EUROPEAN ORGANIZATION FOR NUCLEAR RESEARCH

CERN/EP 80-100

12 June 1980

STUDY OF THE REACTIONS $\pi^- p \rightarrow K^0 \Lambda$ AND

$\pi^- p \rightarrow K^0 \Sigma^0$ AT 3.95 GeV/c

CERN-Collège de France-Madrid-Stockholm Collaboration

P.F. Loverre, R. Armenteros, C. Dionisi(*), Ph. Gavillet, M.J. Losty(**)
and M. Mazzucato

CERN, European Organization for Nuclear Research, Geneva, Switzerland

D. Pennino(*) and L. Dobrzynski
Collège de France, Paris, France

M.R. Pennington(***)
Department of Physics, University of Durham, Durham, U.K.

M. Aguilar Benitez, M.C. Albajar, A. Ferrando, J.A. Rubio and J. Salicio
Junta de Energia Nuclear, Madrid, Spain

I. Sjögren, S. Rodebäck and S.O. Holmgren
Department of Physics, University of Stockholm, Stockholm, Sweden(+)

ABSTRACT

The reactions $\pi^- p \rightarrow K^0 \Lambda$, $K^0 \Sigma^0$ are studied at an incident momentum of 3.95 GeV/c using data from a high statistics bubble chamber experiment corresponding to ~ 90 ev/ μ b. The differential cross sections and hyperon polarizations are presented and compared with existing data from earlier electronic experiments. The data in the forward hemisphere are used to perform an amplitude analysis of the $0^- 1/2^+ \rightarrow 0^- 1/2^+$ hypercharge exchange processes.

Submitted to Zeitschrift für Physik C

(*) Present address: Rome University, Rome, Italy.

(**) Present address: National Research Council, Ottawa, Canada.

(***) Supported by the Science Research Council.

(+) Work supported by the Swedish Natural Science Research Council.

1. INTRODUCTION

As part of a systematic analysis of final states containing at least one seen strange particle decay, we present the results of a study of the associated production reactions

$$\pi^- p \rightarrow K^0 \Lambda \quad (1)$$

and $\pi^- p \rightarrow K^0 \Sigma^0 \quad (2)$

observed at a momentum of 3.95 GeV/c in a hydrogen bubble chamber.

The total and differential cross sections and the $\Lambda(\Sigma^0)$ polarizations are given over the complete angular production range after having paid particular care in the determination of the statistical and systematic uncertainties affecting the data.

The analysis of the data is mainly concentrated in the t region below 1.4 GeV² ($-t$ being the squared four-momentum transferred from the π^- to the K^0) where they are incorporated in a phenomenological study of the amplitudes governing the $0^- 1/2^+ \rightarrow 0^- 1/2^+$ hypercharge exchange processes.

Our measurements are compared with those obtained with higher total statistics in earlier electronic experiments [1,2] each covering only a restricted t range; good agreement will be found in the shapes of the pertaining differential cross sections. The smaller systematic uncertainty in our data leads us nevertheless to use only them in the amplitude analysis. It is also used to resolve an existing discrepancy between the two other experiments which have measured the integrated backward cross section ($u < 3$ GeV², $-u$ being the squared four-momentum transferred from the π^- to the Λ) for reaction (1).

The amplitude analysis assumes, amongst other things, that $I = 3/2$ t channel exchanges are negligible. This assumption we confirm by confronting our $K^0 \Sigma^0$ differential cross section both with data published [3,4,5] for the isospin related reaction $\pi^+ p \rightarrow K^+ \Sigma^+$ at the same momentum and with data obtained in our experiment for the $\pi^- p \rightarrow K^+ \Sigma^-$ reaction.

The two reactions under study are related to the line-reversed processes $K^-p \rightarrow \pi^0\Lambda$ and $K^-p \rightarrow \pi^-\Sigma^+$ for which data from the ACNO collaboration are available [6]. Additional constraints are supplied by SU(3). We use the data for $t < 1.4 \text{ GeV}^2$ to extract information about the intervening vector and tensor amplitudes and the (F/D) ratios for the coupling of an octet of mesons to an octet baryon-antibaryon vertex.

The plan of the paper is the following: in sect. 2 the general characteristics of the experiment are given together with a description of the way the final data have been selected. The determination of the total and differential cross sections and hyperon polarizations is explained in sect. 3, where the comparison with data from other experiments is also made. In sect. 4 the amplitude analysis is presented.

2. DATA

The data come from a high statistics experiment (90 events/ μb) consisting of several exposures of the CERN 2 m hydrogen bubble chamber to π^- beams of momenta: 3.93 GeV/c for 50% of the data, 3.96 GeV/c for 20% and 4.01 GeV/c for the remaining 30%. In each case the momentum dispersion of the beam was $\Delta p/p = \pm 0.5\%$.

Results corresponding to about 15 events/ μb at 3.93 GeV/c have already been published [7]. They are included here, where for simplicity we consider the data as if resulting from a single exposure at 3.95 GeV/c.

The data were obtained from the scanning of events belonging to the zero-prong 2 V^0 or the zero-prong 1 V^0 topologies. Part of the film was scanned twice and from it a scanning efficiency close to 100% was deduced for the events finally selected from analysis.

To ensure good measurability only events within a more restricted volume than that used in the scanning were retained for measurement. In this way also the contamination by off-beam pions could be reduced to a negligible level.

The retained events were measured with IEP's in Madrid and with Spiral Readers in the other laboratories; the standard CERN chain of programs was used for the geometric reconstruction and kinematical fitting of events. Events which failed in the first measurement were examined on the scanning table and if found to be measurable sent for a second measurement, after which the characteristics (nature of V^0 , its projected distance from the interaction point, for example) of failed events were recorded. While the film was on the table bubble density information was used whenever possible to distinguish between different possible mass assignments to a given track. For the reactions considered in this paper, the Λ/K^0 ambiguity was very small and the ionization criterion hardly used.

