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THE USE OF DIMENSIONAL RENORMALIZATION SCHEMES
IN UNIFIED THEORIES

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A B S T R A C T

We present details and clarify some points in a recently proposed modification of the minimal subtraction adapted to unified theories. As an application we give a precise value of the superheavy gauge boson masses in SU(5).

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1. INTRODUCTION

In a recent paper¹⁾ (from now on referred to as paper I), we slightly modified the minimal subtraction scheme²⁾ to incorporate the decoupling features of the Appelquist and Carazzone theorem³⁾. Since then, a growing body of literature⁴⁾⁻⁷⁾ has developed more or less similar approaches and we intend to give, in the present paper, some technical details and present some points left obscure in paper I.

In Section 2, we recall the procedure developed in paper I; Section 3 relates our scheme to the ordinary minimal subtraction scheme used at low energies; Section 4 gives technical details for arbitrary orders; the reader who is convinced of the absence of threshold effects in our procedure can go directly to Section 5, where we use our method to compute a precise value of the superheavy gauge boson masses and of the weak angle within the SU(5) model. Concluding remarks are given in Section 6.

2. THE DECOUPLING SUBTRACTION

In paper I, we discussed two features of the usual minimal subtraction (MS) scheme: the absence of decoupling à la Appelquist and Carazzone³⁾ (every divergent loop, massive or not, contributes to the β function) and the appearance of large logs in the finite contributions (the $\ln(M/\mu)$'s, M being a heavy mass and μ the dimensional regularization scale, are, in the MS scheme, left over). To cure those two unacceptable properties, we defined what we shall, from now on, call a decoupling subtraction (DS):

- i) if $\mu \geq M$, one subtracts the pole $1/\epsilon$ ($\epsilon = 4 - n$; space-time dimension). Therefore, in this region, we keep the minimal subtraction
- ii) if $\mu \leq M$, one subtracts the terms $((1/\epsilon) - \ln(M/\mu))$. Notice that our procedure is continuous at $\mu = M$ *).

It is a straightforward affair to see how the decoupling works. We write the bare coupling g_0 (dimensionful) in terms of the renormalized one g_R (dimensionless)

$$g_0 = \mu^{\epsilon/2} \frac{1}{Z^{1/2}} g_R \quad (2.1)$$

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*) For those who find non-local counter-terms in our procedure, we can phrase it in a different way: have a scheme I corresponding to $1/\epsilon$ subtraction (on the whole range of μ) and a scheme II corresponding to $((1/\epsilon) - \ln(M/\mu))$ subtraction (on the whole range of μ); we work in scheme I when $\mu \geq M$ in scheme II when $\mu \leq M$ and clearly $g_I(M) = g_{II}(M)$ to all orders in perturbation theory.

where μ is the dimensional scale²⁾ already defined (actually, Eq. (1) fixes it) and Z the renormalization constant. Then the β function involves the following derivatives²⁾

$$\begin{aligned} \frac{d}{d \ln \mu} \left(-\frac{1}{\epsilon} \alpha_R \right) &= \frac{d}{d \ln \mu} \left(-\frac{1}{\epsilon} \mu^{-\epsilon} \alpha_0 \right) = \alpha_R & \text{if } \mu \geq M \\ \frac{d}{d \ln \mu} \left[\left(-\frac{1}{\epsilon} + \ln \frac{M}{\mu} \right) \alpha_R \right] &= 0 & (2.2) \\ & \text{if } \mu \leq M \end{aligned}$$

which shows that heavy particles decouple at low energies.

Such a decoupling property is of extreme importance in unified theories where we want the low energy world to be an effective $SU(3) \otimes U(1)$ world, whatever the unifying group is. For example, if a simple group G breaks into a product of simple (or $U(1)$) groups $\otimes_i G_i$ with the DS scheme the effective coupling constant for the group G_i ($\alpha_{DS}^i = (g_{DS}^i)^2/4\pi$) will be expressed in terms of the coupling constant of G ($\alpha = g^2/4\pi$) by

$$\alpha_{DS}^i(\mu)^{-1} = \alpha(\mu)^{-1} + \frac{11}{6\pi} (C_2(G) - C_2(G_i)) \ln \frac{M}{\mu} \quad (2.3)$$

where $C_2(G)$ (respectively $C_2(G_i)$) is the second order Casimir of the group G (respectively G_i), M is the superheavy gauge boson mass and the different effective couplings will join the unifying group coupling strictly at $\mu = M$ (no threshold effects). In Eq. (2.3), for simplicity, we did not take into account heavy fermions or heavy (unphysical or physical) scalar particles (for a complete treatment see next section).

In paper I, it was shown by a diagrammatic analysis for heavy fermions that the summation of the large $\ln M/\mu$ that appear in the MS scheme yields a result of the type of Eq. (2.3). Both procedures are equivalent. Equation (2.3) provides a determination of M in terms of low energy data. For example, with $SU(5)$ breaking to $\otimes_{i=1}^3 G_i = U(1) \otimes SU(2) \otimes SU(3)$, we obtain

$$\begin{aligned} \alpha_{e.m.}^{-1}(M_W) - \frac{8}{3} \alpha_3^{-1}(M_W) &= \alpha_2^{-1}(M_W) + \frac{5}{3} \alpha_1^{-1}(M_W) - \frac{8}{3} \alpha_3^{-1}(M_W) \\ &= \frac{11}{6\pi} \left(\frac{8}{3} C_2(G_3) - C_2(G_2) - \frac{5}{3} C_2(G_1) \right) \ln \frac{M}{M_W} \end{aligned} \quad (2.4)$$

where $\alpha_{e.m.}$ is the QED coupling constant [after breaking of $SU(2) \otimes U(1)$].
Using Λ defined by

$$\alpha_3(M_W) = \frac{4\pi}{\left[\frac{11}{3}C_2(G_3) - \frac{4}{3}T(R)N_f\right] \ln \frac{M_W^2}{\Lambda^2}} \quad (2.5)$$

where N_f is the number of fermions, Eq. (2.4) now reads

$$\ln \frac{M}{M_W} = \frac{\pi}{11} \left[-2 \frac{33-2N_f}{9\pi} \ln \frac{M_W^2}{\Lambda^2} + \alpha_{e.m.}^{-1}(M_W) \right] \quad (2.6)$$

