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CONTRIBUTIONS OF OPERATORS OF TWIST TWO AND HIGHER IN
LARGE N MOMENTS OF STRUCTURE FUNCTIONS

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A B S T R A C T

Though high twist terms are becoming important as x^{-1} , or equivalently, in large n moments, their detection in this regime in deep inelastic lepton scattering needs special caution. The high order terms in the twist two component are strongly dependent on n ; one finds that at $Q^2 \lesssim Q_{**}^2 \simeq \Lambda^2 a_k \exp \{ \alpha_k (\log n)^{2-\frac{1}{k}} (1+b_k/\log n) \}$ the perturbative expansion is invalid, whereas higher twist terms are important at $Q^2 \lesssim Q_*^2 = \Lambda^2 n C$. Since Q_{**}^2 grows very fast with n the necessary requirement for any deep inelastic phenomenological analysis, namely $Q_*^2 \gg Q_{**}^2$, cannot hold for too large moments. The scheme dependence of a_k, α_k and b_k is also discussed.

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Quantum chromodynamics predicts a scaling violation pattern of deep inelastic processes which is governed by logarithms as well as powers of Q^2 ; these are contributions of twist two and higher twist operators ¹⁾ in the operator product expansion. Presently there is no precise theoretical framework in which the relative normalization of these two sources of Q^2 dependence can be satisfactorily calculated ^{2),3)}. Thus, this issue is studied in phenomenological analyses resulting in a wide range of opinions ⁴⁾ as to the importance of higher twist terms. At high enough Q^2 , twist two terms dominate and their Q^2 dependence can be calculated in a perturbative expansion. It is widely believed that at high Q^2 this asymptotic expansion is well behaved and can be trusted up to high orders while being certainly useless at, say, $Q^2 < O(1 \text{ GeV}^2)$. More precise studies of the regime in which the series start to diverge had been carried out ^{5),6)} using estimates of the presently not fully known order $\alpha^2(Q^2)$ terms. Though the exact boundaries of the regime in which high order terms are large is clearly dependent on the chosen normalization and renormalization scheme ^{5),6)}, it is roughly located, in most common schemes, within or next to the low Q^2 region in which it is being claimed that higher twist terms are important. Thus, the behaviour of the twist two perturbative expansion in this region and the occurrence of the higher twist component are two intimately connected problems which will be addressed here.

In the past, much effort had been put in to try and estimate the higher order terms in $g-2$ of the muon and such successful estimates had been achieved in Ref. 7). These estimates could be carried out essentially as a result of the special leading logs structure in solutions of renormalization group equations. Though different in details, there is much in common between the above calculations and the estimates' procedure suggested in Ref. 5). Moreover, it may very well be that the absence in QCD of the problem related to scattering of light by light in Ref. 7) points out that the approximation in the case of the moments of structure functions in QCD is even better than that for $g-2$ in QED ⁸⁾.

The Nachtmann moment ⁹⁾ $M_n^i(Q^2/\Lambda^2)$ of the non-singlet part of the structure function $F_i(x, Q^2)$ in deep inelastic lepton scattering is given by (twist two part) :

$$M_n^i(\alpha_s^2/\Lambda^2) = A_n^i(\bar{g}^2(\alpha_s^2/\Lambda^2))^{\gamma_n^{(0)}/2\beta_0} \sum_{k=0} B_{n,k}^i \bar{g}^{2k}(\alpha_s^2/\Lambda^2) \quad (1.a)$$

or

$$M_n^i(Q^2/\Lambda^2) = A_n^i(g_o^2)^{\gamma_n^{(0)}/2\beta_o} \sum_{k=0} \sum_{j \leq k} C_{k,j}^{n,i} g_o^{2k} (\log g_o^2)^j \quad (1.b)$$