The fraction of failed events after the last measurement was: 5% for the 1 V^0 class and 9% for the 2 V^0 class. Within the limitations imposed by the small number of these events and the incomplete information available on them, we have not found in their examination any striking feature distinguishing them from the sample of accepted events. These will therefore be corrected globally for this loss.

To take into account losses due to decays very close to the interaction point, only V^0 's with a projected decay length greater than 3 mm were accepted for further analysis. This loss and that due to decays outside the fiducial volume were compensated for with the standard weighting procedure. The average weights were 1.12 for K^0 's and 1.14 for Λ 's.

Other corrections to the data are necessary which depend on the topology and final state considered. These constitute the following subsets of reactions (1) and (2).

$$\pi^- p \rightarrow (K^0)\Lambda$$

$$K^0(\Lambda)$$

$$K^0\Lambda$$

$$K^0(\Sigma^0)$$

and

$$K^0\Sigma^0$$

where the brackets indicate that the K^0 or Λ decay is not observed.

The samples in each of the above subsets are obtained from kinematic fits which frequently are not unique. An important fraction of the ambiguities arise from the very low probability level at which kinematical hypotheses are initially accepted; stricter criteria can only be safely established after a critical evaluation of the different probability distributions. In this paper the ambiguities discussed occur as long as the fit probability is bigger than 0.5%. Detection biases may also affect the samples. It is therefore necessary, on the one hand, to establish criteria which allow to resolve the ambiguities and, on the other, to detect and correct the biases. The procedures used to take into account these two facts and the uncertainties they introduce in the data will be discussed now at some length. They help not only to understand the reliability attached to the present data but serve also to illustrate the procedures more generally used in the treatment of data which will be presented in other communications from this collaboration.

2.1 $(K^0)\Lambda$ final state

The missing mass squared (MM^2) recoiling against the Λ is shown in fig. 1 for the region encompassing $MM^2 = M^2(K^0)$. A very clean K^0 signal is apparent over a small background. This comes essentially from the unavoidable contamination from the final state $\Sigma^0(K^0)$ when the effective mass of the K^0 and the γ from the Σ^0 decay fall inside the experimental K^0 distribution. It has been computed using the events of the $\Sigma^0 K^0$ channel and is $< 3\%$ after further purification of the sample has been made by accepting only events with a χ^2 fit probability better than 4% and satisfying $0.165 \text{ GeV}^2 \leq MM^2 \leq 0.335 \text{ GeV}^2$.

It is estimated that the combined effect of the two cuts eliminates about 5% of true $\Lambda(K^0)$ events.

The decay angular distribution of the Λ in its c.m.s. is shown in fig. 2, where θ is the angle made by the pion with the Λ line-of-flight. Deviations from the expected isotropy are seen near both $\cos\theta = +1$ and $\cos\theta = -1$ and correspond to losses of Λ decays giving, respectively, low

momentum protons or pions. These effects have been studied on a much larger sample of Λ 's from all topologies. This study led to the determination of a total correction of the order of 5%; its t dependence has been used in computing differential cross section.

In table 1(a) the final fully corrected number of events for this and the other subsets is given.

2.2 $K^0(\Lambda)$ final state

The only important ambiguity is with the $K^0(\Sigma^0)$ hypothesis involving 622 events i.e. about 77% of the original sample of events. For a large fraction of these events the ratio of χ^2 probabilities is large but many events remain for which this ratio is close to one. Good discrimination between the two hypotheses cannot therefore be obtained on probability grounds alone. For events in which the ratio of probabilities is much larger than one, we have studied the distributions of the pulls of the incident pion momentum for the two hypotheses. The two distributions are well separated and that for the hypothesis with the higher probability is centered at zero and has a standard deviation of about 1 as it is to be expected for events truly belonging to that hypothesis. Considering then all the events independently of the probability ratio, it is noted that the two pull distributions remain good and have only a small overlap. Fig. 3 illustrates these remarks for about 60% of the ambiguous events for which the pull information was available. The loss of true $K^0(\Lambda)$ events and the contamination from $K^0(\Sigma^0)$ events are estimated to be small after applying the cuts shown in the figure since their correct magnitudes are nevertheless difficult to calculate, we have added a 10% uncertainty to the number of events thus selected.

For the remaining 40% of the events with pull information missing, we have considered the missing mass squared, MM^2 , to the K^0 . Although distinctive peaks appear at values corresponding to the Λ and Σ^0 , there is considerable overlap between the two distributions. Only events in the lower half of the Λ distribution ($MM^2 \leq 1.242 \text{ GeV}^2$) have been attributed to the Λ hypothesis.

In table 1(a) the fully corrected number of events is given after detailing the number in the initial and selected samples.

The K^0 decay distribution in its c.m.s. for the selected sample is consistent, as seen in fig. 4, with the expected isotropy. Nevertheless, a loss of events is observed when the distribution of the angle in the laboratory between the decay plane and the optical axis of the cameras is studied. A small loss of events ($\sim 5\%$) is observed for very small angles i.e. for decay planes which are edge on to the cameras. This effect is included amongst the corrections made to the selected sample.

2.3 $K^0\Lambda$ final state

64% (527 events) of the initial sample of events fitting this hypothesis have no other successful fit; the remainder are ambiguous only with the $K^0\Sigma^0$ hypothesis when the decay gamma is very soft. This is reflected in the decay angular distribution of the Λ in the Σ^0 c.m.s. with respect to the direction of the Σ^0 in the overall c.m. This distribution for ambiguous events is compared in fig. 5 with that of the selected $K^0\Sigma^0$ sample described later. It can be seen that attributing all ambiguous events to the $K^0\Lambda$ channel carries with it a negligible contamination.

After excluding events with a χ^2 fit probability smaller than 2% the selected sample consists of 756 events.

2.4 $K^0(\Sigma^0)$ final state

156 events were unique. The others (79% of the initial sample) were ambiguous only with the $K^0(\Lambda)$ hypothesis. The resolution of this ambiguity has already been discussed and here we just remind that beam pulls were used when available and that otherwise events were attributed to the $K^0(\Sigma^0)$ hypothesis when $1.42 \leq MM^2 \leq 1.58 \text{ GeV}^2$, MM^2 being the missing mass squared to the K^0 .

