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WHAT DO THE PRESENT DEEP INELASTIC SCATTERING DATA AND QCD TELL US
ABOUT STRUCTURE FUNCTIONS ?

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A B S T R A C T

The Q^2 behaviour of the moments of gluon distributions as well as flavour singlet and non-singlet combinations of quark distributions are obtained from an analysis of the present electron and muon deep inelastic data. The higher order QCD terms have been taken into account, and two different definitions of parton distributions beyond leading order are discussed. The forms of the distributions which reproduce the moments at a particular value of Q^2 are also given. The extrapolation of the moments to large Q^2 (several hundred GeV^2) are found to be consistent with the preliminary muon data at these high values of Q^2 .

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1. - INTRODUCTION

During the last few years the higher order QCD corrections (subleading in $\log Q^2$) to the Q^2 dependence of the moments of the structure functions in deep inelastic scattering have been calculated ¹⁾⁻⁵⁾. It was found that their contribution can be very important, particularly for the higher moments so that for sufficiently large N (moment number) the perturbation expansion will always break down. An unexpectedly large contribution to the higher order corrections has also been found in muon pair production ⁶⁾.

In the present paper we study the influence of the higher order corrections on the determination of quark and gluon distributions inside a nucleon and on the quality of the QCD fits to the scaling violations for various values of the parameter Λ , which controls the strength of the strong coupling constant. We use as our experimental data the Nachtmann moments obtained from the combined SLAC-FNAL data on structure functions for e-p, e-d, μ -p and μ -d inclusive deep inelastic scattering ⁷⁾.

It is well known that the definition of parton distributions beyond leading order is not unique, and we shall consider here two approaches. In the first one the parton model relation $F_2^{\text{en}}(x, Q^2) = \sum_q e_q^2 x q(x, Q^2)$ is maintained beyond leading order ; the basic formulae relevant to this definition are collected in Section 2, while the description of the fit and the discussion of the quark and gluon distributions thus obtained are contained in Section 3. In the second approach the evolution of the parton distributions is controlled by the anomalous dimensions alone ⁸⁾ ; it is discussed in Section 4 which also contains a description of the corresponding fit. In Section 5 we present our conclusions.

2. - CASE I : BASIC FORMULAE

The formulae for the N^{th} moment of $F_2(x, Q^2)$ are ^{1), 2), 9)}

$$\begin{aligned} M_2^N(Q^2)_{p(n)} &= \int_0^1 dx x^{N-2} F_{2, p(n)}(x, Q^2) \\ &= F_{2, NS}^N(Q^2)_{p(n)} + Q_2^N(Q^2) \end{aligned} \quad (1)$$

(the subscripts p and n stand for proton and neutron, respectively) where the Q^2 dependence of the flavour non-singlet term $F_{2,NS}^N(Q^2)_{p(n)}$ is given by

$$F_{2,NS}^N(Q^2)_{p(n)} = F_{2,NS}^N(Q_0^2)_{p(n)} \left[1 + \frac{\bar{\alpha}(Q^2) - \bar{\alpha}(Q_0^2)}{4\pi} A_{2,NS}^N \right] \left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right]^{\gamma_0^N/2\beta_0} \quad (2)$$

with

$$\bar{\alpha}(Q^2) = \bar{\alpha}(Q_0^2) \left[1 - \frac{\beta_1}{4\pi\beta_0} \bar{\alpha}_0(Q^2) \ln\left(\ln \frac{Q^2}{\Lambda^2}\right) \right] \quad (3)$$

and

$$\bar{\alpha}_0(Q^2) = \frac{4\pi}{\beta_0 \ln \frac{Q^2}{\Lambda^2}} \quad (4)$$

and

$$\begin{aligned} \gamma_0^N &= \frac{8}{3} \left[1 - \frac{2}{N(N+1)} + 4 \sum_{j=2}^N \frac{1}{j} \right] \\ \beta_0 &= 11 - \frac{2}{3} N_f \\ \beta_1 &= 102 - \frac{38}{3} N_f \end{aligned} \quad (5)$$

N_f is the number of active ^{*)} flavours and the coefficients of $A_{2,NS}^N$ are given in Table I of Ref. 2). The singlet contribution $Q_2^N(Q^2)$ can be split into a part depending only on itself at a reference value Q_0^2 and a part, $G_2^N(Q_0^2)$, depending on the gluon distribution at Q_0^2 .

