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COMPOSITE VECTOR FIELDS AND TUMBLING GAUGE THEORIES

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A B S T R A C T

In non-Abelian gauge theories with a non-real fermion content it is possible that vector channels become more attractive than the most attractive scalar channel. It is speculated that the appearance of a vector condensate may signal the presence of massless composite vector fields which gauge a flavour symmetry of the underlying model, as is suggested by earlier work on non-linear spinor models.

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1. INTRODUCTION

The presence of fundamental scalar fields in grand unified theories leads in general to unnatural conditions on the parameters of the theory. A possible solution to this problem arises if one considers the Higgs fields as composite. This idea dates back to the very first models of spontaneous symmetry breaking¹⁾. It assumes the occurrence of some non-perturbative phenomenon, which is notoriously difficult to establish theoretically. Recently, a more phenomenological approach has been suggested, largely based on our present understanding of chiral-symmetry-breaking in the strong interactions. In QCD, this breaking is caused by a non-vanishing vacuum expectation value $\langle \bar{\psi}\psi \rangle \neq 0$, corresponding to a condensate of tightly bound fermion-antifermion pairs, and leads, because of the Nambu-Goldstone mechanism, to the existence of massless pseudoscalar bound states: the pions. Introducing a new gauge interaction among new constituents (techni-, meta-, primed, etc., quarks) with a running coupling constant which becomes of order unity at energies in the TeV range, it is possible to generate the Higgs bosons of the Weinberg-Salam model in a similar way²⁾. Subsequently it was pointed out³⁾ that if the fermion content of the new theory is not real, then the scalar condensate cannot be a singlet under the gauge group G , so that it necessarily breaks G down to some subgroup $G' \subset G$: this is a phenomenon usually denoted as "tumbling".

What we would like to emphasize in this paper is that in tumbling gauge theories one should not only consider scalar channels, but also the possibility of vector channels, as these may be more attractive, at least according to the simple criteria employed. In Section 2 we calculate the ratio of "attractivity" in the scalar and vector channels, a_v/a_s , for a wide class of models. We find that in general $a_v/a_s \geq 1$, i.e. the vector channel is preferred to the scalar channel as far as group factors are concerned. On the other hand, we also show that for the class considered the ratio is actually bounded.

How does a vector condensate manifest itself? At first sight it appears to be an unacceptable nuisance, as it breaks sacred Lorentz invariance. As we will discuss in Section 3, this is not necessarily so. We recall some results obtained in spinor theories with quartic self-interaction, where this idea has been investigated quite extensively. In principle, it seems possible that the vector condensate generates massless vector fields, which gauge a flavour symmetry of the underlying theory, being Nambu-Goldstone bosons of broken Lorentz invariance. A remarkable aspect of this mechanism is that the presence of the Lorentz-symmetry-breaking condensate escapes observability *in the effective gauge theory* through a subtle conspiracy between the choice of the direction η^μ of the condensate and the resulting (axial type) gauge-fixing condition $\eta^\mu A_\mu = 0$. Along these lines one may imagine that the formation of a vector condensate signals the appearance of new effective gauge fields.

In Section 4 we briefly discuss the effect of various anomalies. We conclude with a brief discussion.

2. SCALAR VERSUS VECTOR CONDENSATE

Let us consider an asymptotically free theory with gauge group G , and some massless left-handed fermions in a not necessarily irreducible representation R of G which is asymptotically free. We use left-handed two-component Weyl spinors, i.e. objects that transform as $(2,1)$ under the homogeneous Lorentz group. Candidates for a scalar condensate are the irreducible components of $R \times R$, because the scalar $(1,1) \subset (2,1) \times (2,1)$. Raby et al.³⁾ have argued that the scalar condensate will form in the most attractive channel (MAC), which is determined by looking at the strength of the force in the single gluon exchange approximation. Two irreducible components R_1 and R_2 of R will form a condensate in the representation R' ($\subset R_1 \times R_2$) if the quantity

$$a_s = C(R') - C(R_1) - C(R_2) \quad (1)$$

is the most negative for that particular combination $R_1 R_2$, and R' . Here $C(R_i)$ is the eigenvalue of the quadratic Casimir operator defined by $\sum_a T^a(R_i) T^a(R_i) = C(R_i)$ and the $T^a(R_i)$ form the representation of the generators acting on R_i . If the representation R is not real (i.e. $R \neq R^*$), then R' cannot be a singlet under G leading to the effect of tumbling, where the condensate breaks the group.