Finally, we end this section with a simple remark that will prove to be important. We are perfectly authorized to subtract $\ln M\kappa/\mu$ (with κ a constant) instead of $\ln M/\mu$. Then the couplings join together at $\mu = \kappa M$ and Eq. (2.4) now reads

$$\alpha_{(e.m.)}^{-1}(M_W) - \frac{8}{3} \alpha_{3(\kappa)}^{-1}(M_W) = \frac{11}{\pi} \ln \frac{\kappa M}{M_W} \quad (2.7)$$

Of course, this gives the same value for M as (2.4) because α_3 and α_1 changed when we change our subtraction procedure. Actually, if we write $\alpha_{3(\kappa)}$ in terms of $\Lambda_{(\kappa)}$ [cf. Eq. (2.5)], we have

$$\ln \frac{\Lambda_{(\kappa)}}{\Lambda} = - \frac{\frac{11}{3} (C_2(G) - C_2(G_3))}{\frac{11}{3} C_2(G_3) - \frac{4}{3} T(R) N_f} \ln \kappa \quad (2.8)$$

A question immediately arises. We already have, at low energy, a renormalization procedure, let us say the minimal subtraction scheme. To which value of κ does it correspond? In other words, for which value of κ , do the minimal and the decoupling subtractions coincide, in the low energy region?

3. MINIMAL AND DECOUPLING SUBTRACTIONS

We first work in the MS scheme ignoring the existence of the unifying group G . To fix our notations, the part of the Lagrangian that we are interested in is:

$$\begin{aligned} \mathcal{L} = & - \frac{(Z_3)_{MS}}{4} (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) (\partial^\mu A^{\nu a} - \partial^\nu A^{\mu a}) \\ & + i (Z_2)_{MS} \bar{\psi} \not{\partial} \psi + g_{MS} (Z_1)_{MS} \bar{\psi} \not{A}^a t^a \psi + \dots \end{aligned} \quad (3.1)$$

where the A_μ^a 's are the renormalized gauge fields of the group G_i , the ψ 's are the renormalized fermion fields and the Z_ℓ 's are renormalization constants. The renormalized coupling g_{MS} of group G_i is expressed in terms of the bare coupling g_0^i by:

$$g_{MS} = (Z_3)^{1/2} \frac{(Z_2)_{MS}}{(Z_1)_{MS}} g_0^i \mu^{-\epsilon/2} \quad (3.2)$$

We now compute the Green's function given in Fig. 1a, renormalized by the diagrams of Fig. 1b. The result for the unrenormalized Green's function is

$$\Gamma_u = g_0^i \left[1 + g_0^{i^2} \left(\frac{A}{\epsilon} + A' \right) \right] \quad (3.3)$$

which gives, for the renormalized one

$$\Gamma_R = (Z_2)^{-1}_{MS} (Z_3)^{-1/2}_{MS} \Gamma_u = g_{MS} \left[1 + (g_{MS})^2 A' \right] \quad (3.4)$$

If we now take into account the existence of superheavy fields, it is clear from the discussion in Section 2 that we must use a decoupling subtraction. g_0^i is now g_0 , the coupling of the unifying group G and, through the diagrams of Fig. 2, the superheavy fields give extra renormalization constants $\tilde{Z}_\ell$ to the Green's function. Defining

$$(Z_\ell)_{DS} = \tilde{Z}_\ell (Z_\ell)_{MS} \quad (3.5)$$

we have

$$g_{DS} = (Z_3)_{DS}^{1/2} \frac{(Z_2)_{DS}}{(Z_1)_{DS}} g_0 \mu^{-\epsilon/2} \quad (3.6)$$

The unrenormalized Green's function is now:

$$\Gamma_u = g_0 \left[1 + g_0^2 \left(\frac{A}{\epsilon} + A' + \frac{B}{\epsilon} - B \ln \frac{M}{\mu} + B' \right) \right] \quad (3.7)$$

Where the $\ln M/\mu$ term is precisely the term subtracted in the DS scheme as previously discussed. Then

$$\Gamma_R = (Z_2)^{-1}_{DS} (Z_3)^{-1/2}_{DS} \Gamma_u = g_{DS} \left[1 + g_{DS}^2 (A' + B') \right] \quad (3.8)$$

Comparing (3.4) and (3.8) we obtain

$$g_{DS} = g_{MS} [1 - g_{MS}^2 B'] \quad (3.9)$$

Therefore if $B' \neq 0$, the two schemes do not coincide. Two approaches are then possible. In the first one, we extract Λ_{MS} from experiment and translate it into Λ_{DS} thanks to Eq. (3.9) ^{*}). The second one uses the remark made at the end of Section 2 : we can subtract $(1/\epsilon) - B \ln M\kappa_0/\mu$ where $\kappa_0 = e^{-B'/B}$, in which case we have

$$\Gamma_R = g_{DS} [1 + g_{DS}^2 A'] \quad (3.10)$$

and $g_{DS} = g_{MS}$; then the DS and MS schemes coincide at low energy. From now on, we shall adopt this approach.

One should note that B' is a constant up to terms of order k^2/M^2 where k is a typical external momentum. If we want MS and DS schemes to coincide exactly, we are then led to non-local counter-terms in the DS scheme, whose effect, however, is negligible (order k^2/M^2) at low energy ^{4),6)}. Alternatively, since the constant term in B' is without ambiguity, we can use it to define κ_0 and the counter-terms in DS are local; in this case DS and MS schemes coincide only up to terms of order k^2/M^2 and the comparison between them should be made far from the superheavy threshold.

Our procedure for comparing two different renormalization schemes via renormalized Green's functions may seem a little unusual. The reason is that the usual comparison through the bare coupling constant is impossible here because in one scheme it is the coupling g_0^i of the group G_i , in the other it is g_0 of the group G . We could also have made the comparison by using S matrix elements (e.g., fermion-fermion scattering via one boson exchange).