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^{*)} As a good approximation a flavour is considered active if $Q^2 > m_{\psi_i}^2$, where m_{ψ_i} is the mass of the lowest lying vector meson consisting of a quark and an antiquark of this flavour.

$$\begin{aligned}
 Q_2^N(Q^2) = & \left\{ \left[A_{11}^N + A_{12}^{(2),N} \frac{\bar{\alpha}(Q^2)}{4\pi} + A_{13}^{(2),N} \frac{\bar{\alpha}(Q_0^2)}{4\pi} \right] \left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right]^{\gamma_+^N/2\beta_0} \right. \\
 & \left. + \left[A_{21}^N + A_{22}^{(2),N} \frac{\bar{\alpha}(Q^2)}{4\pi} + A_{23}^{(2),N} \frac{\bar{\alpha}(Q_0^2)}{4\pi} \right] \left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right]^{\gamma_-^N/2\beta_0} \right\} Q_2^N(Q_0^2) \\
 & - \left\{ \left[B_{11}^N + B_{12}^{(2),N} \frac{\bar{\alpha}(Q^2)}{4\pi} + B_{13}^{(2),N} \frac{\bar{\alpha}(Q_0^2)}{4\pi} \right] \left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right]^{\gamma_+^N/2\beta_0} \right. \\
 & \left. + \left[B_{21}^N + B_{22}^{(2),N} \frac{\bar{\alpha}(Q^2)}{4\pi} + B_{23}^{(2),N} \frac{\bar{\alpha}(Q_0^2)}{4\pi} \right] \left[\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right]^{\gamma_-^N/2\beta_0} \right\} G_2^N(Q_0^2)
 \end{aligned}
 \tag{6}$$

where the coefficients A_{ij} and B_{ij} are obtained from the errata ⁹⁾ to Ref. 2). It should be noted that these numbers as well as $A_{2,NS}^N$ were calculated ¹⁾ using the dimensional regularization scheme with minimal subtraction renormalization prescription.

For the deuterium data the moments are taken as a simple sum of those for the proton and neutron separately (this introduces a small error since the Fermi motion of the nucleons within the nucleus is hereby neglected)

$$M_2^N(Q^2)_d = M_2^N(Q^2)_p + M_2^N(Q^2)_n
 \tag{7}$$

$Q_2^N(Q^2)$ is (up to a numerical factor) the N^{th} moment of the flavour singlet combination of quark and antiquark distributions $q_s(x, Q^2)$ ¹⁰⁾, and it is the same for the proton and neutron.

$$\begin{aligned}
 q_s(x, Q^2) = & u(x, Q^2) + \bar{u}(x, Q^2) + d(x, Q^2) + \bar{d}(x, Q^2) + s(x, Q^2) + \bar{s}(x, Q^2) \\
 & + T (c(x, Q^2) + \bar{c}(x, Q^2)) + \dots
 \end{aligned}
 \tag{8}$$

