Ref.TH.2989-CERN

A STABILITY CRITERION FOR TUMBLING PATTERNS

D. Amati and M.A. Virasoro

CERN -- Geneva

A B S T R A C T

We discuss the tumbling of gauge theories using the Nambu-Jona Lasinio model as a theoretical tool. By analyzing the stability of possible vacua, we find a pattern of symmetry breaking that complements the most attractive channel criterion when this leads to ambiguous predictions.

Ref.TH.2989-CERN
19 November 1980

The fate of a non-Abelian gauge theory with massless fermions, as the energy is decreased and the theory passes from the asymptotic freedom regime to the infrared divergent one has been recently discussed¹⁾. When the fermions belong to non-real representations of the gauge group, it has been argued that the theory may pass through different stages (tumbling), at each of which we are left with an effective theory of a smaller subgroup of massless gauge bosons and fermions.

The analysis done relies on the assumption that the behaviour of the gauge theory as a function of the energy can be simulated by the running of the coupling constant. Then, when this becomes sufficiently large, a condensate is formed which is responsible for the breaking of the symmetry. We shall adopt this attitude even if it implies a certain inconsistency in the language (as the actual meaning of "massless bound states at small distances"), hoping that a more careful analysis will not affect the results.

In this note we will analyze on a more dynamical basis the formation of the condensate and the fate of the symmetry. We shall then find a more detailed structure of the tumbling process. Where the "Most Attractive Channel" (MAC) criterion leads to unambiguous predictions, our approach reproduces them; on the other hand, our analysis gives a more detailed insight in those cases where the MAC criterion is ambiguous or does not specify the breaking pattern.

Let us first recall that in the view of Ref. 1), also adopted here, the condensation appears as a phase transition when a coupling constant overcomes a critical value g_c . This can be visualized as a scalar bound state getting down to zero mass for a sufficiently strong attraction. The condensation of the would-be tachyon (for $g > g_c$) is the cause of spontaneous symmetry breaking. For a still stronger coupling, another bound state could, in its turn, become a tachyon. This should, however, be analyzed in the presence of the first condensate, which has a stabilizing effect by giving mass to a first set of fermions and/or by screening the forces between them. Therefore a very small difference in attraction determines the direction of the tumbling. One should keep in mind, however, that in principle, massive fermions may still participate in a condensate if the attraction is large enough. We have checked this in a model with two coupling constants that can be varied arbitrarily, and found a first order phase transition separating the regimes controlled by one or the other condensate. The rules of the game called "tumbling" assume that such a first order phase transition does not occur on the track of the running coupling constant.

If the relevant physics depends only on the neighbourhood of $g = g_c$, where two massless fermions bind into a massless boson, then the attractive potential will act through its spatial average, and therefore the same physics will follow from a short-range force as from a long-range one. This serves as partial justification for using the Nambu-Jona Lasinio²⁾ (NJL) Lagrangian as a model for experimenting with the ideas of tumbling. We will rederive the NJL equations in a way which is useful when there is an internal group and the condensate is not a singlet. The Lagrangian is written in the following way:

$$\mathcal{L} = u^{*i} \sigma^\mu_{ij} \partial_\mu u_j - H_{kl}^{ij} u_i \varepsilon u_j u^{*k} \varepsilon u^{*l} \quad (1)$$

where u, u^* are two component Weyl spinors. In Minkowsky space, u^* is the complex conjugate of u . Both σ_μ and ε are chosen to be:

$$\sigma_\mu = \mathbb{1}, \vec{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (2)$$

$$\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad (3)$$

Under the internal group G the $u(u^*)$ transforms like a possibly reducible representation $R(R^*)$. We are interested in the case where R is not equivalent to R^* . Therefore the condensate bilinear in the fermion fields cannot be both a singlet under G and a Lorentz scalar. The matrix H_{kl}^{ij} is an invariant tensor and can be expressed as:

$$H_{kl}^{ij} = \sum_r g^{(r)} P^{(r)ij}_{kl}, \quad g^{(r)} \geq 0 \quad (4)$$

where $P^{(r)}$ is the projector which, acting on the symmetrized product $u^i \otimes u^j$, projects the representation $R^{(r)}$. The Fierz identities:

$$\begin{aligned}
 u^{*1} \sigma_\mu u_2 \quad u^{*3} \sigma_\mu u_4 &= u^{*1} \sigma_\mu u_4 \quad u^{*3} \sigma_\mu u_2 = \\
 &= -2 u^{*1} \varepsilon u^{*3} u_2 \varepsilon u_4
 \end{aligned} \tag{5}$$

should be noticed.