where ¹⁰⁾ $g_o^2 = (\beta_o \log(Q^2/\Lambda^2))^{-1}$. Presently existing two-loop calculations ¹¹⁾ give the value of the order g_o^2 term in Eq. (1.b). Out of the seventeen different terms that add up to form the full three-loop contribution [order g_o^4 in Eq. (1.b)] fourteen were exactly known from the two-loop calculations [see the analogous case in Ref. 7)] and thus an estimate had been made in Ref. 5) of the full g_o^4 term and discussed there. [Meanwhile a new calculation ¹²⁾ of the β function to three loops enables us to further know one of the three missing terms ⁵⁾ bringing the estimate to an even better accuracy ¹³⁾.] As mentioned, the relative magnitude of the one, two-loop and estimated third loop contributions in Eq. (1) are clearly dependent on the renormalization scheme used ¹⁴⁾ as well as on the normalization of the expansion parameter g_o^2 (choosing Λ). Indeed, using the estimates of the g_o^4 term in Eq. (1.b), Abbott ⁶⁾ had pointed out that if the expansion parameter g_o^2 is normalized so that $g_o^2(Q_o^2) = \bar{g}_{\overline{MS}}^2(Q_o^2)$, by choosing $\Lambda^2 = \Lambda^2(Q_o^2)$, the perturbative expansion is well behaved up to much lower Q^2 than in most schemes in which Λ is kept independent of Q_o^2 . Other methods had been suggested ¹⁴⁾ as well to minimize the effect of higher order terms which are viewed as theoretical uncertainties attached to the presently exactly calculated order g_o^2 in Eqs (1). The understanding of these uncertainties in the known twist-two perturbative expansion is clearly necessary if one wants to study phenomenologically the size of the higher twist terms.

The full QCD prediction to $M_n^i(Q^2/\Lambda^2)$ has the approximate form ¹⁾

$$M_n^i(Q^2/\Lambda^2) = M_n^{i, \tau=2}(Q^2/\Lambda^2) \left[1 + n \frac{\Lambda^2}{Q^2} R_{n,1}^i + n^2 \left(\frac{\Lambda^2}{Q^2} \right)^2 R_{n,2}^i + \dots \right] \quad (2)$$

where $M_n^{\tau=2}$ is given in Eqs (1), Λ is the common scale of a few hundred MeV and $R_{n,k}^i(Q^2)$ are functions of order one, slowly-varying functions of n , k and Q^2 . The factor n^k in front of the relative strength of the high twist terms $R_{n,k}^i$ represents a combinatorial factor of the high twist operator in the operator product expansion. Thus the region $Q^2 < Q_*^2$ in which the higher ($\tau > 2$) twist terms in M_n^i start to be important grows linearly with n

$$Q_{**}^2 \simeq n \Lambda^2 c \quad (3)$$

where c is of order one. Eq. (3) is exhibiting, of course, the fact that higher twist terms are important in the $x \rightarrow 1$ region of the deep inelastic structure functions and one thus is advised to look for them in this regime or, equivalently, in large n moments ^{1),3),4)} where they will successfully compete with the leading twist two contribution represented by the expansion in Eqs (1). This asymptotic expansion certainly breaks down at $Q^2 < O(1 \text{ GeV}^2)$; more precisely, at some $Q^2 = Q_{**}^2$ the high order terms, say order g_0^{2k} , start to be large and the asymptotic expansion is worthless beyond this term. The value of Q_{**}^2 at which the k^{th} order term in Eq. (1.b) is of the order of the leading term is determined from ¹⁵⁾

$$\left| g_0^{2k} \sum_{j=0}^k C_{k,j}^{n,i} (\log g_0^2)^j \right| \simeq 1 \quad (4)$$

The n dependence of the coefficients of g_0^{2k} in Eqs (1) is of the form ^{11),16)}

$$A_{0,k} (\log n)^{2k} + A_{1,k} (\log n)^{2k-1} + A_{2,k} (\log n)^{2k-2} + \dots \quad (5)$$