The selected sample consists of 261 events (table 1(a)).

2.5 $K^0 \Sigma^0$ final state

349 events ($\sim 43\%$ of the initial sample) fitted uniquely this channel. 34% were ambiguous with the $K^0 \Lambda$ hypothesis. As described before, these events were attributed to the $K^0 \Lambda$ channel with a loss of genuine $K^0 \Sigma^0$ events estimated to be negligibly small.

There is as well an ambiguity ($\sim 23\%$ of the events) with the $1C$ channel $\Lambda K^0 \pi^0$. A combined study of the χ^2 fit probability and missing mass squared, MM^2 , to the K^0 system shows that accepting only events satisfying $|MM^2| \leq 0.015 \text{ GeV}^2$ and $P(\chi^2) \geq 0.04$ provides a good $K^0 \Sigma^0$ sample with a loss of about 2% of good events. The MM^2 distribution for the final sample is shown in fig. 6.

The distribution of the angle made by the Λ with respect to the Σ^0 direction in the Σ^0 c.m.s. is shown in fig. 5 for the 472 events in the selected sample. It is consistent with being isotropic.

The hatched events in the figure correspond to those ambiguous with the $K^0 \Lambda$ hypothesis; their extreme peaking near $\cos\theta = +1$ justifies the decision made earlier on to attribute them to the $K^0 \Lambda$ channel.

The final number of corrected events is given as usual in table 1(a).

3. TOTAL AND DIFFERENTIAL CROSS SECTIONS AND POLARIZATIONS

As shown in table 1(b), the ratios of the corrected number of events from the pertinent subsets agree with the well established branching ratios $(K^0 \rightarrow \pi^+ \pi^-)/(K^0 \rightarrow \text{neutrals})$ and $(\Lambda \rightarrow p \pi^-)/(\Lambda \rightarrow n \pi^0)$ as given by the particle data group [8]. We will therefore add without further correction the events in the different subsets to obtain the final samples for the channels $K^0 \Lambda$ and $K^0 \Sigma^0$.

The reaction cross sections were computed on the basis of the topological sensitivities. These were obtained directly on about 40% of the film from a beam count on every one out of thirty photographs. For the

remaining 60% of the film the normalization was obtained from the numbers of events in the different V^0 topologies relative to those observed in the part of the film where the beam count was made.

In addition to the statistical uncertainty in the data there is an overall systematic uncertainty (scale error), arising mainly from the normalization procedure, which we estimate to be $\sim 4\%$.

After correcting for unobserved final states and using the data of table 1(a), we obtain for the total cross sections

$$\sigma(\pi^- p \rightarrow K^0 \Lambda) = (64.4 \pm 2.8) \mu b$$

$$\sigma(\pi^- p \rightarrow K^0 \Sigma^0) = (39.7 \pm 2.2) \mu b$$

values which are higher than, although not in disagreement with, those given [7] from the earlier data of this experiment: $(58 \pm 3) \mu b$ and $(33 \pm 4) \mu b$, respectively.

The differential cross sections have been computed after normalization to the total cross sections. In the computation account has been taken of the small t dependent corrections for the loss of Λ 's decaying into low momenta pions or protons. They are given in table 2 as a function of $t' = t - t_{\min}$ ($u' = u - u_{\min}$) and displayed in figs 7(a) and 8(a).

The hyperon polarizations, P , have been determined using

$$P \pm \Delta P = \frac{1}{\alpha} \left[\frac{\langle \cos \theta \rangle}{\langle \cos^2 \theta \rangle} \pm \sqrt{\frac{3}{N}} \right] \text{ for the } \Lambda \text{ [6] and}$$

$$P \pm \Delta P = -\frac{9}{\alpha} \langle \cos \theta_1 \cdot \cos \theta_2 \rangle \pm \frac{3}{\alpha} \sqrt{\frac{1}{N}} \text{ for the } \Sigma^0.$$

with $\cos \theta = (\hat{B}_i \cdot \hat{B}_f) \cdot \hat{q}_1$, $\cos \theta_1 = (\hat{B}_i \cdot \hat{B}_f) \cdot \hat{q}_2$ and $\cos \theta_2 = \hat{q}_1 \cdot \hat{q}_2$, B_i and B_f being the initial and final baryon momenta in the c.m.s., $q_1(q_2)$ the proton (Λ) momentum in the $\Lambda(\Sigma^0)$ rest frame, α the Λ weak decay parameter and N the number of events.

We now give a brief description of figs 7 and 8 and compare at the same time our results with those obtained in three counter experiments at 4 GeV/c. The first [1] studied the reactions $K^0\Lambda$ and $K^0\Sigma^0$ up to t' values of 1.2 GeV^2 , the two others [2a,b], the $K^0\Lambda$ reaction for the u' region below 2 GeV^2 . Their $d\sigma/dt'$ ($d\sigma/du'$) distributions have been plotted^(*) together with ours in figs 7 and 8. At this energy only the Λ polarization in the backward hemisphere of the $K^0\Lambda$ reaction has been measured in these experiments [2]. It is plotted in fig. 7(b), where we have also plotted for comparison the polarization measured in the forward direction at 5.0 GeV/c by the group of ref. [1].

Although the agreement of our data with the counter experiments results appears to be very satisfactory the following comments can be made.

$K^0\Lambda$ reaction - Ward et al. [2b] have measured the backward cross section ($0.0 \leq u' \leq 0.6 \text{ GeV}^2$ for reaction (1) to be $\sigma_B = (3.07 \pm .32)\mu\text{b}$; they pointed out that this result disagrees with that of Beusch et al. [2a] who obtained $\sigma_B = (1.7 \pm 0.3)\mu\text{b}$ in the same u range. The disagreement arises from the overall normalization, the shape of the u dependence being almost the same in the two experiments. Our measurement of the same quantity gives $\sigma_B = (1.6 \pm 0.3)\mu\text{b}$ in good agreement with the result of Beusch et al. It has to be pointed out that, at variance with the electronic experiments where the computation of the overall normalization is a delicate task, our error on σ_B is essentially statistical.