We transform (1) into an action bilinear in the fermions, and then integrate over the latter to obtain the effective action for the order parameter $u_k \varepsilon u_\ell$:

$$i S_{\text{eff}} = \int \frac{d^4 p}{(2\pi)^4} 2 \operatorname{tr} \ln (p^2 \delta_i^j - M_{ik} M^{*kj}) + i M_{ij} (H^{-1})_{kl}^{ij} M^{*kl} \tag{6}$$

where

$$\begin{aligned}
 M_{ij} &= \langle u_k \varepsilon u_\ell \rangle H_{ij}^{kl} \\
 M^{*ij} &= \langle u^{*k} \varepsilon u^{*\ell} \rangle H_{kl}^{ij}
 \end{aligned} \tag{7}$$

In Euclidean space the $S_{\text{eff}}^{(E)}$ is:

$$S_{\text{eff}}^{(E)} = - \int \frac{d^4 p}{(2\pi)^4} 2 \operatorname{tr} \ln (p^2 \delta_i^j + M_{ik} M^{*kj}) + M_{ij} (H^{-1})_{kl}^{ij} M^{*kl} \tag{8}$$

The first term favours breaking the symmetry (i.e., $M_{ij} \neq 0$). In fact, it may be written in the form:

$$- \int \frac{d^4 p}{(2\pi)^4} 2 \sum_k \operatorname{tr} \ln (p^2 + m_k^2) \tag{9}$$

where m_k , the eigenvalues of the matrix M , are the dynamical masses acquired by the fermions. The second term, on the other hand, stabilizes the symmetric ($M_{ij} \equiv 0$) vacuum.

Using the same cut-off as in NJL²⁾ we obtain:

$$S_{eff}^{(E)} = -\frac{1}{16\pi^2} \text{tr} \left[(\Lambda^4 \mathbf{1} - (M M^*)^2) \ln(\Lambda^2 \mathbf{1} + M M^*) + \right. \\ \left. + (M M^*)^2 \ln(M M^*) + \Lambda^2 M M^* \right] + M_{ij} (H^{-1})_{kl}^{ij} M^{*kl} \quad (10)$$

As discussed before, we are interested in solutions near the critical points, i.e., for small masses. Then, rescaling $M_{ij} = m \mu_{ij}$ with $m \ll \Lambda$:

$$S_{eff}^{(E)} = \mu_{ij} m^2 \left((H^{-1})_{kl}^{ij} - \frac{\Lambda^2}{8\pi^2} \delta_k^i \delta_l^j \right) \mu^{*kl} - \\ - \frac{m^4}{8\pi^2} \left(\ln \frac{m^2}{\Lambda^2} \right) \mu_{ij} \mu^{*jk} \mu_{kl} \mu^{*li} \quad (11)$$

The stationary points are given by:

$$\mu_{ij} = H_{ij}^{kl} \frac{\Lambda^2}{8\pi^2} \left[\mu_{kl} + \frac{2m^2}{\Lambda^2} \mu_{kp} \mu^{*pq} \mu_{ql} \ln \frac{m^2}{\Lambda^2} \right] \quad (12)$$

This equation has no solution besides the trivial one ($\mu_{ij} = 0$) if all coupling constants in Eq. (12) are smaller than $g_c = 8\pi^2/\Lambda^2$. One or more solutions may appear as soon as one of them overcomes g_c . If different representations of μ_{ij} (different condensates $R^{(i)}$) leave the same subgroup invariant, then all of them will turn out to be different from zero, so that the solution will be a definite mixture of all of them (and not a pure $R^{(i)}$).

In analyzing Eq. (12), we found that it may happen that when a coupling (call it $g^{(1)}$) overcomes the critical value g_c , ($g^{(j)} < g_c$, $j \neq 1$), several distinct solutions appear together. This is due to the fact that a condensate (or a set of them) may possess non-equivalent orientations on the group manifold, that induce different breakings of the group G . This implies that at the critical value different vacua may appear, and the discrimination among them necessitates a stability criterion. This implies determining which of the non-trivial solutions

minimizes in absolute value the effective action, and this, as discussed before, in the immediate vicinity of the critical point. This is a simple matter, because at the stationary point, the effective action is simply proportional to the quartic invariant and, therefore, the chosen solution is the one with the largest sum of the fourth powers of the generated masses in the vicinity of the critical value. This stability quantity is easy to compute, due to the fact that it is a lowest order in $g^{(1)} - g_c$. This turns out to be independent of all other $g^{(i)}$ and therefore not dependent on actual values of coupling constants (i.e., details of strong interactions).