T is a threshold factor arising from the fact that the charm quark is massive so that this term is not present below the energy for producing a pair of charmed quarks. The quantity $F_{2,NS}^N(Q^2)$ in Eq. (2) is the moment of the following combination¹⁰⁾ of quark distributions

$$q_{Ns}(x, Q^2) = \frac{1}{3} q_3(x, Q^2) + \frac{1}{\sqrt{27}} q_8(x, Q^2) + \frac{1-2T}{\sqrt{27}} \sqrt{2} q_{15}(x, Q^2) \quad (9)$$

where

$$q_3(x, Q^2) = \frac{1}{2} \left(u(x, Q^2) + \bar{u}(x, Q^2) - d(x, Q^2) - \bar{d}(x, Q^2) \right) \quad (10)$$

$$q_8(x, Q^2) = \frac{1}{\sqrt{12}} \left(u(x, Q^2) + \bar{u}(x, Q^2) + d(x, Q^2) + \bar{d}(x, Q^2) - 2s(x, Q^2) - 2\bar{s}(x, Q^2) \right) \quad (11)$$

$$q_{15}(x, Q^2) = \frac{1}{\sqrt{24}} \left(u(x, Q^2) + \bar{u}(x, Q^2) + d(x, Q^2) + \bar{d}(x, Q^2) + s(x, Q^2) + \bar{s}(x, Q^2) - 3c(x, Q^2) - 3\bar{c}(x, Q^2) \right) \quad (12)$$

In terms of these combinations of quark distributions the structure function $F_2(x, Q^2)$ is given by¹⁰⁾

$$\begin{aligned} F_2(x, Q^2) &= \frac{4}{9} (u(x, Q^2) + \bar{u}(x, Q^2)) + \frac{1}{9} (d(x, Q^2) + \bar{d}(x, Q^2)) \\ &\quad + \frac{1}{9} (s(x, Q^2) + \bar{s}(x, Q^2)) + \frac{4T}{9} (c(x, Q^2) + \bar{c}(x, Q^2)) \\ &= \frac{3+2T}{18} q_3(x, Q^2) + q_{Ns}(x, Q^2) \end{aligned} \quad (13)$$

In order to obtain the moments of the quark and gluon distributions at any value of Q^2 we need to know their values at some fixed value Q_0^2 . These quantities are not directly calculable in QCD since one does not know the wave functions of quarks and gluons inside the hadrons and the bound state problem is connected with the large distance behaviour of QCD which is so far unresolved. Thus the theory can only predict the Q^2 behaviour of the moments of the parton distributions.

3. - CASE I : THE FIT

In the following we shall approximate $F_{2,NS}^N(Q^2)_p = -F_{2,NS}^N(Q^2)_n$, which amounts to taking the threshold factor $T = 1$ in Eqs. (8) and (9) and neglecting the difference between the strange and charmed quark distributions in the hadron. The latter quantity was measured in neutrino experiments and was found to be consistent with zero¹¹⁾. With these simplifications in mind we shall use as parameters the quark and gluon moments $q_V^N(Q_0^2)$, $q_S^N(Q_0^2)$ and $g^N(Q_0^2)$ related to the previously introduced quantities by

$$\begin{aligned} q_V^N(Q_0^2) &= 6 \cdot F_{2,NS}^N(Q_0^2) = u^N(Q_0^2) + \bar{u}^N(Q_0^2) - d^N(Q_0^2) - \bar{d}^N(Q_0^2) \\ q_S^N(Q_0^2) &= \frac{18}{5} F_{2,S}^N(Q_0^2) \\ g^N(Q_0^2) &= \frac{18}{5} G_2^N(Q_0^2) \end{aligned} \tag{14}$$

There is in the fit a large number of free parameters relative to the number of data points, but we shall see that it is possible to parametrize the parton distributions with a reduced number of parameters and with simple forms. The influence of the higher order contributions can be seen from the comparison of the dependence on Λ of the χ^2 distribution of fits done with or without the higher order corrections. As it was noted in Ref. 3) any change in the scale parameter Λ is equivalent in subleading order to a shift of the coefficients $A_{2,NS}^N$, A_{ij}^N and B_{ij}^N by an amount proportional to the one-loop anomalous dimensions. Thus any attempt to extract the value of Λ from experimental data with only leading order formulae in QCD has no meaning and when considering the higher order corrections one must state the renormalization prescription for which these corrections have been calculated before Λ becomes meaningful. As mentioned before we work in the minimal subtraction scheme so that the Λ 's refer to $\Lambda_{\overline{MS}}$ (the more popular $\Lambda_{\overline{MS}}$ can be obtained by multiplying $\Lambda_{\overline{MS}}$ by 2.66).