The constant κ_0 is universal (i.e., independent of the group content of the theory). The calculation of its value is therefore not a new one (see, e.g., Ref. 8) where the finite parts of the above Green's function were calculated in a $SU(2)_L \otimes SU(2)_R \otimes U(1)$ model for the study of the W_R decoupling). For the sake of completeness, we give in Appendix A some details of the computation. We give the result in the same form as Eq. (2.3) ^{**)}:

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^{*}) For example, such a translation should be done in paper I where all results were given in terms of Λ_{DS} .

^{**)} The only difference is that we now work in a truncated scheme ($\overline{DS}$): we include $\gamma/2 - \ln 4n/2$ in the $1/\epsilon$ pole ⁹⁾.

$$\alpha_{\overline{S}}^i(\mu)^{-1} = \alpha(\mu)^{-1} + \frac{1}{2\pi} \left[(C_2(G) - C_2(G_i)) \left(\frac{11}{3} \ln \frac{M}{\mu} - \frac{1}{6} \right) - \frac{1}{6} (C_2(G) - C_2(G_i)) \ln \frac{M}{\mu} - \frac{1}{6} \sum_R T_R^i(R_H) \ln \frac{M_R}{\mu} - \frac{4}{3} \sum_F T_F^i(R) \ln \frac{M_F \sqrt{2}}{\mu} \right] \quad (3.11)$$

Notations are defined in Appendix A. In the first line, we have artificially separated the contribution of the unphysical Higgs $(-1/6 \ln M/\mu)$ from that of the superheavy gauge bosons $(11/3 \ln M/\mu)$. This result is exactly the one obtained by Weinberg in Ref. 4). This looks rather encouraging because his approach involves a somewhat different method (effective field theory), although it is based on the same grounds (decoupling of heavy fields in an MS type renormalization procedure^{*)}). People who are used to compute finite parts of Green's functions might wonder why the constant terms in Eq. (3.11) are universal and do not depend for example on the Green's function chosen to calculate them (as in a momentum subtraction computation) or on the gauge. The reason is that such constants are intimately related to the pole term in ϵ (and therefore to the coefficients of the β function). Typically, the integration of the loop momentum in any of the diagrams shown in Fig. 2 gives the result

$$A(\epsilon) \Gamma(\epsilon) \left(\frac{M}{\mu} \right)^{-\epsilon} + o\left(\frac{k^2}{M^2}\right)$$

where $A(\epsilon)$ is a constant (depending on ϵ) and k is a typical external momentum. The fact that the part which is not proportional to $\Gamma(\epsilon)$ is of order k^2/M^2 is just a reflection of the Appelquist and Carazzone theorem³⁾. Expanding $A(\epsilon)$: $A(\epsilon) = A + A'\epsilon$ gives us the constant term that we are looking for. As an example, the $\sqrt{2}$ in the heavy fermion term comes from the fact that spin space has dimension $2^{n/2} = 2^{2-\epsilon/2}$.

4. HIGHER ORDERS

We hope that, by now, the reader is convinced that our method is just a trivial modification of the minimal subtraction. For those who are not yet convinced that this is true in higher orders, we develop, in this section, our subtraction procedure to all orders of perturbation theory. This allows us, in passing, to give a precise definition of the mass parameters (M, M_h, M_f) that we have used in the preceding sections.

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^{*)} See, however, the remark at the end of Section 4.

We will closely follow the method of 't Hooft²⁾ and start with some reminders. Let α_B and M_B be a bare coupling and a bare mass, μ the dimensional scale. We can express them in terms of the renormalized coupling α_R and mass $M_R = X_R \mu$ in the minimal subtraction scheme:

$$\alpha_B = \mu^{4-n} \left[\alpha_R + \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \frac{a_{ij} \alpha_R^j}{(n-4)^i} \right] \quad (4.1a)$$

$$M_B = \mu X_R \left[1 + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{b_{ij} \alpha_R^j}{(n-4)^i} \right] \quad (4.1b)$$

The β and γ_M functions are then:

$$\beta(\alpha_R) = \mu \frac{d\alpha_R}{d\mu} \Big|_{n=4} = - \sum_{j=2}^{\infty} (j-1) a_{1j} \alpha_R^j \quad (4.2a)$$

$$\gamma_M(\alpha_R) = \frac{\mu}{X_R} \frac{dX_R}{d\mu} \Big|_{n=4} = -1 - \sum_{j=1}^{\infty} j b_{1j} \alpha_R^j \quad (4.2b)$$

and the a_{ij}, b_{ij} ($i \neq 1$) are given functions of the a_{1j}, b_{1j} . For example,

$$a_{i+1,j} = \sum_{l=2}^{j-i} \frac{(l-1)(j-l+1)}{(j-1)} a_{1l} a_{i,j-l+1} \quad (4.3)$$

We want to study the decoupling of the field of mass M . To this purpose, we define $a_{ij}^{(1)}$ as the value of the a_{ij} parameters if the field of mass M was absent. We then notice that the a_{ij} 's, b_{ij} 's can be made X_R dependent as long as we do not change the coefficient of the poles $1/(n-4)^i$. This is not true in the MS scheme because a_{ij}, b_{ij} are X_R independent, but nothing prevents such a dependence. Actually, this is exactly what happens to our decoupling subtraction which we define to all orders by:

$$\alpha_B = \mu^{4-n} \left[\alpha'_R + \sum_{i=1}^{\infty} \sum_{j=i+1}^{\infty} \frac{a'_{ij}(n) \alpha_R'^j}{(n-4)^i} \right] \quad (4.4a)$$

$$M_B = \mu X'_R \left[1 + \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \frac{b'_{ij}(n) \alpha_R'^j}{(n-4)^i} \right] \quad (4.4b)$$

with

$$a'_{1j}(n) = a_{1j}^{(1)} + x_R'^{(j-1)(n-4)} (a_{1j} - a_{1j}^{(1)}) \quad (4.5a)$$

$$b'_{1j}(n) = b_{1j} (x_R')^{j(n-4)} \quad (4.5b)$$

As for the other parameters, we have, for $i \neq 1$

$$a'_{ij}(n) = a_{ij}^{(1)} + \sum_{k=2}^j (x_R')^{(k-1)(n-4)} a_{ij}^{(k)} \quad (4.6a)$$

$$b'_{ij}(n) = \sum_{k=1}^j (x_R')^{k(n-4)} b_{ij}^{(k)} \quad (4.6b)$$

with the constraints^{*)}

$$a_{ij} = \sum_{k=1}^j a_{ij}^{(k)} \quad b_{ij} = \sum_{k=1}^j b_{ij}^{(k)} \quad (4.7)$$