The same sort of argument will solve other accidental degeneracies of MAC, as, for instance, different channels having the same attractiveness. This could be represented in Eq. (12) with:

$$g^{(1)} = g^{(2)} \sim g_c \quad 0 < g_i < g_c, \quad i \neq 1, 2 \quad (13)$$

We shall illustrate this case with an example later on. As before, different solutions appear when $g^{(1)} = g^{(2)} = g_c$, and the stability criterion [larger sum of $(\text{mass})^4$] determines the physical solution. As a rule, all possible condensates compatible with the invariant subgroups will be formed and will appear in well-defined combinations.

Although we derived these results in a particular model, they are much more general because they depend exclusively on Eq. (11) for $S_{\text{eff}}^{(E)}$ which corresponds to an obvious expansion in powers of the matrix $M_{ij} (M^*)^{jk}$ with the possible presence of logarithms which are quantitatively irrelevant when checking stability.

As an example, let us refer to the $SU(3)$ case with quarks in the representation 6. Symmetric combinations of $6 \otimes 6$ transform as $\bar{6}$ and 15, so that H in Eq. (4) will depend on two couplings ($g_{\bar{6}}$ and g_{15}). The MAC criterion predicts $g_{\bar{6}} > g_{15}$ and therefore a condensate in $\bar{6}$. The residual group is, however, not determined because there are inequivalent orientations possible for the condensate that would lead to $SO(3)$, $SU(2)$ or $SO(2)$. Our Eq. (12) shows indeed that as soon as a critical line ($g_{\bar{6}} = g_c, g_{15} < g_{\bar{6}}; g_{15} = g_c, g_{\bar{6}} < g_{15}$) is overcome, a solution for each of the aforementioned breaking patterns actually appears. Each solution implies a condensate which is a mixture of $\bar{6}$ and 15. As discussed for the general case, it is very easy to evaluate the mass pattern in the vicinity of

the critical line, and therefore to determine which is the stable solution. In this case, it turns out that $SO(3)$ is the most stable vacuum not dependent on where the critical line has been crossed. All quarks get masses and transform as a $5 + 1$ representation of $SO(3)$. The relation between the quintuplet and singlet masses depends of course on g_8 and g_{15} . Another example appearing in the literature together with the one discussed before⁵⁾ is the case in which quarks belong to $6 + \bar{3}$. The MAC criterion would give equal attractiveness in $6 \otimes 6 \rightarrow \bar{6}$ and $6 \otimes \bar{3} \rightarrow 3$, so that we would come upon the case described by Eq. (13). In this case, the stable solution corresponds to breaking $SU(3)$ into $SU(2)$ with all nine quarks acquiring masses and organized as three doublets and three singlets.

We would like to end this letter with a word of caution. There are no compelling arguments for the choice of the bilinear Lorentz scalar as the order parameter. An alternative possibility, discussed a long time ago in the literature³⁾, is that the Lorentz vector could condensate, even more so for the case discussed here of non-real representations for the fundamental fermions, due to the fact that the vector bilinear $u_i \sigma_\mu u^{i*}$ is always a singlet of the group, and therefore belongs to the most attractive channel from the group representation's point of view. Such a condensate would not break either the group or chiral symmetry. On the other hand, it is a controversial matter whether or not it would spontaneously break Lorentz invariance, or give rise to massless vector bosons which would produce an effective gauge theory⁴⁾. But in one way or the other, it is apparent that a vector condensate will dramatically affect the interaction. A real understanding of the condensation phenomena cannot be obtained without the treatment of the competition of mutually excluding order parameters. Problems of analogous nature have been successfully tackled in the study of a model for metal-insulator transition⁶⁾. This is a matter that deserves further investigation, but needs a much more detailed description of the dynamics than the one introduced here.

ACKNOWLEDGEMENT

We thank P. Sikivie for interesting discussions on the ideas of tumbling, and in particular on the ambiguities of the MAC criterion.

REFERENCES

- 1) S. Raby, S. Dimopoulos and L. Susskind, "Tumbling gauge theories", Stanford preprint ITP-653 (1980).
- 2) Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345.
- 3) J.D. Bjorken, Ann. Phys. (N.Y.) 24 (1963) 174;
T. Eguchi, Phys. Rev. D14 (1976) 2755;
F.A. Bais and J.-M. Frère, "Composite vector fields and tumbling gauge theories", CERN preprint TH.2911 (1980)
- 4) T. Banks and A. Zaks, "Composite gauge bosons in four-Fermi theories", Tel Aviv University preprint TAUP 889-80 (1980).
- 5) S. Dimopoulos, S. Raby and P. Sikivie, Stanford preprint ITP-664 (1980).
- 6) C. Castellani, C. di Castro, D. Feinberg and J. Ranninger, Phys. Rev. Lett. 43 (1979) 1957. We thank G. Parisi for calling our attention to this paper.