Returning to our fundamental fermion fields q , which transform as $(R, 2, 1)$, we may construct the quantity $i\sigma_2 q^*$, which is a right-handed field. [Remember that if we have the helicity operator $\vec{\sigma} \cdot \vec{p} / |\vec{p}| = h$ act on q with $hq = -q$, then one has $h(i\sigma_2 q^*) = i\sigma_2 q^*$.] So $i\sigma_2 q^*$ transforms as $(R^*, 1, 2)$, implying that with this convention $q^\dagger i\sigma_2 \sigma_\mu q$ transforms like $(R \times R^*, 2, 2)$, i.e. a Lorentz vector. It should, furthermore, be noted that $R \times R^*$ always contains a singlet! If we now define the attractivity of a vector channel in analogy with the scalar case as

$$a_v = C(R') - C(R_1) - C(R_2)$$

then the most attractive vector channel always has

$$a_v = -2 C(R_m)$$

where R_m is the irreducible component of R with the largest Casimir. We used the fact that $R' = \underline{1}$ with $C(\underline{1}) \equiv 0$ and $C(R_m) = C(R_m^*)$. Obviously $a_v \geq a_s$ for the maximally attractive channels, where the equal sign applies if R is real.

To decide which of the various scalar channels is the most attractive, it is reasonable just to compare the values of the respective a_s , but if one wants to compare the difference between scalar and vector channels the situation is more complicated. Even in the single-gluon exchange approximation, the Hartree equations for the vector and scalar condensate (or the Bethe-Salpeter equation for the bound state) have a different form {in fact for a four-fermion interaction $[\bar{\psi}\gamma^\mu(1 - \gamma_5)\psi]^2$, with massless fermions, the relevant Bethe-Salpeter equations at zero relative momentum differ by a factor of two}. In general, one expects, however, that the spin-dependent contribution to the force favours binding in the scalar channel, so that for a vector to become more attractive a_v should certainly be larger than a_s (see the QCD example in Section 3). From group considerations alone, a_v/a_s can be arbitrary large, and thus vector condensates have to appear in some models whatever the spin suppression may be. A more careful analysis shows, however, that this ratio is rather small in all cases we will consider hereafter*).

A tumbling gauge theory is one which has the following properties:

- a) asymptotic freedom,
- b) no anomalies,
- c) non-real fermion content.

Condition (c) limits our attention to $SU(N)$ ($N \geq 3$), $SO(4N + 2)$ and $E(6)$. Whereas each condition separately allows for arbitrary large a_v/a_s values, the combination of them provides a stringent bound on a_v/a_s . It should be mentioned though that for the $SU(N)$ groups, we have restricted ourselves to models with fermions in basic representations only (i.e. fully antisymmetric tensor representations). The results are listed in Table 1. We illustrate its content by considering the second row. Take an $SU(N)$ model for $N \geq 6$, with fermions in $\frac{1}{2}(N - 3)(N - 6)$ times a $[1]$ (the fundamental representation) and a single $[N - 3]$ (the fully antisymmetrical tensor of rank $N - 3$). This theory is anomaly free. The column labelled AF tells us that the theory is asymptotically free for $N \leq 17$. Now there are two possible cases:

- i) If we choose two families, i.e. doubled the fermion content, and $N \leq 12$, then the scalar MAC would be the $[N - 6] \subset [N - 3] \times [N - 3]$. This leads with the vector channel in the $[0] \subset [N - 3] \times [N - 3]$ to $a_v/a_s = (N - 3)/3$ and with the vector channel $[0] \subset [1] \times [1]$ to $a'_v/a_s = (N - 1)/9$.

*) Through the self-interaction of the gluons it is also possible to have a scalar singlet bound state in the two-gluon channel. Pure group considerations would favour a condensate in this channel, because the Casimir of the adjoint representation is usually larger than those of the fermion representations. In this case too it is not possible to make a comparison at the level of the Hartree equations.

- ii) If we have a single family, then the scalar $[N - 6]$ vanishes by symmetry, the $[N - 2] \subset [N - 3] \times [1]$ becomes the scalar MAC, which is also the case in general if $N \geq 12$. This yields $a_V/a_S = 3$ and $a'_V/a_S = (N - 1)/(N - 3)$.

From the last columns of the table it is apparent that in the $SU(N)$ models the vector channel never becomes much more attractive than the most attractive scalar channel ($a_V/a_S \leq 3$), so that, even if a vector condensate is possible in principle, the critical (unknown) value for a_V/a_S may never be reached.