Assuming that the β function of α_R' is given by the $a_{ij}^{(1)}$ parameters:

$$\beta(\alpha_R') = \mu \frac{d\alpha_R'}{d\mu} \Big|_{n=4} = - \sum_{j=2}^{\infty} a_{1j}^{(1)} \alpha_R'^j \quad (4.8)$$

completely determines the a'_{ij} in terms of the $a_{ij}^{(1)}$, in a similar way as one could write Eq. (4.3) for the a_{ij} 's. In particular

$$a'_{i,i+1}(n) = (a'_{i2}(n))^i \quad (4.9)$$

We do not write the general relation because of its complexity but it can be worked out order by order.

It is easy to express α_R' by using 't Hooft's method²⁾: by a suitable change of parameter, we write Eq. (4.4a) under the form of Eq. (4.1a); because of the uniqueness of the series of Eq. (4.1a) [no $(n-4)$ terms], the new parameter is α_R . We obtain

$$\alpha_R' = \frac{\alpha_R}{1 + A_{12}(\ln x_R') \alpha_R + \sum_{j=3}^{\infty} \sum_{k=1}^{j-2} A_{jk} (\ln x_R')^k \alpha_R^{j-1}} \quad (4.10)$$

^{*)} These constraints assure that the coefficients of the poles $1/(n-4)^i$ do not change.

with

$$\begin{aligned} A_{1j} &= (j-1) (a_{1j} - a_{1j}^{(4)}) \\ A_{j,j+2} &= 2 \frac{(-1)^{j+1}}{j} \left[\left(\frac{a_{13}}{a_{12}} - \frac{a_{13}^{(4)}}{a_{12}^{(4)}} \right) a_{12}^{(4)} + \frac{a_{13}^{(4)}}{a_{12}^{(4)}} (a_{12} - a_{12}^{(4)})^j \right] \\ &\text{etc.} \end{aligned} \quad (4.11)$$

Now, what is X'_R ? To see this, we just apply the same method: we write Eq. (4.4b) under the form of Eq. (4.1b) and compare the constant terms. This gives

$$X'_R = \frac{X_R}{1 + \sum_{j=1}^{\infty} \alpha_R^j \sum_{k=1}^j B_{kj} (l_n X_R)^k} \quad (4.12)$$

where

$$\begin{aligned} B_{11} &= b_{11} \quad B_{1j} = j b_{1j} - \sum_{l=1}^{j-1} B_{1,j-l} l b_{1l} \\ j B_{jj} &= [b_{11} - a_{12} (j-1)] B_{j-1,j-1} \\ (k+1) B_{k+1,j} &= - \sum_{l=1}^{j-k-1} B_{k+1,j-l} (k+1) l b_{1l} + \sum_{l=1}^{j-k} [B_{k,l+k-1} (j-l-k+1) b_{1,j-l-k+1} - a_{1,l+k} B_{k,j-l} l (j-l)] \end{aligned} \quad (4.13)$$

We then have

$$\mu \frac{dX'_R}{d\mu} = -X'_R \quad (4.14)$$

Therefore the mass $M = X'_R \mu$ is "frozen" by our decoupling subtraction. The scale μ_0 where this occurs is given by the equation [see Eq. (4.12)]

$$X_R(\mu_0) = 1 \quad (4.15)$$

Notice that $M = \mu_0$ is neither the mass defined as the pole nor the constituent mass (see e.g., paper I).

Of course, the same properties apply if one replaces X_R by κX_R in Eq. (4.5) and the following. This was the basis of Section 3. However, it seems from Eq. (4.10) that the same constant κ should occur to all orders. This would be a further proof of the universality of this constant. However, we did not check this property explicitly.

We now come back to the lowest orders. Keeping first only terms of order $(\alpha_R \ln X'_R)^n$ in Eq. (4.10), we obtain

$$\alpha'_R = \frac{\alpha_R}{1 + (a_{12} - a_{12}^{(4)}) (l_n X'_R) \alpha_R} \quad (4.16)$$

This is nothing other than Eq. (3.11). It shows in particular that Eq. (3.11), although computed at the one-loop level, is an implicit resummation of the leading logarithms and therefore that its range of validity is not restricted by $\ln X'_R \ll 1$. This is of great importance for our GUTs applications. Notice that $\alpha_R(\mu)$ for $\mu \leq M$ is merely a parameter and there is no need to worry if α_R is large: the perturbation expansion at low energy is in terms of α'_R only.

If we now include terms of order $\alpha_R(\alpha_R \ln X'_R)^n$ the situation becomes a little more complicated. In Eq. (4.10), we must sum the series $\sum_{j=3}^{\infty} A_{j-2,j} (\ln X'_R)^{j-2} \alpha_R^{j-1}$. This is easily done by using the explicit values of $A_{j-2,j}$ given in (4.16):

$$\begin{aligned} \alpha_R^{1-1} = \alpha_R^{-1} + (a_{12} - a_{12}^{(1)}) \ln X'_R + 2 \left(\frac{a_{13}}{a_{12}} - \frac{a_{13}^{(1)}}{a_{12}^{(1)}} \right) \ln [1 + a_{12} \alpha_R \ln X'_R] \\ + 2 \frac{a_{13}^{(1)}}{a_{12}^{(1)}} \ln [1 + (a_{12} - a_{12}^{(1)}) \alpha_R \ln X'_R] \end{aligned} \quad (4.17)$$

Finally, we would like to comment on a recent paper by Hagiwara and Nakazawa¹⁰⁾. They raise a potential problem in the applicability of minimal subtraction to two-loop calculations involving mixed (light and heavy) loop diagrams. In Appendix B, we show that there is no such problem with the DS scheme.