The small statistical weight of our data prevents us however, from giving new information on the shape of the differential cross section in the backward region. For a discussion of the latter in terms of baryon exchange we refer the reader to e.g. [2b] and to the there included references.

(*) To facilitate visualization, we have limited in the plot the data points of refs [1,2]. This has been done by interpolating and combining to suitable t' values the published values; the errors have been properly reduced.

In the forward hemisphere the differential cross section shows a fast exponential decrease with a break at $t' \sim 0.45 \text{ GeV}^2$; this break appears to be associated with a change in the sign of the Λ polarization, which is close to -1 beyond $t' \sim 0.6 \text{ GeV}^2$. Our forward data, which covers now the complete forward hemisphere, indicates the possible existence of a small shoulder close to $t' \sim 1 \text{ GeV}^2$ and followed by a second exponential decrease of gentler slope than that in the very forward t' region.

Comparing our data in the forward hemisphere to that of ref. [1] it has to be noted that our statistical accuracy, although substantially worse at small t' values, becomes slightly better as t' increases. The overall systematic uncertainty is larger in the counter experiment ($\pm 10\%$) and it is therefore not possible to gain in precision by combining the results of the two experiments. The smaller systematic uncertainty of our data and the fact that they cover the wider t' region are the reasons why we will use them only in the amplitudes analysis to follow.

$K^0 \Sigma^0$ reaction - We note first the evidence from our data for a backward peak which is about three times smaller than that in the $K^0 \Lambda$ reaction. In the forward hemisphere the differential cross section falls exponentially down to $t' \sim 0.4 \text{ GeV}^2$ at a faster rate than in the $K^0 \Lambda$ reaction and our data shows a second exponential fall after the flattening off of the cross section in the t' region between about 0.5 and 1.5 GeV^2 . The Σ^0 polarization, of which this is the first measurement in this energy region, is consistent with being close to zero below $t' \sim 0.4 \text{ GeV}^2$ and rises to about 0.75 beyond the first break in the differential cross section. Within large errors it agrees with that measured by [3] in the reaction $\pi^+ p \rightarrow K^+ \Sigma^+$ at $4.0 \text{ GeV}/c$ as it is to be expected if $I = 1/2$ meson exchange predominates.

We will keep only our data for further analysis for the same reasons given before.

In the amplitude analysis the assumption will be made that $I = 3/2$. t channel exchanges can be neglected. Although this is a generally accepted experimental fact, we briefly describe how this is confirmed by our data by themselves or combined with $\pi^+ p \rightarrow K^+ \Sigma^+$ results.

If $I = 1/2$ and $I = 3/2$ t channel exchanges operate in forward $K\Sigma$ production, with an obvious notation, the amplitudes for the three reactions can be written:

$$A(\pi^- p \rightarrow K^0 \Sigma^0) = \frac{\sqrt{2}}{3} T^{3/2} + \frac{\sqrt{2}}{3} T^{1/2}$$

$$A(\pi^- p \rightarrow K^+ \Sigma^-) = T^{3/2}$$

$$A(\pi^+ p \rightarrow K^+ \Sigma^+) = \frac{1}{3} T^{3/2} - \frac{2}{3} T^{1/2}$$

The reaction $\pi^- p \rightarrow K^+ \Sigma^-$ has been studied in our experiment. Although the number of events observed in the forward direction is too small to determine the differential cross section, the total cross section for $t' < 2 \text{ GeV}^2$ has been found to be $(0.21 \pm 0.04) \mu\text{b}$. From it we derive the following ratio

$$\left| \frac{T^{3/2}}{T^{1/2}} \right| = 0.05 \pm 0.01.$$

In principle a variation with t of the $I = 3/2$ contribution can be studied by comparing the differential cross sections of the $K^0 \Sigma^0$ and $K^+ \Sigma^+$ reactions, which would be in the ratio 1 : 2 if only $I = 1/2$ exchange intervened. In fig. 9 we have replotted, after multiplying by 2, our data in the forward hemisphere together with the data available from different experiments [3,4,5] on the reaction $\pi^+ p \rightarrow K^+ \Sigma^+$ at 4.0 GeV/c. With the possible exception of the data very close to $t = 0$ - on which we comment below - the agreement over the complete forward hemisphere is good and consistent with the negligibly small $I = 3/2$ contribution determined above. Nevertheless, a quantitative estimate is not possible because of the systematic uncertainties in the data. As an illustration, we have chosen to represent the data of ref. [5] by the hatched area instead of by the published data points at 4.0 GeV/c. This area covers the values given in [5] for the differential cross sections at 3.75, 4.0 and 4.25 GeV/c. This was done because of the large systematic uncertainty ($\pm 20\%$) affecting these data and which manifests itself through the fact that the values at 4.25 GeV/c are, on the average, higher than at 4.0 GeV/c.

4. AMPLITUDE ANALYSIS

Having presented data on the two reactions $\pi^- p \rightarrow K^0 \Lambda$ and $\pi^- p \rightarrow K^0 \Sigma^0$ at 3.95 GeV/c, it is natural to ask what can we learn from a comparison of these with other pseudoscalar production processes involving the π , η and η' . In particular, can we determine the amplitudes for these channels? Our reactions are simply related by isospin to their line-reversed processes $K^- p \rightarrow \pi^0 \Lambda$ and $K^- p \rightarrow \pi^- \Sigma^+$, respectively, where, by factorization of crossed channel exchanges, any symmetry applies to each vertex separately. Moreover, these hypercharge exchange reactions are in turn all related by SU(3), and hence can then determine the production of the pseudoscalar octet member, η_8 . However, how the η_8 is itself connected to the physical η and η' is far from certain [9,12]. In fact as much as can be deduced from the recent data within the most simple mixing scheme [13] has already been discussed by Marzano et al. [6b]. Of course it may be that the η_8 is not just a mixture of η and η' but also of their radial excitations. This assumption led Cohen and Lipkin [12] to propose the following relation