For the orthogonal groups the situation turns out to be not much better. With fermions in the 16-dimensional spinor representation of $O(10)$ one obtains $a_V/a_S = 5/3$. In $O(18)$ with the 256 one finds $a_V/a_S = 153/119$. Taking one 64-dimensional spinor in $O(14)$ one has the circumstance that in $64 \times 64 = 14 + 364 + \dots$ the 14 actually vanishes by symmetry, so that the 364 would form leading to $a_V/a_S = 91/25$. As for the exceptional group $E(6)$ with the 27-dimensional spinor one has $a_V/a_S = 2$, which completes the analysis for the most interesting cases.

3. AN INTERPRETATION OF THE VECTOR CONDENSATE

What are the physical consequences of the presence of the vector condensate? A comprehensive analysis of this situation has been made by Björken⁴⁾, who, in complete analogy with the Nambu-Jona-Lasinio model, considered a pure spinor model with a $[\bar{\psi}\gamma^\mu\psi]^2$ interaction. Choosing an appropriate cut-off procedure the possibility arises that $\langle \bar{\psi}\gamma^\mu\psi \rangle = \eta^\mu$, where η^μ is taken to be some constant Lorentz vector, guaranteeing energy-momentum as well as current conservation. Summing the bubble graphs in the $\bar{\psi}\gamma^\mu\psi$ channel a massless photon propagator is obtained as one would get from the one-loop calculation of the polarization tensor $\Pi_{\mu\nu}$ in ordinary QED in the axial gauge $\eta^\mu A^\mu = 0$. The equivalence of this theory and QED persists to all orders in perturbation theory (up to corrections of order $1/G\Lambda^2$, where G is the strength of the Fermi coupling and Λ the cut-off). This result is usually interpreted as follows⁵⁾. The composite massless vector field arises as a consequence of the Nambu-Goldstone theorem applied to the case of spontaneously broken Lorentz invariance. As the field is coupled to a conserved current, one of the components decouples from the physical spectrum, leaving us with two physical components, i.e. a gauge field. More remarkable is the fact that it is exactly the effective gauge invariance which protects the breaking of Lorentz invariance from detection. This can be seen from the way the gauge condition $\eta^\mu A^\mu = 0$, which appears, is linked to the direction η^μ of the condensate. Alternatively, one could think of η^μ as a constant background potential, which has no physical implications. The coupling constant of the effective theory is found to be $\alpha \propto 1/\log(\Lambda/m)$, where m is the mass or typical momentum transfer of the fermions. Note that it is independent of the Fermi coupling constant G . This

suggests an alternative way of interpreting a large difference in coupling strength between different interactions.

Since a vector condensate does not break chiral symmetry, it does not generate masses for the fermions, and this has important consequences if one wants to combine these ideas with tumbling as we will see later.

The Björken model has been generalized to non-Abelian theories⁶⁾. In this case, one starts with, for example, an $SU(N)$ global symmetry through an interaction of the form $(\bar{\psi}\gamma^\mu\lambda^a\psi)^2$. In the case of non-Abelian theories, not only the vector boson propagator (as in Björken's model) but also the 3- and 4-gluon vertices must be generated out of divergent (but suitably cut-off) contributions from the original four-fermion interaction. The obvious candidates for this purpose are respectively the fermion triangle and square. The relative weight and Lorentz structure of these effective vertices must furthermore comply with the usual Ward identities. While this seems an extremely stringent requirement, it is amusing to note that it is automatically fulfilled by the lowest-order contributions. One can easily check it by direct calculation, or more simply by noting that exactly the same graphs contribute to the renormalization of 3- and 4-point vertices also in the usual formulation of gauge theories with fermions and therefore their divergent parts have to present the correct weight and Lorentz structure.

Different methods of handling this case have been presented in the literature, either via functional integration, or explicit calculation à la Nambu-Jona-Lasinio.

Just as the Nambu-Jona-Lasinio model for the pion was successfully carried over to QCD, we like to speculate that also the Björken model or some generalization thereof may in certain cases be interpreted as resulting from an underlying strongly interacting gauge theory. The formation of a vector condensate in the underlying theory would then manifest itself through the effective gauging of a flavour symmetry rather than through the breaking of Lorentz invariance! As is apparent from Table 1, requiring the theory to be anomaly-free leads in most cases to the introduction of various identical irreducible multiplets, so that a global $U(N)$ symmetry is automatically present. This $U(N)$ may be gauged, as the colour forces are flavour blind (i.e. tasteless).