5. APPLICATIONS

We now use our method to compute the mass of the superheavy gauge bosons in an SU(5) scheme.

5.1 The electromagnetic coupling in the Glashow-Weinberg-Salam model¹¹⁾

We first apply the ideas developed above to the breaking of $SU(2) \otimes U(1)$ down to $U(1)_{e.m.}$. In a first step, we do not have to bother with the grand unification; the only tribute that we must pay to it is to keep a general relation between charge and group generators¹²⁾

$$Q = T_3 - c T_0 \quad (5.1)$$

where T_i ($i = 1, 2, 3$) are the weak isospin generators of $SU(2)$ and T_0 the generator of $U(1)$. If all leptons and quarks appear in generations $[(\nu, e, u_i, d_i), (\nu_\mu, \mu, c_i, s_i), (\nu_\tau, \tau, t_i, b_i), \dots, i = 1, 2, 3, \text{ being the colour index}]$, then¹²⁾
 $c = \sqrt{5/3}$.

The situation is slightly different from the one developed in the preceding section because the group that we are starting with is $SU(2) \otimes U(1)$ which is not simple. The generalization is, however, straightforward and we give it

without any further details. The electric coupling can be expressed in terms of the couplings of the groups SU(2) and U(1) as follows:

$$\alpha_{e.m.}^{-1}(\mu) = \alpha_2^{-1}(\mu) + C^2 \alpha_1^{-1}(\mu) + \frac{1}{2n} \left[\frac{1}{3} \left(2 \ln \frac{M_W}{\mu} - 1 \right) - \frac{4}{3} \sum_F Q_F^2 \ln \frac{M_F \sqrt{2}}{\mu} \right] \quad (5.2)$$

Here, F subscript refers to heavy fermions (that is $M_F \geq \mu$) and the $\alpha_i(\mu)$ $i = 1, 2$ are the U(1) and SU(2) couplings with effective (that is not decoupled) W, Z, physical Higgs and heavy fermions. In other words:

$$\begin{aligned} \mu \frac{d\alpha_1}{d\mu} &= -\frac{\alpha_1(\mu)^2}{2n} \left[-\frac{1}{6C^2} - \frac{4}{3} \sum_f T_f^{(1)}(R) \right] \\ \mu \frac{d\alpha_2}{d\mu} &= -\frac{\alpha_2(\mu)^2}{2n} \left[\frac{22}{3} - \frac{1}{6} - \frac{4}{3} \sum_f T_f^{(1)}(R) \right] \end{aligned} \quad (5.3)$$

where we have included the effect of one complex Higgs doublet¹¹⁾ ($T_3 = \pm 1/2$, $T_0 = -1/20$) and the fermions transform according to the possibly reducible representation (R).

Using from Eq. (5.1) [see also for an example Eq. (A.15)]

$$\sum_f T_f^{(1)}(R) + C^2 \sum_f T_f^{(1)}(R) = \sum_f Q_f^2 \quad (5.4)$$

one can check that Eq. (5.2) provides the right β function for $\alpha_{e.m.}(\mu)$:

$$\mu \frac{d\alpha_{e.m.}^{-1}}{d\mu} = -\frac{2}{3n} \sum_\psi Q_\psi^2$$

where ψ refers to the light fermions ($M_\psi \leq \mu$). On the other hand, the constant terms in (5.2) are by now familiar, showing once more their universality.

Finally, we need a boundary condition. In the MS (or DS) scheme, it is provided in a very simple way by the Thompson limit:

$$\alpha_{e.m.}(m_e \sqrt{2}) = \alpha = \frac{1}{137.036} \quad (5.5)$$

To see this, we adopt the method of Coquereaux¹³⁾. We write

$$\alpha_{e.m.}(\mu) = \frac{\alpha_0}{1 + \frac{\alpha_0}{\pi} \int_0^1 dx \, 2x(1-x) \ln \frac{2m_e^2}{\mu^2}} \quad (5.6)$$

where α_0 is an integration constant and $\sqrt{2} m_e \leq \mu \leq m_f \sqrt{2}$; we write the photon vacuum polarization tensor

$$\begin{aligned} i \Pi_{\mu\nu}(q) &= -i (g_{\mu\nu} q^2 - q_\mu q_\nu) \Pi(q^2) \\ \Pi(q^2, \mu^2) &= \frac{\alpha(\mu)}{\pi} \left[- \int_0^1 dx \, 2x(1-x) \log 2 \frac{m_e^2 - x(1-x)q^2}{\mu^2} \right] \end{aligned} \quad (5.7)$$

α is then defined by¹³⁾

$$\alpha = \lim_{q^2 \rightarrow 0} \alpha_{\text{e.m.}}(\mu) \frac{1}{1 + \Pi(q^2, \mu^2)} \quad (5.8)$$

which gives $\alpha = \alpha_0$ and therefore Eq. (5.5).

5.2 A complete one-loop calculation of M_X in the SU(5) model

We now allow the couplings to evolve up to the grand unification. Thus α_1 and α_2 are given by Eq. (3.11). We assume that there are no superheavy fermions, that is no fermions which have not been taken into account in the preceding section. Thus,

$$\begin{aligned} \alpha_{\text{e.m.}}^{-1}(\mu) &= (1+C) \alpha^{-1}(\mu) + \frac{1}{2\pi} \left[\frac{17}{9} \left(21 \ln \frac{M_X}{\mu} - 1 \right) \right. \\ &\quad \left. + \frac{1}{3} \left(21 \ln \frac{M_W}{\mu} - 1 \right) - \frac{1}{3} \sum_R Q_R^2 \ln \frac{M_R}{\mu} - \frac{4}{3} \sum_{\substack{F \\ m_F > m_e}} Q_F^2 \ln \frac{M_F \sqrt{2}}{\mu} \right] \end{aligned} \quad (5.9)$$