$$\begin{aligned} \sigma(K^- p \rightarrow \Lambda \eta) + 2. \sigma(K^- p \rightarrow \Lambda \eta') &= \\ &= 0.9 \sigma(\pi^- p \rightarrow K^0 \Lambda) + 1.1 \sigma(K^- p \rightarrow \pi^0 \Lambda) \end{aligned} \quad (3)$$

which they showed to be satisfied for integrated forward cross sections. However, the use of differential cross sections leads to the result shown in fig. 10. There we have plotted the two sides of the equality as a function of t using, for the K^- reactions, the data of ref. [6b] at 4.2 GeV/c; our data on the $K^0 \Lambda$ reaction are corrected for the difference in the c.m. energy^(*). The plot clearly indicates a difference between the two members of eq. 3, also in the very forward region where the equality should work better; moreover the difference is t dependent. Thus, it turns out that it is impossible to make use in our analysis of the $\Lambda \eta$ and $\Lambda \eta'$ reactions without "ad hoc" assumptions for the mixing scheme and without the introduction of a t dependent R_T [6b] (R_T is the ratio of singlet/octet coupling $\eta_1 K K^{**} / \eta_8 K K^{**}$).

(*) The procedure adopted for the correction will be explained in connection with the amplitude analysis to follow.

We shall therefore compare our data on $\pi^- p \rightarrow K^0 \Lambda, K^0 \Sigma^0$ with just $K^- p \rightarrow \pi^0 \Lambda, \pi^- \Sigma^+$ at similar momenta: these will be sufficient to perform an amplitude analysis. High statistics data are, in fact, available at 4.2 GeV/c on the latter two reactions from our parent ACNO collaboration [6b], so these are the data we shall use. In figs 11 and 12 we plot the differential cross sections and polarizations for these four related processes for $t_{\min} < t < 1.4 \text{ GeV}^2$.

Neglecting, as experiments indicate we should, the isospin 3/2 content of the hypercharge exchange, we have two classes of isospin 1/2 exchange contributing to each of these processes, namely Tensor (T) and Vector (V) exchange. These are to be identified with the K^{**} trajectory, and its attendant daughters and cuts, and with the K^* trajectory and its appendages, respectively. For each of these two exchanges, we have two corresponding helicity amplitudes: Non flip (N) and Flip (F). In terms of these four basic amplitudes $T_i(s, t), V_i(s, t)$, with $i = N, F$, for each of the Λ and Σ reactions considered, we have the amplitudes

$$A_i(\pi^- p \rightarrow K^0 \Lambda) = \sqrt{2}[T_i^\Lambda + V_i^\Lambda]$$

$$A_i(K^- p \rightarrow \pi^0 \Lambda) = T_i^\Lambda - V_i^\Lambda$$

$$A_i(\pi^- p \rightarrow K^0 \Sigma^0) = \sqrt{2}[T_i^\Sigma + V_i^\Sigma] \quad (4)$$

$$A_i(K^- p \rightarrow \pi^- \Sigma^+) = 2[T_i^\Sigma - V_i^\Sigma]$$

for $i = N, F$. In terms of these the observables, the differential cross section and polarization, are given by

$$\frac{d\sigma}{dt} = |A_N|^2 + |A_F|^2 \quad (5)$$

$$P \frac{d\sigma}{dt} = 2 \text{Im}(A_N A_F^*).$$

SU(3) symmetry relates the exchange amplitudes for the Λ and Σ reactions in terms of the corresponding (F/D) ratios for the coupling of an octet of mesons to an octet baryon-antibaryon vertex. Conveniently defining the quantities γ_i^T, γ_i^V ($i = N, F$), in terms of the appropriate $(F/D)^{T,V}$, generically by

$$\gamma = \sqrt{3} \frac{(F/D - 1)}{(3F/D + 1)} \quad (6)$$

we have

$$T_i^\Sigma = \gamma_i^T \quad T_i^\Lambda, \quad V_i^\Sigma = \gamma_i^V \quad V_i^\Lambda \quad (7)$$

From measurements of just the differential cross section and polarization for each of these processes, the tensor and vector, non flip and flip amplitudes cannot be extracted without some assumption. It is however generally believed that to a good approximation the flip amplitudes are dominated by a simple exchange degenerate Regge pole, e.g. [14]. Making such an assumption means

$$\begin{aligned} T_F^{\Lambda, \Sigma}(s, t) &= -F^{\Lambda, \Sigma}(s, t)(1 + e^{-i\pi\alpha(t)}) \\ V_F^{\Lambda, \Sigma}(s, t) &= -F^{\Lambda, \Sigma}(s, t)(-1 + e^{-i\pi\alpha(t)}) \end{aligned} \quad (8)$$

where $F^{\Lambda, \Sigma}$ are real implying $\gamma_F^T = \gamma_F^V$. This is not expected to apply to the non flip terms because of absorption, e.g. [14]. In the present case of hypercharge exchange we use for the trajectory the form

$$\begin{aligned} \alpha(t) &= 1 + \frac{(t - m_{K^*}^2)}{K} \\ &\quad \frac{(m_{K^{**}}^2 - m_{K^*}^2)}{K} \\ &\approx 0.37 + 0.79t. \end{aligned} \quad (9)$$

since the experiment only determines the relative phases of amplitudes this assumption, (eq. 8), for the flip amplitudes fixes the overall phase of all

of them, except in the forward direction where the flip contributions kinematically vanish. We are therefore free to choose this phase and we do this by requiring the non flip amplitudes in the forward direction to be in the same quadrant as the simple Regge phases implied by eq. (8) so that^(*)

$$\begin{aligned}\phi_N^T(t=0) &\simeq \frac{\pi}{2} (2 - \alpha(0)) \\ \phi_N^V(t=0) &\simeq \frac{\pi}{2} (1 - \alpha(0)).\end{aligned}\tag{10}$$

This choice corresponds to a positive imaginary part for the forward amplitude of $\pi^- p \rightarrow K^0 \Lambda$, a process with an uncrossed duality diagram.