We have fitted the data using the standard computer program FUMILI¹²⁾ improved for our purposes. We first found out that letting Λ vary as a free parameter the other parameters are obtained with large correlation factors (especially the gluon moments). These correlations are strongly reduced if the fit is done with a fixed Λ for values of Λ between 0.01 and 2.01 GeV. This procedure does not determine the value of Λ from the data, but it serves for

the examination of the quality of the QCD description for different values of Λ and gives a good idea of the effects of the higher order corrections. In order to see possible still higher order (or non-perturbative) effects we analyze in this way each moment independently. We have plotted χ^2 per degree of freedom, $\chi_{df}^2 = \Sigma((\text{data-theory})/(\text{experimental error}))^2/(\text{number of data points} - \text{number of parameters})$ as a function of Λ for $N = 2, 4, 6, 8$ and 10 for the case with leading logarithmic fits (Fig. 1) and for the case with the higher order corrections (Fig. 2a). The reference momentum was taken to be $Q_0^2 = 5 \text{ GeV}^2$ for both cases. Comparison of the figures shows that the interval of Λ which permits a description of the data with $\chi_{df}^2 \leq 1$ is reduced from $\Lambda = 0.7 - 1.1 \text{ GeV}$ for the leading logarithmic case to $\Lambda = 0.3 - 0.6 \text{ GeV}$ when higher order corrections are included. One also sees the insensitivity of the fits in both cases to the different values of Λ within the allowed intervals ^{3), 7), 13)}. More experimental data with smaller error bars in the region $Q^2 = 40 \text{ GeV}^2$ would be very helpful for a more accurate determination of Λ . Furthermore, the variation of the values $\Lambda_{\min}^N$ at which the χ_{df}^2 curves achieve their minimum becomes smaller after inclusion of the second order corrections. Thus one might expect that taking into account the next higher order corrections (and possibly non-perturbative effects) would make this variation negligible.

Equations (1), (2) and (6) enable us to study the Q^2 dependence of the moments of quark and gluon distributions. For this purpose we have performed simultaneous fits for all the moments with the same value of Λ . We have checked the independence of the quality of the fits on different values of the reference momentum Q_0^2 : taking $Q_0^2 = 5, 10, 20, 40$ and 80 GeV^2 the plots of χ_{df}^2 against Λ remain almost unchanged with the minimum of χ_{df}^2 at $\Lambda = 0.83 \text{ GeV}$ for leading order QCD and $\Lambda = 0.45 \text{ GeV}$ for the parametrization with next-to-leading QCD corrections. The quality of the fit obtained in the latter case can be seen in Fig. 3 where it is compared to the data of Anderson et al. ⁷⁾. The curves obtained with different values of Q_0^2 are not distinguishable and only the $Q_0^2 = 5 \text{ GeV}^2$ case is shown. Because of the stability of the fit the variation with Q_0^2 of the fitting parameters $q_V^N(Q_0^2)$, $q_S^N(Q_0^2)$ and $g^N(Q_0^2)$ corresponds almost exactly to the Q^2 dependence of the moments of the distribution functions. This dependence is shown in Fig. 4, where the error bars indicate up to what accuracy these parameters can be extracted from the experimental data ^{*)}. It should be noted that if we

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*) Our results at $Q_0^2 = 10 \text{ GeV}^2$ agree with the fitted values of Anderson ⁷⁾.