It had been argued in Ref. 17) that large corrections exposing the singular infra-red structure of QCD of the type $\bar{g}^2(A_{0,k}(\log n)^{2k})$ can be extracted from the perturbative expansion and resummed. The rest of the coefficients $A_{i,k}$ ($i \geq 1$) are slowly-varying functions of Q^2 and, in general, scheme dependent. Thus, from Eqs (4) and (5), one finds

$$Q_{**}^2 \simeq \Lambda^2 a_k \exp\{f_{k,n}\} \quad (6)$$

where

$$f_{k,n} = \alpha_k (\log n)^{2 - \frac{1}{k}} [1 + b_k (\log n)^{-1} + \dots] \quad (7)$$

$$\alpha_k = \frac{|A_{1,k}|^{\frac{1}{k}}}{\beta_0}, \quad b_k = \frac{1}{k} \left(\frac{A_{2,k}}{A_{1,k}} \right)$$

a_k effectively represents the lower order $(\log n)^{-k}$ terms and behaves at large n and k as ¹⁸⁾

$$a_k \approx \exp \left\{ \frac{\alpha_k}{k} \left[\frac{A_{3,k}}{A_{1,k}} - \frac{1}{2} \left(\frac{A_{2,k}}{A_{1,k}} \right)^2 \right] \right\} \quad (8)$$

One thus finds that due to the strong n dependence ¹⁹⁾ of the coefficients in the perturbative expansion of the twist two component in Eqs (1), Q_{**}^2 in Eq. (6) grows very fast with n even for $k=2$ where numerical analysis is possible ²⁰⁾.

In phenomenological studies higher twist components have to be added to the twist two component in regimes where the high order terms in Eqs (1) are under control, namely, it is necessary that $Q_*^2 \gg Q_{**}^2$. Comparing Eqs (3) and (6), it is clear that such an analysis is possible provided one does not go to very high n ($x \rightarrow 1$). High twists are indeed important in this regime but the fast growing Q_{**}^2 will approach Q_*^2 , which grows only linearly with n . Perturbation expansion for the twist two component will then break down at Q^2 close to the regime in which the higher twist started to be important, making a phenomenological analysis impossible. Of course, at the end, in the limit $x \rightarrow 1$ as we enter the pseudo-elastic, elastic regime, the whole analysis is different as all twists are important.

Thus, an analysis of high twist terms is possible provided we work in such a scheme in which large enough n moments will exist for which still $Q_*^2 \gg Q_{**}^2$. The values of a_k , α_k and b_k in different schemes are of much interest. Presently, one can study the value of Q^2 at which the three-loop contribution starts to be too large, e.g., in the scheme of Ref. 5) one finds

$$Q_{**}^2 \approx 0.92 \Lambda^2 \exp \left\{ 0.64 (\log n)^{3/2} \left[1 + \frac{1.17}{\log n} \right] \right\}$$

One should note that already at Q^2 above Q_{**}^2 but close to it, the third loop correction is becoming uncomfortably large (Q^2 around 4 GeV^2 in this scheme). Better schemes can be found in which the divergence of the perturbative expansion occurs at lower Q^2 . Using the estimated values of the three-loop contribution, a_2 , α_2 and b_2 were calculated for different schemes and given in the Table. Only in the last scheme of the Table (scheme IV) can one obtain a Q_{**}^2 small enough so that high twist terms can be comfortably studied at low Q^2 even in high n moments.

In conclusion, one finds that the high n behaviour of moments plays a major role in determining our ability to study high twist terms, the strong n dependence of the expansion coefficients puts string constraints on choosing an appropriate scheme.

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Scheme	a_2	α_2	b_2
I) Ref. 5) in MS scheme	0.27	0.64	3.12
II) Ref. 5) in $\overline{\text{MS}}$ scheme	0.18	0.90	2.15
III) Ref. 6) in MS scheme	0.14	1.15	1.35
IV) Ref. 6) in $\overline{\text{MS}}$ scheme	0.24	1.02	0.51

TABLE : At large n the value of $Q^2 = Q_{**}^2$ at which the three-loop contribution is of the order of the leading term is given by

$$Q_{**}^2 \approx a_2 \Lambda^2 \exp \left\{ \alpha_2 (\log n)^{3/2} \left(1 + \frac{b_2}{\log n} \right) \right\}$$

$\{a_2, \alpha_2, b_2\}$ are given here in different schemes.

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