In previous amplitude analyses it has been assumed that some or all the F/D ratios (and hence the γ 's of eq. 6) are independent of t eg. [15, 16]. This corresponds to the assumption that the exchange amplitudes for the Σ reactions are just a constant multiple of those for the analogous Λ processes. It is now clear that there is little justification for this, particularly as in many schemes, for example in the framework of the dual topological unitarization [17], the SU(3) properties of couplings change as we move along the exchanged trajectory. What is more it can be easily seen that the constancy is not supported by the data. With the flip amplitudes being exchange degenerate, eq. (8), the ratio of cross sections at a common energy

$$\frac{2 \frac{d\sigma}{dt} (\pi^- p \rightarrow K^0 \Sigma^0) - \frac{d\sigma}{dt} (K^- p \rightarrow \pi^- \Sigma^+)}{2 \frac{d\sigma}{dt} (\pi^- p \rightarrow K^0 \Lambda) - 4 \frac{d\sigma}{dt} (K^- p \rightarrow \pi^0 \Lambda)}$$

is equal to $\gamma_N^T \gamma_N^V$. In fig. 13 this ratio is shown. It is seen that it is rather strongly t dependent. This means that we must allow the γ 's ($\gamma_N^T, \gamma_N^V, \gamma_F^{T,V}$) to be functions of t .

(*) The data determine $\cos(\phi_N^T - \phi_N^V)$ in the forward direction, so eq's (10) are only approximate; only the sector for each phase being defined.

To take account of the 5% difference in c.m. energy of the π^-p data at 3.95 GeV/c and the K^-p results at 4.2 GeV/c, we can adopt a number of options. One possibility is just to ignore this, since SU(3) symmetry is not expected to work to greater accuracy. A second alternative is to assume the energy dependences of a Regge pole model. At 4 GeV/c this will not be exact, but may be a reasonable approximation. A third option is to compare the cross sections at equal Q values as suggested by Meshkov et al. [18] and scale both $d\sigma/dt$ and $P d\sigma/dt$ by the factor $(s/9 \text{ GeV}^2) \cdot (p_{in}^*/p_f^*)$, where 9 is the squared c.m. energy of K^-p scattering at 4.2 GeV/c. In all cases these corrections are relatively small and make a negligible difference. For definiteness, the results quoted are for the Meshkov scaling factor.

A second source of energy dependence and reaction dependence at low momenta is that t_{min} is different. We shall regard the amplitudes of eq. (4) as functions of t rather than t' aside from a kinematic factor of $\sqrt{t'}$ in each of the flip amplitudes.

In fig. 14 our results are given. We show the real and imaginary parts of the tensor and vector non flip and flip amplitudes for the Λ and Σ reactions, which smoothly interpolate the differential cross sections of fig. 11 and polarizations of fig. 12^(*). Typical errors on these amplitudes, reflecting the uncertainties in the experimental data, are also shown in fig. 14. The curves in figs 11, 12 and 13 are the fits given by the smooth amplitudes of fig. 14. It is seen that they provide an excellent representation of the data on all four reactions. For both Λ, Σ reactions, the tensor non flip contribution is the largest for small t . The imaginary part of this amplitude has a distinct zero at $t \approx .15 \text{ GeV}^2$, while its real part vanishes for $t \approx 0.6 \text{ GeV}^2$. For the Σ processes, this tensor non flip amplitude is very small indeed for $t > 0.7 \text{ GeV}^2$, so that the vector component dominates the non flip amplitude there even more than for the Λ reactions. It is thus γ_N^T that has the most dramatic t dependence of the F/D ratio and is mainly responsible for the behaviour of the ratio of fig. 13.

(*) The flip amplitudes have been drawn with $t_{min} = 0$.

The signs of the γ 's are not determined, only that of the products $\gamma_N^T \cdot \gamma_N^V$, $\gamma_N^T \cdot \gamma_F^{T,V}$, which for small t are positive and negative respectively. In drawing fig. 14, we have taken $\gamma_N^T(t=0) > 0$, so that $\gamma_N^V(t) > 0$ but $\gamma_F^{T,V}(t) < 0$.

Although $\gamma_N^T(t)$ is strongly t dependent, for small t it is very close in value to that for vector exchange, γ_N^V . Thus for small t : $\gamma_N^T \simeq \gamma_N^V \simeq 0.8$, $\gamma_F^{T,V} \simeq \mp 1$, which correspond to

$$(F/D)_N^{T,V} \simeq -3.5, (F/D)_F^{T,V} \simeq 0.15$$

if $\gamma_N^T(0) > 0$, or

$$(F/D)_N^{T,V} \simeq 0.2, (F/D)_F^{T,V} \simeq -2$$

for $\gamma_N^T(0) < 0$. These are close to the values found in the analysis of Irving, Martin and Michael [15] and of Navelet and Stevens [16] who take γ_N^T to be positive. However, at larger t , for example at 0.9 GeV^2 γ_N^T vanishes, while $\gamma_N^V \pm 0.8$, $\gamma_F^{T,V} \simeq \mp 0.6$.

Although our amplitudes are in general not dissimilar to those of Irving et al. [15] and of Navelet and Stevens [16], who extracted their amplitudes by making strong assumptions about the t dependence or independence of the F/D ratios, there are significant differences, particular in the relative amounts of tensor and vector non flip components. The existence of data on all four related reactions from a single collaboration at almost the same momentum has allowed a more detailed amplitude analysis for t from 0 to 1.4 GeV^2 at $3.95 \text{ GeV}/c$.