Writing Eq. (5.9) for $\mu = m_e \sqrt{2}$, we still have one unknown quantity, namely $\alpha^{-1}(m_e \sqrt{2})$. We therefore need another boundary condition. As usual, we take the QCD parameter Λ (in the $\overline{\text{MS}}$ scheme and for four effective flavours):

$$\alpha_3(m_b) = \frac{12\pi}{25 \ln(m_b^2/\Lambda^2)} \quad (5.10)$$

Using Eq. (3.11) once more yields

$$\begin{aligned} \frac{25}{6\pi} \ln \frac{m_b}{\Lambda} &= \alpha^{-1}(m_b) + \frac{1}{2\pi} \left[\frac{1}{3} \left(21 \ln \frac{M_X}{m_b} - 1 \right) \right. \\ &\quad \left. - \frac{1}{6} \sum_R T_R^{(3)}(R_u) \ln \frac{M_R}{m_b} - \frac{4}{3} \sum_{\substack{F \\ m_F \geq m_b}} T_F^{(3)}(R) \ln \frac{M_F \sqrt{2}}{m_b} \right] \end{aligned} \quad (5.11)$$

It is then easy to obtain the final answer ($C^2 = 5/3$)

$$\begin{aligned} (22-1) \ln \frac{M_X}{m_e} &= 2\pi \alpha^{-1} + \frac{8}{3} \frac{25}{3} \ln \frac{\Lambda}{m_e} - 7 \ln \frac{M_W}{m_e} + \frac{4}{3} + \frac{32}{9} \ln 2 \\ &\quad + \frac{4}{3} \sum_\Psi Q_\Psi^2 \ln \frac{M_\Psi}{m_e} + \frac{4}{3} \sum_F \left[Q_F^2 - (1+C) T_F^{(3)}(R) \right] \ln \frac{M_F}{m_e} \\ &\quad + \frac{1}{6} \sum_R \left[2Q_R^2 - (1+C) T_R^{(3)}(R_u) \right] \ln \frac{M_R}{m_e} \end{aligned} \quad (5.12)$$

where

$$\psi = (e, u_i, d_i; \mu, c_i, s_i; i=1,2,3)$$

$$F = (\tau, t_i, b_i; \dots)$$

As has been noticed by many previous authors¹⁴⁾⁻¹⁶⁾ if all the members of a Higgs representation have a mass equal to M_X , if it contains non-physical Higgs, they do not contribute to Eq. (5.12). From now on, we take the so-called minimal SU(5)^{12),17)}: SU(5) is broken down to SU(3) $\otimes$ U(1) with a 24 and a 5. We also assume that all the physical Higgs of the 24 have a mass equal to M_X : their contribution to the right-hand side therefore compensates the (-1) in the left-hand side (coming from the non-physical Higgs). The same cancellation is not possible with the 5 = (3,1) + (1,2) since the doublet gives a mass to the W. If the mass of the triplet is M_X , they give $-1/3 \ln M_X/m_e$. Finally assuming that $F = (\tau, t_i, b_i; i=1,2,3)$ we have:

$$\begin{aligned} \left(22 + \frac{1}{3}\right) \ln \frac{M_X}{m_e} = & 2\pi\alpha' + \frac{8}{3} \times \frac{25}{3} \ln \frac{1}{m_e} - 7 \ln \frac{M_W}{m_e} + \frac{4}{3} \\ & + \frac{32}{9} \ln 2 + \frac{4}{3} \sum_{\psi} Q_{\psi}^2 \ln \frac{M_{\psi}}{m_e} - \frac{4}{3} \ln \frac{m_b}{m_{\tau}} \end{aligned} \quad (5.13)$$

One should note that the top quark decouples from Eq. (5.13). We now need the value of the fermion mass parameters to give a value of M_X . As shown in Section 4, they are given by (see Eq. (4.15) and the following remark)

$$M_{\psi} = M(\mu_0) \quad (5.14)$$

where μ_0 is the solution of:

$$\sqrt{2} M(\mu_0) = \mu_0 \quad (5.15)$$

For the leptons, this is the mass with some corrections of order $\alpha_{e.m.}$ that we shall neglect. For the heavy quarks, this is a sort of constituent mass¹⁸⁾. We take

$$m_c = 1.5 \text{ GeV}, \quad m_b = 5.0 \text{ GeV}$$

For light quarks, perturbation theory is no longer reliable and the masses in (5.13) are only parameters which account for strong interaction effects. We extract their value from the computation of Ellis et al.¹⁹⁾ who use data for R in a dispersion relation:

$$m_u = 15 \text{ MeV}, \quad m_d = 20 \text{ MeV}, \quad m_s = 150 \text{ MeV}$$

The comparison goes as follows: we start with a central value

$$\alpha_{e.m.}^{-1}(m_e^2, m_e^2, M_W^2) \Big|_{\text{mom.}} = 128.64$$

[see Eq. (2.23) of Ref. 19)], then translate it into the $\overline{\text{MS}}$ scheme¹³⁾:

$$\begin{aligned} \alpha_{e.m.}^{-1}(M_W) \Big|_{\overline{\text{MS}}} &= \alpha_{e.m.}^{-1}(m_e^2, m_e^2, M_W^2) \Big|_{\text{mom.}} - \frac{1}{\pi} \sum_{\psi} Q_{\psi}^2 \left[\frac{5}{9} - \frac{1}{3} \ln 2 \right] \\ &= \alpha_{e.m.}^{-1}(m_e^2, m_e^2, M_W^2) \Big|_{\text{mom.}} - 0.826 = 127.81 \end{aligned} \quad (5.16)$$

This is exactly the value obtained by computing $\alpha_{e.m.}^{-1}(M_W) \Big|_{\overline{\text{MS}}}$ with the above choice of mass parameters.

The low value of these parameters (compared to what one would naively expect^{14),15)}) seems to be a reflection of the strong interaction corrections to threshold crossing. Anyway, using the constituent masses ($m_u = m_d = 300$ MeV, $m_s = 500$ MeV) would make a difference of 1 in $\alpha_{e.m.}^{-1}(M_W) \Big|_{\overline{\text{MS}}}$ which reflects approximately the uncertainty on this quantity¹⁹⁾.

