figure 7), we see that there is a difference between the two curves. The QCD distribution at small $\sqrt{x_{12}}$ is larger than the Abelian distribution. This effect is mainly due to the gluon propagator of Figures 2f and 2g.

Under the same assumptions we give in Figure 8 the distribution of the opening angle between the two gluons. As is to be expected, the average opening angle for the QCD reaction is smaller than for the Abelian case. The difference is of the order of 15° .

Finally, in Figure 9 we give the opening angle distribution between the quark and antiquark. Here also we see a difference between the two models. From Figs 7 - 9 it looks as if gluon identification will allow distinction between QCD and the Abelian model. One should realize, however, that as explained before, it is now necessary to fold the distributions with the fragmentation functions. Although we have not done this here, it is clear that this will diminish the effect. Also, a not completely reliable gluon identification will make the effect smaller. One should keep in mind also that the total useful four-jet cross-section is of the order of 2 pb at $\sqrt{s} = 30$ GeV and $\xi = 0.25$.

From the calculations presented here it is clear that it will be difficult to establish the gauge theory nature of QCD by looking at real four-jet events. The difference between QCD and the Abelian model becomes visible in a region where the gluon propagators of Figure 2g and 2h become large, which means small ξ . But then, as we have seen, perturbation theory to lowest order is no longer reliable. In the limiting case where the invariant mass of the two-gluon system becomes very small, one is effectively looking at gluon fragmentation which can be studied beyond lowest order perturbation theory¹⁰⁾

If one can distinguish gluons from quarks experimentally, the prospects are better. We will probably have to wait, however, for e^+e^- machines of much higher centre-of-mass energies than presently available.

REFERENCES

- 1) J. Ellis, M.K. Gaillard and G. Ross, Nucl. Phys. B111 (1979) 253.
- 2) R. Brandelik et al., Phys. Lett. 86B (1979) 243 (TASSO Collaboration).
- 3) D.P. Barber et al., Phys. Rev. Lett. 43 (1979) 830 (MARK-J Collaboration).
- 4) Ch. Berger et al., Phys. Lett. 86B (1979) 418 (PLUTO Collaboration).
- 5) S. Orito, talk presented at the Lepton-Photon Conference (Chicago, 1979).
DESY report 79-77 (JADE Collaboration)
- 6) H. Newman, talk presented at CERN (MARK-J Collaboration).
- 7) A. Ali et al., Phys. Lett. 82B (1979) 285.
- 8) A. Ali et al., DESY preprint 79-54.
- 9) H. Strubbe, Comp. Phys. Comm. 8 (1974) 1.
- 10) K. Koller, T. Walsh and P. Zerwas, Phys. Lett. 82B (1979) 263.
- 11) See A. De Rújula, J. Ellis, R. Petronzio, G. Preparata and W. Scott, CERN
preprint TH. 2778, top of p. 29.
- 12) A. De Rújula, J. Ellis, F.G. Floratos and M.K. Gaillard, Nucl. Phys. B138
(1978) 387.

Figure captions

- Fig. 1 : Feynman diagrams for $q\bar{q}g$ production.
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- Fig. 3 : Feynman diagrams for $q\bar{q}q\bar{q}$ production.
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for the two models.

FIG. 1

FIG. 2

FIG. 3

FIG. 5

FIG. 7

FIG. 8

FIG. 9