neglect the higher order corrections the errors for the $N = 8$ and 10 gluon moments become of order 100% at all values of Q_0^2 . Thus no information on their Q^2 behaviour and on the x dependence of the gluon distribution can be extracted from the leading order approximation. Even with the higher order corrections included the errors for the gluon moments are of order 10-50%. This is not surprising as the measured structure functions do not depend directly on the gluon distributions but only their Q^2 dependence does. As the scaling violations are small the determination of the gluon distribution from the data is inaccurate. Figure 4 shows that for $N \geq 4$ all the moments of the distributions decrease as Q^2 increases and that this decrease is more rapid as N grows. For the second moment only $q_V^2(Q^2)$ decreases with increasing Q^2 whereas $q_S^2(Q^2)$ and $g^2(Q^2)$ remain practically constant in the interval $5 \text{ GeV}^2 \leq Q^2 \leq 80 \text{ GeV}^2$.

The moments of the parton distributions obtained in the $Q_0^2 = 5 \text{ GeV}^2$ fit with the higher order corrections (Fig. 4) were fitted with simple analytical expressions of the form $C \cdot x^\alpha (1-x)^\beta$. The valence distribution $q_V(x)$ was normalized to one and has thus only two parameters, whereas the singlet quark distribution $q_S(x)$ has three free parameters. For the gluon distribution a good fit could not be obtained in this way and the errors and correlation factors were very large. However, fixing the power of x to zero we obtain a good χ_{df}^2 and the results of this fit are given by

$$q_V^I(x, Q_0^2=5) = C (1-x)^{2.47 \pm 0.21} / x^{0.22 \pm 0.10} \quad (\chi_{df}^2 = 0.06) \quad (15)$$

$$q_S^I(x, Q_0^2=5) = (2.29 \pm 0.18) (1-x)^{2.26 \pm 0.05} / x^{0.87 \pm 0.04} \quad (\chi_{df}^2 = 0.06) \quad (16)$$

$$g^I(x, Q_0^2=5) = (13.1 \pm 3.1) (1-x)^{3.13 \pm 0.28} \quad (\chi_{df}^2 = 0.03) \quad (17)$$

The values of the quark and gluon moments obtained above (Fig. 4a,b,c) allow us to make predictions for the values of the structure function moments at large Q^2 values accessible at some current experiments. Taking the average value of best Λ ($\Lambda = 0.3 \text{ GeV}$) obtained from fitting the moments separately we use Eqs. (1)-(6) to extrapolate our fit to the range $Q^2 > 40 \text{ GeV}^2$ for $N = 4, 6$. This is shown in Fig. 5, where the solid line indicates the region of the fit. We have also plotted preliminary data points from the NA4 experiment¹⁴⁾. We see that within

$1\frac{1}{2}$ standard deviations they are consistent with the theoretical curve ^{*)}. A fit to these preliminary data alone shows that they can be well described (i.e. $\chi^2_{df} \leq 1$) with values of Λ in the interval $\Lambda = 0.01 - 0.1$ GeV. We remark that for $N = 4, 6$ the extrapolation of the fit with the higher order corrections included coincides with the extrapolation of the fit based on leading order formulae with $\Lambda = 0.6$. This illustrates the possibility of absorbing the higher order corrections into a redefinition of Λ and having a different Λ for each moment [see Ref. 3)].

4. - CASE II : FORMULAE AND FIT

We consider in this section an alternative definition of parton distributions in which the corrections to the coefficient functions have been explicitly factored out ⁸⁾, i.e., the non-singlet part of the structure function is parametrized as (for e-p scattering)

$$F_{2,NS}^N(Q^2) = q_V^N(Q^2) c_{2,NS}^N(1, \bar{\alpha}(Q^2)) \quad (18)$$

where

$$c_{2,NS}^N(1, \bar{\alpha}(Q^2)) = \frac{1}{6} \left(1 + \frac{\bar{\alpha}(Q^2)}{4\pi} \bar{B}_{2,NS}^N \right) \quad (19)$$

while $q_V^N(Q^2)$ satisfies an Altarelli-Parisi type equation (with the β function and the anomalous dimensions calculated up to two-loop) whose solution is

$$q_V^N(Q^2) = q_V^N(Q_0^2) \left(1 + Z_{NS}^N \frac{\bar{\alpha}(Q^2) - \bar{\alpha}(Q_0^2)}{4\pi} \right) \left(\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right)^{\gamma_0^N/2\beta_0} \quad (20)$$