REFERENCES

- [1] C.E.W. Ward et al., Phys. Rev. Lett. 31 (1973) 1149;
The numerical values have been taken from A. Lesnik, 1973,
University of Chicago Ph.D. Thesis ANL/HEP 7364.
- [2] (a) W. Beusch et al., Nucl. Phys. B19 (1970) 546;
(b) C.E. Ward et al., Phys. Rev. D16 (1977) 2041.
- [3] S.M. Pruss et al., Phys. Rev. Lett. 23 (1969) 189.
- [4] K.S. Han et al., Phys. Rev. Lett. 24 (1970) 1353.
- [5] A. Bashian et al., Phys. Rev. D4 (1971) 2667.
- [6] (a) G.G.G. Massaro et al., Phys. Lett. 66B (1977) 385;
(b) F. Marzano et al., Nucl. Phys. B123 (1977) 203.
- [7] M. Abramovich et al., Nucl. Phys. B27 (1971) 477.
- [8] Particle Data Group, Phys. Lett. 75B (1978) 1.
- [9] N. Isgur, Phys. Rev. D12 (1975) 3770;
N. Isgur, Phys. Rev. D13 (1975) 122.
- [10] A. de Rujula, H. Georgi and S.L. Glashow, Phys. Rev. D12 (1975)
147;
H. Fritzsch and P. Minkowski, Nuovo Cim. 30A (1975) 393.
- [11] C. Rosenzweig, Phys. Rev. D13 (1976) 3080;
S. Okubo, Phys. Rev. D16 (1977) 2336.
- [12] I. Cohen and H.J. Lipkin, Nucl. Phys. B151 (1979) 16.
- [13] G. Alexander, H.J. Lipkin and F. Scheck, Phys. Rev. Lett. 17
(1966) 412.
- [14] B. and F. Schrempp, Nucl. Phys. B96 (1975) 307.
- [15] A.C. Irving, A.D. Martin and C. Michael, Nuov. Cim. 16A
(1973) 573.
- [16] H. Navelet and P.R. Stevens, Nucl. Phys. B104 (1976) 171.
- [17] G.F. Chew and C. Rosenzweig, Phys. Rep. 41C (1978) 1.
- [18] S. Meshkov, G.A. Snow and G.B. Yodh, Phys. Rev. Lett. 12 (1964)
87.

TABLE CAPTIONS

- Table 1 (a) Number of events, referring to the subsets defined in the text, involved in the different stages of the analysis of the reactions $\pi^- p \rightarrow K^0 \Lambda$, $K^0 \Sigma^0$.
- (b) Comparison of the ratio of events from the pertinent subsets with the P.D.G. values of the $(K^0 \rightarrow \pi^+ \pi^-)/(K^0 \rightarrow \text{neutrals})$ and $(\Lambda \rightarrow p \pi^-)/(\Lambda \rightarrow n \pi^0)$ branching ratios.
- Table 2 Differential cross section and Λ polarization for the reaction $\pi^- p \rightarrow K^0 \Lambda$ at 3.95 GeV/c in both the forward and backward hemispheres.
- Table 3 Differential cross section and Σ^0 polarization for the reaction $\pi^- p \rightarrow K^0 \Sigma^0$ at 3.95 GeV/c in both the forward and backward hemispheres.

TABLE 1(a)

Number of events in successive samples

Channel	Raw ^(a)	Selected ^(b)	Final ^(c)
1. $(K^0)\Lambda$	2261	1880	2316 ± 104
2. $K^0\Lambda$	821	756	1090 ± 48
3. $K^0(\Lambda)$	810	428	638 ± 54
4. $K^0\Sigma^0$	811	472	692 ± 39
5. $K^0(\Sigma^0)$	755	261	386 ± 36

- (a) The raw number is the total number of events giving a successful fit ($P(\chi^2) > 0.5\%$) to the hypothesis considered.
- (b) The selected samples are those obtained after the selections described in the text have been applied.
- (c) These are final samples obtained after all losses and contaminations have been corrected for. It should be noted that in order to compute the branching ratios given in table 1(b), we have subtracted from channels (1), (3) and (5) the estimated number of events in which a $K^0 \rightarrow \pi^+\pi^-$ or $\Lambda \rightarrow p\pi^-$ decay occurred outside the fiducial volume.

TABLE 1(b)

	P.D.G. Value	Experimental value
$\frac{K^0 (\Lambda)}{K^0 \Lambda}$	.56	.59 $\pm$.06
$\frac{K^0 \Lambda}{(K^0)\Lambda}$	.52	.47 $\pm$.03
$\frac{K^0 (\Sigma^0)}{K^0 \Sigma^0}$	.56	.57 $\pm$.06

TABLE 2

t' range [GeV ²]	$d\sigma/dt'$ [$\mu\text{b}/\text{GeV}^2$]	t' range [GeV ²]	Polarization
0.000 - 0.025	296 $\pm$ 17	0.00 - 0.05	$+.11 \pm .11$
0.025 - 0.050	302 $\pm$ 17	0.05 - 0.10	$+.25 \pm .13$
0.050 - 0.075	221 $\pm$ 15	0.10 - 0.20	$+.13 \pm .11$
0.075 - 0.100	193 $\pm$ 14	0.20 - 0.40	$+.22 \pm .13$
0.10 - 0.15	157 $\pm$ 8	0.40 - 0.60	$-.28 \pm .21$
0.15 - 0.20	106 $\pm$ 7	0.6 - 1.0	$-1.18 \pm .20$
0.20 - 0.25	79.6 $\pm$ 5.6	1.0 - 2.0	$-.95 \pm .20$
0.25 - 0.30	55.5 $\pm$ 4.9		
0.3 - 0.4	30.7 $\pm$ 2.7		
0.4 - 0.5	25.0 $\pm$ 2.2		
0.5 - 0.6	16.8 $\pm$ 2.0		
0.6 - 0.7	13.8 $\pm$ 1.8		
0.7 - 0.8	11.9 $\pm$ 1.6		
0.8 - 0.9	11.3 $\pm$ 1.6		
0.9 - 1.0	10.1 $\pm$ 1.5		
1.0 - 1.2	8.1 $\pm$.9		
1.2 - 1.4	6.3 $\pm$.8		
1.4 - 1.6	3.9 $\pm$.6		
1.6 - 1.8	2.4 $\pm$.5		
1.8 - 2.0	2.0 $\pm$.4		
2.0 - 3.0	.6 $\pm$.2		
u' range [GeV ²]	$d\sigma/du'$ [$\mu\text{b}/\text{GeV}^2$]	u' range [GeV ²]	Polarization
0.0 - 0.1	8.7 $\pm$ 1.3	0.0 - 0.5	$.84 \pm .36$
0.1 - 0.2	3.0 $\pm$ 0.8		
0.2 - 1.0	0.6 $\pm$ 0.2		
1.0 - 3.0	0.2 $\pm$ 0.1		