As a hypothetical example let us consider massless two-flavour QCD with the assumption that the vector condensate would form even though $a_v/a_s = 1$. Under $SU(3)_{\text{colour}} \times SU(2)_3 \times SU(2)_{\bar{3}}$ we have left-handed fermions transforming as a $(3,2,1)$ and a $(\bar{3},1,2)$. The celebrated scalar condensate which breaks chiral symmetry is the $(1,2,2) \subset (3,2,1) \times (\bar{3},1,2)$ with $a_s = C(1) - C(3) - C(\bar{3}) = -8/3$. The two vector condensates would transform as $(1,1,1)_3 \subset (3,2,1) \times (\bar{3},2,1)$ and as $(1,1,1)_{\bar{3}} \subset (\bar{3},1,2) \times (3,1,2)$. The formation of these condensates would give rise to an effective gauging of $SU(2)_3 \times SU(2)_{\bar{3}} \times U(1)_{3-\bar{3}}$. We have assumed that the

$U(1)_{3+\bar{3}}$ is broken by the axial anomaly (as will be discussed in the next section). Since the vector condensate would not generate masses for quarks or gluons, the scalar condensate would form as well. The massless Goldstone pions would now be eaten by the newly generated vector particles, leaving us with an effective $SU(2)_{3-\bar{3}} \times U(1)_{3-\bar{3}}$ theory. Needless to say this picture is in total disagreement with the experimental facts, which we at this stage have interpreted as an indication that the vector condensate will not form if $a_v/a_s = 1$.

As a possibly more realistic example, let us discuss the standard $SU(5)$ tumbling model with two flavours. The fermions transform as $(5,2,1) + (\bar{10},1,2)$ under $SU(5) \times [SU(2)_5 \times SU(2)_{\bar{10}} \times U(1)_{3N_5-N_{\bar{10}}}]$, where the global symmetry group is given between the brackets. The vector channel $(1,1,1) \subset (\bar{10},1,2) \times (10,1,2)$ is more attractive than the scalar MAC $(5,1,3) \subset (\bar{10},1,2) \times (\bar{10},1,2)$ with a ratio $a_v/a_s = 3/2$. The vector condensate which we assume to form at some scale M_0 gives rise to the effective gauging of $SU(2)_{\bar{10}}$. Then at M_1 the scalar condensate would form which breaks $SU(5) \rightarrow SU(4)$ and also $SU(2)_{\bar{10}} \rightarrow U(1)_{\bar{10}}$. Fermions decompose as $5 \rightarrow 4 + 1$ and $\bar{10} \rightarrow 6 + \bar{4}$. The 6 takes part in the condensate and gets a mass $O(M_1)$, as do the $SU(5)/SU(4)$ gauge bosons. The $SU(2)/U(1)$ bosons get a mass $O(\sqrt{\alpha} M_1)$, where α is the fine-structure constant of the effective theory. At an energy scale well below M_1 we have the following symmetry $SU(4) \times U(1)_{\bar{4}} \times [SU(2)_4 \times SU(2)_1 \times U(1)_{N_4-N_{\bar{4}}+2N_1}]$, where the last $U(1)$ factor is generated by the anomaly-free linear combination $Q' = 1/5(2Q - X)$ with $Q = 3N_5 - N_{\bar{10}} = 3N_4 + 3N_1 - N_{\bar{4}} - N_6$ and the diagonal $SU(5)/SU(4)$ generator $X = N_4 - 4N_1 + 3N_{\bar{4}} - 2N_6$. The surviving fermions transform as $(4,0,2,1,1) + (\bar{4},1,0,1,-1) + (1,0,1,2,2)$. Next a scalar condensate in the $1 \subset 4 \times \bar{4}$ forms, producing three Goldstone bosons of which only one gets eaten to break the $U(1)_{\bar{4}}$. The 4 and $\bar{4}$ obtain a mass $O(M_2)$, whereas the $U(1)_{\bar{4}}$ gauge particle gets a mass $O(\sqrt{\alpha} M_2)$. We are now left with $SU(4) \times [SU(2)_1 \times U(1)_{N_1}]$ and two massless fermions (neutrinos) in the $(1,2,1)$. The pure tumbling model without taking vector condensates into account would have produced five Goldstone particles, in our scheme three of these are actually used to generate masses for the new gauge particles. Such a Higgs mechanism in the effective theory might provide a way to eliminate unwanted Goldstone bosons from tumbling theories.