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^{*)} It should be mentioned that apart from the problem of threshold effects connected with the excitation of new heavy flavours as Q^2 increases, Eq. (2) does not allow to extrapolate $F_{2,NS}^N(Q^2)$ to too high values of Q^2 . This is because taking Q_0^2 small the higher order correction term can in principle change the sign of $F_{2,NS}^N(Q^2)$ at large Q^2 and N ($N = 8, 10$).

The constants Z_{NS}^N and $\bar{B}_{2,NS}^N$ are given in Ref. 3) and γ_0^N , β_0 and $\bar{\alpha}(Q^2)$ are as before. The non-singlet quark distribution $q_V^N(Q_0^2)$ will now be fitted to the data by

$$F_{2,NS}^N(Q^2) = q_V^N(Q_0^2) \frac{1}{6} \left(1 + \frac{\bar{\alpha}(Q^2)}{4\pi} \bar{B}_{2,NS}^N \right) \left(1 + \frac{\bar{\alpha}(Q^2) - \bar{\alpha}(Q_0^2)}{4\pi} Z_{NS}^N \right) \left(\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right)^{\gamma_0^N/2\beta_0} \quad (21)$$

Unlike in the case of Section 2 we work here in the $\overline{MS}$ scheme where the terms $\frac{1}{2}\gamma_0^N(\ln 4\pi - \gamma_E)$ appearing in the calculation of $B_{2,NS}^N$ have been absorbed into a redefinition of Λ , and this gives a slightly larger value of Λ for the best fit. The correspondence between the two parametrizations is easy to establish : comparing Eqs. (2) and (20) one finds

$$q_V^{(I)N}(Q_0^2) = q_V^{(II)N}(Q_0^2) \left(1 + \frac{\bar{\alpha}(Q_0^2)}{4\pi} \bar{B}_{2,NS}^N \right) \quad (22)$$

where the superscripts I and II refer to the parton distribution definitions of Section 2 and of this section, respectively. The $\bar{B}_{2,NS}^N$ are small at small N but increase with N reaching the value 20 for $\bar{B}_{2,NS}^{10}$. The two parametrizations will therefore differ substantially at large N , i.e., at large x for the inverted functions with $q_V^{(II)}(x, Q_0^2)$ falling below $q_V^{(I)}(x, Q_0^2)$.

We turn now to the more complicated singlet case. The gluon and singlet quark distributions are defined by the decomposition

$$Q_2^N(Q^2) = \frac{5}{18} \left\{ q_s^N(Q^2) \left(1 + \frac{\bar{\alpha}(Q^2)}{4\pi} \bar{B}_{2,s}^N \right) + \frac{\bar{\alpha}(Q^2)}{4\pi} g^N(Q^2) \bar{B}_{2,g}^N \right\} \quad (23)$$

where the singlet quark and gluon moments satisfy the following evolution equations

$$q_s^N(Q^2) = \left(q_s^N(Q_0^2)(1 - d_N) - g^N(Q_0^2) \tilde{\alpha}_N \right) \left(\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right)^{\gamma_+^N/2\beta_0} H_{+4}^N(Q^2, Q_0^2) \\ + \left(q_s^N(Q_0^2) d_N + g^N(Q_0^2) \tilde{\alpha}_N \right) \left(\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right)^{\gamma_-^N/2\beta_0} H_{-7}^N(Q^2, Q_0^2) \quad (24)$$

$$g^N(Q^2) = \left(g^N(Q_0^2) \alpha_N - q_S^N(Q_0^2) \varepsilon_N \right) \left(\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right)^{\gamma_+^N/2\beta_0} H_{+G}^N(Q^2, Q_0^2) \\ + \left(g^N(Q_0^2) (1 - \alpha_N) + q_S^N(Q_0^2) \varepsilon_N \right) \left(\frac{\bar{\alpha}(Q^2)}{\bar{\alpha}(Q_0^2)} \right)^{\gamma_-^N/2\beta_0} H_{-G}^N(Q^2, Q_0^2) \quad (25)$$