We can now give a precise value for M_X ; for example, if $\Lambda_{\overline{\text{MS}}} = 0.4$ GeV (one loop level), then $M_X = 2.30 \times 10^{15}$ GeV.

5.3 Two-loop calculation of M_X

Including two-loop effects is not a problem. It only requires the use of equations of the type (4.17) instead of (4.16). Actually, we first translate (4.17) in order not to use the parameter $\alpha_R = \alpha(\mu)$ which blows up at low energies but $\alpha(M)$ which is small.

$$\alpha'^{-1}(\mu) = \alpha^{-1}(M) - a_{12}^{(1)} \ln \frac{M}{\mu} + 2 \frac{a_{13}^{(1)}}{a_{12}^{(1)}} \ln \left[1 - a_{12}^{(1)} \alpha(M) \ln \frac{M}{\mu} \right] \quad (5.17)$$

This expression is not a surprise: it only states that $\alpha'(\mu)$ starts from $\alpha(M)$ at $\mu = M$ and evolves down with a low energy β function: $\beta = -a_{12}^{(1)} \alpha'^2 - 2a_{13}^{(1)} \alpha'^3$ [the proof of which was precisely our starting point: Eq. (4.10)]. In the QCD case, we recall that²⁰⁾

$$a_{12}^{(1)} = \frac{1}{2\pi} \left(11 - \frac{2}{3} N_f \right) \quad a_{13}^{(1)} = \frac{1}{16\pi^2} \left(102 - \frac{38}{3} N_f \right) \quad (5.18)$$

where N_f is the number of flavours. Of course, in the case that we are interested in, we have second-order terms connecting the different coupling constants.

The modification of (5.17) is straightforward:

$$\alpha_i'^{-1}(\mu) = \alpha_i^{-1}(M) - \frac{\beta_0^i}{2\pi} \ln \frac{M}{\mu} + \sum_{j=1}^3 \frac{1}{4\pi} \frac{\beta_1^{ij}}{\beta_0^i} \ln \left[1 - \frac{\beta_0^j}{2\pi} \alpha_j(M) \ln \frac{M}{\mu} \right] \quad (5.19)$$

where

$$\mu \frac{d\alpha_i^{-1}(\mu)}{d\mu} = \frac{\beta_0^i}{2\pi} + \sum_{j=1}^3 \frac{\beta_1^{ij}}{8\pi^2} \alpha_j(\mu)$$

The β_1^{ij} 's are given in Ref. 14). As an input, we take the usual strong interaction parameter Λ , defined at the two-loop level, in the $\overline{MS}$ scheme and for four effective flavours. In other words

$$\alpha_s^{-1}(Q^2) = \frac{25}{12\pi} \ln \frac{Q^2}{\Lambda^2} + \frac{154}{100\pi} \ln \ln \frac{Q^2}{\Lambda^2} \quad 2m_c^2 \leq Q^2 \leq 2m_b^2 \quad (5.20)$$

Using an equation of the type (4.17), we can deduce from it $\alpha_s^{-1}(\kappa M_W)$. The other input is α from which we obtain, following the method of subsection 5.1^{*)}, $\alpha_{e.m.}^{-1}(\kappa M_W) = 127.9$ ($\kappa = e^{-1/21}$, $m_t = 30$ GeV). Finally, M_X is given by

$$\begin{aligned} \frac{11 + \frac{1}{6}}{\pi} \ln \frac{M_X}{M_W} &= \alpha_{e.m.}^{-1}(\kappa M_W) - \frac{8}{3} \alpha_s^{-1}(\kappa M_W) - \frac{1}{4\pi} \ln \left[1 + \frac{2}{\pi} \alpha(\kappa M_X) \ln \frac{M_X}{M_W} \right] \\ &- \frac{2}{5\pi} \ln \left[1 - \frac{5}{3\pi} \alpha(\kappa M_X) \ln \frac{M_X}{M_W} \right] + \frac{24}{7\pi} \ln \left[1 - \frac{7}{2\pi} \alpha(\kappa M_X) \ln \frac{M_X}{M_W} \right] \end{aligned} \quad (5.21)$$

In this formula, we have included the Higgs contribution (+ 1/6) only to the first order and assumed the existence of only one complex Higgs doublet to break $SU(2) \times U(1)$ (if there are N_H of them, 1/6 should be turned into $N_H/6$). The numerical result is then

Λ GeV	M_X GeV	$\alpha^{-1}(\kappa M_X)$
0.5	8.2×10^{14}	41.17
0.4	6.5×10^{14}	41.23
0.3	4.8×10^{14}	41.30
0.2	3.18×10^{14}	41.40
0.1	1.56×10^{14}	41.56

^{*)} This method is based on one-loop calculations. There is no second loop computation possible because α_s blows up at low energy. On the other hand, the procedure of Ref. 19) (and therefore our mass parameters) is a non-perturbative one and therefore includes, in an effective way, higher orders.

The uncertainties in the result are the following:

i) the main one comes from the uncertainty on $\alpha_{e.m.}^{-1}(\kappa M_W)$. Assuming $\delta\alpha_{e.m.}^{-1}(\kappa M_W) = \pm 0.5$ [see above and Ref. 19)], we obtain roughly $\delta M_X \approx \pm 1 \times 10^{14}$ GeV. Note that M_X increases when $\alpha_{e.m.}^{-1}(\kappa M_W)$ increases.

ii) as already noticed, the top quark mass decouples from the one-loop calculation. At the two-loop level, the uncertainty is therefore quite small: $\delta M_X \approx 0.1 \times 10^{14}$ GeV if $m_t \leq 50$ GeV (M_X decreases when m_t increases).

iii) since we have neglected the three loop effects, one could express the log-log terms in (5.21) in terms of $\alpha_1(\kappa M_W)$; this does not change the final result. This shows that the next order should not change much the values of $M_X^{14)}$.

iv) in (5.21), we have assumed that all Higgs masses are of the order of M_X , except for the weak doublet breaking $SU(2) \otimes U(1)$. Of course, other assumptions are possible and would modify the determination of $M_X^{16)}$ (for example, M_X decreases if N_H increases).