Instead of dwelling further on the consequences and possibilities of this idea, we would like to make some more comments on its feasibility. First of all it is our impression that even within the framework of the four-fermion models the quoted results have not been rigorously shown to be correct. In the first place the actual proof of a solution to the Hartree equations corresponding to a vector condensate has not been given⁷⁾. But even assuming the existence of such a solution, demonstrating the equivalences of these different theories

involves a rather intricate juggling with highly divergent quantities. From the physical point of view, it is puzzling how a local symmetry appears to be realized where it is not manifestly present on the fundamental level. In this respect, there seems to be an essential difference between these and the two-dimensional $CP(N-1)$ models, where a gauge field appears to be generated dynamically as well. In the cases at hand the local symmetry is achieved through a judicious choice of the cut-off. In our picture everything is fixed by the underlying interaction, so is the cut-off Λ (which is real). This has the advantage that $\alpha \approx 1/\log(\Lambda/m)$ may be a sensible quantity. A problem may be that the gauge- and Lorentz-symmetry-breaking effects which are of order $(1/G\Lambda^2)$ set in at relatively low energies; on the other hand, it is conceivable that G would become very large. Even accepting the fact that the breaking of Lorentz invariance is undetectable in the effective theory it may be detectable through other interactions (notably gravity). Needless to say, further investigation of these points is desired.

4. ANOMALIES

So far we have only been concerned with the anomalies in the underlying strong interaction theory. That this sector be anomaly-free is required by the renormalizability and thus consistency of the model. The question is different for the effective gauge theory, since, by definition, this theory has only a meaning for momenta smaller than the cut-off Λ . In the scheme presented there are two types of anomalies to worry about:

- i) anomalies induced in the effective theory by the underlying theory;
- ii) anomalies entirely within the effective theory.

The first type arises because certain global symmetries of the original model may be violated through Adler-Bell-Jackiw anomalies. These can have important consequences for the observable spectrum of low-lying states (η -mass). Here they would generate an "infinite" mass term for the would-be gauge fields which couple to the corresponding anomalous currents, through the diagram of Fig. 1⁸⁾. An infinite mass term presumably implies that these vectors do not bind in the first place.

More problematic seems the second type of anomalies. These are necessarily present if the relevant global symmetry is $U(N)$ with $N \geq 3$, because by definition we have only a single fundamental representation under that global group. Here we have to assume that the dynamics allows at most some maximal anomaly-free subgroup of the flavour group to be gauged. Neglecting $U(1)$ factors which might appear one has $Sp(2N) \subset SU(2N)$ and $SO(2N+1) \subset SU(2N+1)$ or $[\otimes SU(2)]^N \subset SU(2N(+1))$. It is then necessary, however, that the vectors corresponding to the generators of the coset space do not bind. Merely giving them a

mass is not sufficient, because through the diagram of Fig. 1 that mass would proliferate to the other fields as well. $O(N)$ and $Sp(N)$ symmetries are not the ones we observe in Nature. $SU(2) \otimes SU(2) \otimes U(1)$ is of potential interest though, but in that case colour $SU(3)$ must be part of the underlying theory, which survives after tumbling. In such models colour and weak gauge bosons would differ essentially, the former being fundamental, the latter composite. We have worked out some of the previously-mentioned theories according to our rules, but have so far not succeeded in constructing a realistic example.

5. CONCLUSIONS

In theories with non-real fermion content it is conceivable that a vector condensate is formed. For a wide class of models we have compared the group theoretical factors for the vector and scalar channels and find their ratio to be between one and four. Subsequently, we have speculated that a flavour symmetry of the underlying theory may be gauged effectively as a consequence of the vector condensate. To promote this speculation to a more respectable level, either further theoretical investigations are needed, or a realistic model should be produced.

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Table 1
Comparison between vector and scalar channels in $SU(N)$ theories

	[1]	[2]	[N-2]	[N-3]	[N-4]	[N-5]	AF	a_V/a_S	a'_V/a_S	Remarks
$N \geq 4$	$N-4$		1				$4 \leq N$	$(N-2)/2$	$(N-1)/4$	$N \leq 6$
$N \geq 6$	$\frac{1}{2}(N-3)(N-6)$			1			$6 \leq N \leq 17$	2	$(N-1)/(N-2)$	$N \geq 6$
$N \geq 8$	$\frac{1}{6}(N-3)(N-4)(N-8)$				1		$8 \leq N \leq 10$	$(N-3)/3$	$(N-1)/9$	$N \leq 12, 2 \text{ fam.}$
$N \geq 10$	$\frac{1}{24}(N-1)(N-4)(N-5)(N-10)$					1	$N = 10$	3	$(N-1)/(N-3)$	$N \leq 12, 1 \text{ fam.}$ $N \geq 12$
$N \geq 6$		$\frac{1}{6}(N-3)(N-6)$		$N-4$			$6 \leq N \leq 7$	$(N-4)/4$	$(N-1)/16$	
$N \geq 8$		$\frac{1}{6}(N-3)(N-4)(N-8)$			$N-4$		$N = 8$	-	-	

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FIGURE CAPTION

Fig. 1 : Mass generation through anomaly.

FIG. 1