The parameters to be fitted are $q_S^N(Q_0^2)$ and $g^N(Q_0^2)$ while all other quantities are calculable in QCD and can be found in the review by Buras ³⁾.

We would like to make the following comments at this point. Each of Eqs. (24) and (25) can be written in a form similar to Eq. (6) and the numerical coefficients of $\bar{\alpha}(Q^2)$ and $\bar{\alpha}(Q_0^2)$ are rather small (except for a few coefficients appearing in the fourth moments of the gluon equations when one assumes four active flavours ³⁾). As $\bar{B}_{2,G}^N$ are also small the correction terms are mainly controlled by $\bar{B}_{2,S}^N = \bar{B}_{2,NS}^N$. Thus in this case the evolution of the parton distributions will not be very different from the leading order calculation although the normalization at Q_0^2 will be ¹⁵⁾. One should also note that as $g^N(Q_0^2)$ appears always multiplied by a small coefficient it will not be well-determined from the data. From Eqs. (24) and (25) one finds that $g^2(Q^2) + q_S^2(Q^2)$ is independent of Q^2 and should be normalized to one if it is to be interpreted as parton momentum conservation inside the hadron. This constraint will not be imposed in our fit but we shall check that it is satisfied within the error bars.

We parallel now the discussion of Section 3 on the fit. In Fig. 2b we present the χ_{df}^2 as a function of Λ for each moment separately and we find that $\chi_{df}^2 \leq 1$ for all moments in the range $\Lambda = 0.5 - 0.9$ GeV. This slightly higher value of Λ compared to the results of Section 3 (for case I) is due to the different regularization scheme (now $\overline{MS}$) ³⁾. Using the same value of Λ for all N we fit the moments of the parton distributions at different Q_0^2 to find the best Λ value. The quality of the fit is similar as in case I in the interval $5 \text{ GeV}^2 \leq Q_0^2 \leq 80 \text{ GeV}^2$ and we find $\chi_{df}^2 = 24.3/75$ at $\Lambda = 0.60$ for the best fit. Figure 3 shows that the fits to the moments differ slightly for large N so that at large Q^2 the curve for case II is above the case I curve. This is true also for the extrapolations to large Q^2 values (Fig. 5) the difference between the two cases being, however, less than the error bars of the preliminary data. The parton distribution moments for case II are shown in Figs 4d,e,f and as expected they coincide with case I for $N = 2$ moments whereas the higher moments of q_V

and q_S are smaller for case II. The high gluon moments decrease more slowly with increasing Q^2 in case II so that in x space the gluon distribution for case II is steeper at small Q^2 but the situation is reversed at higher Q^2 . Thus it seems that scale violation in the gluon distribution is stronger for the case I definition of parton distributions beyond leading order. However, the large error bars in the gluon moments (especially at high N values) prevent a more quantitative analysis of this statement.

A similar fit as was done in Section 3 to the moments of the parton distributions gives at $Q_0^2 = 5 \text{ GeV}^2$ the following analytical forms

$$q_V^{\text{II}}(x, Q_0^2 = 5) = C (1-x)^{3.12 \pm 0.24} / x^{0.12 \pm 0.11} \quad (\chi_{df}^2 = 0.03) \quad (26)$$

$$q_S^{\text{II}}(x, Q_0^2 = 5) = (4.85 \pm 0.66) (1-x)^{3.08 \pm 0.10} / x^{0.57 \pm 0.07} \quad (\chi_{df}^2 = 0.07) \quad (27)$$

$$g^{\text{II}}(x, Q_0^2 = 5) = (15.1 \pm 4.5) (1-x)^{3.27 \pm 0.41} \quad (\chi_{df}^2 = 2.33) \quad (28)$$