TABLE 3

t' range [GeV ²]	$d\sigma/dt'$ [$\mu\text{b}/\text{GeV}^2$]	t' range [GeV ²]	Polarization
0.00 - 0.05	197 $\pm$ 16	0.0 - 0.1	-.09 $\pm$.31
0.05 - 0.10	126 $\pm$ 13	0.1 - 0.2	-.04 $\pm$.45
0.10 - 0.15	93 $\pm$ 10	0.2 - 1.0	+.68 $\pm$.42
0.15 - 0.20	57 $\pm$ 7.8	1.0 - 2.0	+.81 $\pm$.48
0.2 - 0.3	26.9 $\pm$ 3.8		
0.3 - 0.4	11.3 $\pm$ 2.4		
0.4 - 0.5	7.0 $\pm$ 1.9		
0.5 - 0.7	7.4 $\pm$ 1.4		
0.7 - 1.0	7.7 $\pm$ 1.2		
1.0 - 1.5	7.7 $\pm$ 0.9		
1.5 - 2.0	5.1 $\pm$ 0.7		
2.0 - 3.0	0.8 $\pm$ 0.2		
u' range [GeV ²]	$d\sigma/du'$ [$\mu\text{b}/\text{GeV}^2$]		
0.0 - 0.5	1.1 $\pm$.4		
0.5 - 2.85	0.12 $\pm$.05		

FIGURE CAPTIONS

- Fig. 1 Missing mass squared distribution for the reaction $\pi^- p \rightarrow \Lambda + \text{neutrals}$ at 3.95 GeV/c. The hatched area represents the estimated contribution from the $K^0 \Sigma^0$ final state. The events contributing to the $K^0 \Lambda$ final state after the selection described in the text are indicated by the dotted histogram.
- Fig. 2 C.m. decay angle distribution for the Λ from the ΛK^0 and $\Lambda(K^0)$ subsets. The hatched area refers to the $\Lambda(K^0)$ subset only.
- Fig. 3 Scatter plot of the pull of $1/p$ ($p \equiv$ beam momentum) for the $K^0(\Lambda)$ and $K^0(\Sigma^0)$ ambiguous events. The events contained in the regions defined by the hatched area are assigned to the $K^0(\Lambda)$ and $K^0(\Sigma^0)$ samples.
- Fig. 4 C.m. decay angle distribution for the K^0 from the $K^0 \Lambda$, $K^0(\Lambda)$, $K^0 \Sigma^0$ and $K^0(\Sigma^0)$ subsets. The hatched area refers to the $K^0 \Sigma^0$ and $K^0(\Sigma^0)$ subsets only.
- Fig. 5 C.m. decay angle distribution for the Σ^0 from the $K^0 \Sigma^0$ subset. The figure includes the events satisfying the selection criteria described in the text together with those ambiguous with the $K^0 \Lambda$ final state (hatched area).
- Fig. 6 Missing mass squared distribution for the $K^0 \Sigma^0$ subset after applying the selections described in the text.

FIGURE CAPTIONS (Cont'd)

- Fig. 7 (a) The differential cross section and

 (b) polarization for the reaction $\pi^- p \rightarrow K^0 \Lambda$ at
 3.95 GeV/c.
- Fig. 8 (a) The differential cross section and

 (b) Σ^0 polarization for the reaction $\pi^- p \rightarrow K^0 \Sigma^0$
 at 3.95 GeV/c.
- Fig. 9 Comparison of the $\pi^- p \rightarrow K^0 \Sigma^0$ differential cross section
 in the forward hemisphere with the isospin related reaction
 $\pi^+ p \rightarrow K^+ \Sigma^+$ from the refs [3,4,5].
- Fig. 10 Test with differential cross sections of the Cohen and
 Lipkin sum rule [12] at 4.2 GeV/c.
- Fig. 11 The differential cross section, $d\sigma/dt$, as a function of t .
 The open squares are our data on $\pi^- p \rightarrow K^0 \Lambda$ at 3.95 GeV/c,
 the solid circles are $K^- p \rightarrow \pi^0 \Lambda$ at 4.2 GeV/c from
 ref. [6b], the open circles are our $\pi^- p \rightarrow K^0 \Sigma^0$ at
 3.95 GeV/c and the solid triangles $K^- p \rightarrow \pi^- \Sigma^+$ at
 4.2 GeV/c from ref. [6b]. The curves are the fits given by
 the amplitudes of fig. 14.
- Fig. 12 The polarization, P , as a function of t . The data points and
 curves are as for fig. 11.

FIGURE CAPTIONS (Cont'd)

- Fig. 13 The ratio of differences of the differential cross sections discussed in the text, where $\Delta\sigma \equiv d\sigma/dt$. Only the final states are labelled, the initial particles being π^-p or K^-p as appropriate.
- Fig. 14 The real and imaginary parts of the tensor and vector, non flip and flip, amplitudes are shown for the Λ and Σ reactions, as defined by eqs (4, 5), deduced by smoothly interpolating the data of figs 11, 12. For the non flip amplitudes, the imaginary parts are shown by solid curves, the real parts by a dashed line. The flip amplitude is taken to be exchange degenerate, so the imaginary parts are the same for tensor and vector exchange and are drawn solid; the real parts differ and are shown dashed and dotted respectively. Representative errors are indicated by the points - these reflect just the experimental uncertainties of figs 11, 12.

Fig. 1

Fig. 2

Fig. 3

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig.8

Fig. 9

Fig.10

Fig.11

POLARISATION

Fig.12

Fig.13

Fig.14