5. - CONCLUSIONS

Thus we conclude that while the present $e(\mu)$ deep inelastic scattering data on the moments of structure functions ⁷⁾ do not permit us to determine the exact value of Λ inside the range $\Lambda = 0.3-0.6 \text{ GeV}$ (case I, $\overline{\text{MS}}$ scheme) or $\Lambda = 0.5-0.9 \text{ GeV}$ (case II, $\overline{\text{MS}}$ scheme), we can extract information about the parton distributions for the two definitions of these distributions beyond leading order. Comparison between the non-singlet quark distributions of case I and case II shows that at $Q_0^2 = 5 \text{ GeV}^2$ the case II distribution is somewhat steeper than for case I. This is in good agreement with the result of Bielas and Buras ¹⁶⁾. The gluon distribution can be determined (with large errors) and it is rather flat $\sim (1-x)^3$ at $Q_0^2 = 5 \text{ GeV}^2$ in both cases I and II. However, already at $Q_0^2 = 20 \text{ GeV}^2$ the gluon has a $(1-x)^5$ behaviour, which is consistent with the results from neutrino experiments ¹⁶⁾. One can understand the stronger scaling violations in gluon distribution by comparing the anomalous dimensions,

which control (at the leading logarithmic level) the Q^2 evolution of the parton distributions. Neglecting quark-gluon mixing the relevant evolution variable for gluon is $6\cdot\bar{S}$ [$\bar{S} = \ln(\ln(Q^2/\Lambda^2)/\ln(Q_0^2/\Lambda^2))$], whereas the corresponding variable for quark is $\frac{4}{3}\cdot\bar{S}$. The numbers 6 and $\frac{4}{3}$ are the colour factors for the three-gluon vertex and quark-gluon vertex, respectively.

We have not studied in this paper the effects of higher-twist terms in QCD as this would introduce new free parameters in the fits. As these higher-twist effects may affect the QCD fits considerably ¹⁷⁾, we hope to return to this subject later.

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FIGURE CAPTIONS

- Figure 1   The χ^2 per degree of freedom curves for different moments
($N = 2, 4, 6, 8$ and 10) for the fit with the leading order QCD formulae.
- Figure 2   The χ^2 per degree of freedom curves for different moments for the
fit with the higher order QCD corrections.
a) parton distribution definition of case I ;
b) parton distribution definition of case II.
- Figure 3   The Q^2 dependence of the moments of the structure function F_2 for
proton (Fig. 3a) and for deuterium (Fig. 3b) targets. The data points
are from Ref. 7), and the solid line is the higher order QCD fit with
case I parton distribution definition and the dashed line the same
with case II parton distributions.
- Figure 4   The Q^2 dependence of the moments of the parton distributions
 q_V^I (Fig. 4a), q_S^I (Fig. 4b) and $gluon^I$ (Fig. 4c) obtained from
the fit with higher order corrections and case I definition of the
distributions beyond leading order. The same is presented for
case II definition of parton distributions in Figs. 4d (q_V^{II}),
Fig. 4e (q_S^{II}) and Fig. 4f ($gluon^{II}$).
- Figure 5   The extrapolation for the $N = 4, 6$ moments of the structure function
 $F_2(x, Q^2)$ using the QCD formulae with higher order corrections. The
dashed line is for case I and the dash-dotted line for case II. The
experimental points shown are from the SLAC-FNAL deuterium measure-
ments 7) and preliminary results from the NA4 experiment at CERN 14).

FIG.1

FIG. 2a

FIG. 3a

FIG. 3b

FIG. 4b

FIG. 4c

FIG. 4d

FIG. 4e

FIG.4 f

FIG. 5