The whole result can be expressed in a single expression:

$$M_X = (\Lambda)^{1.03} \times 1.67 \times 10^{15} \times (1 \pm 0.14) \text{ GeV} \quad (5.22)$$

where Λ is the QCD parameter, in the $\overline{MS}$ scheme, parametrizing the two-loop level, with four effective flavours and expressed in GeV and the uncertainty in the Higgs sector is not included in the error [see (iv)].

This result is in agreement with the one obtained by Marciano⁵⁾ who has developed a procedure very similar to ours. It is somewhat higher (about 50%) than the calculations based on the momentum subtraction scheme^{14),19)}.

5.4 $\sin^2 \theta_W$

As emphasized by Marciano¹⁵⁾, $\sin^2 \theta_W(\kappa M_W)$ is the more relevant quantity for comparison with present experiments. If we assume that the mass of the physical Higgs breaking $SU(2) \otimes U(1)$ is smaller than $M_W^*)$, then at the two-loop level, in the minimal $SU(5)$

$$\begin{aligned} \alpha_2^{-1}(\kappa M_W) = & \alpha_5^{-1}(\kappa M_W) + \frac{23}{12\pi} \ln \frac{M_X}{M_W} - \frac{1}{32\pi} \ln \left[1 + \frac{2}{\pi} \alpha(\kappa M_X) \ln \frac{M_X}{M_W} \right] \\ & + \frac{1}{16\pi} \ln \left[1 - \frac{5}{3\pi} \alpha(\kappa M_X) \ln \frac{M_X}{M_W} \right] - \frac{19}{14\pi} \ln \left[1 - \frac{7}{2\pi} \alpha(\kappa M_X) \ln \frac{M_X}{M_W} \right] \end{aligned} \quad (5.23)$$

---

*) We did not have to make such an assumption in the preceding subsection because this Higgs does not contribute to the electromagnetic or strong interactions (which we use to determine M_X).

and

$$\sin^2 \theta_w(\kappa M_W) = \frac{\alpha_e^{-1}(\kappa M_W)}{\alpha_{e.m.}^{-1}(\kappa M_W)} \quad (5.24)$$

This gives

$\Lambda_{\overline{MS}}$ GeV	M_X GeV	$\sin^2 \theta(\kappa M_W)$
0.5	8.2×10^{14}	0.2060
0.4	6.5×10^{14}	0.2073
0.3	4.8×10^{14}	0.2090
0.2	3.2×10^{14}	0.2113
0.1	1.56×10^{14}	0.2153

The main uncertainty, coming from $\alpha_{e.m.}^{-1}(\kappa M_W)$, is very small. Of course, increasing the number of weak scalar doublets would change the result (increase it¹⁵⁾). The above table can be summarized in the formula

$$\sin^2 \theta_w(\kappa M_W) = 0.202 \pm 0.0001 - 5.77 \times 10^{-3} \ln \Lambda \quad (5.25)$$

where Λ is expressed in GeV.

6. CONCLUSIONS

We have emphasized the details of our subtraction procedure to convince the reader that the choice of the renormalization scheme plays an important role in the study of the decoupling of heavy particles. We have restricted the examples of application of our method to the calculation of the mass M_X of the super-heavy gauge bosons in the minimal SU(5) grand unification scheme; we found a central value of 6.5×10^{14} GeV for M_X ; the corresponding value of $\sin^2 \theta_w(M_W)$ is 0.207. Of course, the same computation can be generalized to more fancy unifying groups or schemes of breaking. The method can also be applied to the effect of heavy quarks in QCD.

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Appendix A

We give some details on the calculation that leads to the determination of κ , [see Eq. (3.11)].

We consider an N dimensional gauge group G breaking into a product of simple groups $\otimes G_i$ (of dimension M_i). We first fix the notations:

i) The C_{abc} 's are the gauge group structure constants

$$\sum_{b,c} C_{abc} C_{dbc} = C_2(G) \delta_{ad} \quad (A.1)$$

$a, b, c \dots$ are general group indices; $\alpha, \beta, \gamma \dots$ will refer to G_i gauge fields (which remain massless) and $A, B, C \dots$ to superheavy gauge bosons (which decouple).

The only structure constants that mix light and heavy fields are of the form $C_{\alpha BC}$. It is then clear that:

$$\sum_{BC} C_{\alpha BC} C_{\delta BC} = \delta_{\alpha\delta} [C_2(G) - C_2(G_i)] \quad (A.2)$$

ii) We have a set of scalar particles described by the Hermitian field H which transform according to the representation (R_H) , perhaps reducible, of G ; the T^a 's are the matrices of the group generators in the representation (R_H) . The symmetry breaking leaves the vacuum $\langle H \rangle = v$ invariant under $\otimes G_i$. There are $(\sum_i M_i)$ generators T^a satisfying $T^a v = 0$, and $(N - \sum_i M_i)$ generators T^A corresponding to the massive gauge fields: for them, $T^A v \neq 0$. The $T^A v$'s span a $(N - \sum_i M_i)$ dimensional space S . Since

$$T^\alpha (T^A v) = [T^\alpha, T^A] v = i C_{\alpha AB} T^B v \quad (A.3)$$

the T^α coincide, in the space S , with the generators in the adjoint representation. Therefore we have, from (A.2)

$$\text{Tr}|_S (T^\alpha T^\delta) = \delta_{\alpha\delta} [C_2(G) - C_2(G_i)] \quad (A.4)$$

where $\text{Tr}|_S$ means that the trace is taken between states in S .

We define $T(R_H)$ by

$$T(R_H) \delta_{ab} = \text{Tr}(T^a T^b) \quad (A.5)$$

Now let $\{h\}$ be a subset of physical Higgs fields which transforms according to the irreducible representation $(R_1^H, \dots, R_i^H, \dots)$ of $\otimes G_i$. We can define, in a way similar to (A.5), $T(R_i^H)$ for the representation R_i^H of the group G_i and $T_{\{h\}}^i(R_H)$ by

$$\text{Tr}_{\{R\}} (T^\alpha T^\beta) = \left(\prod_{j \neq i} n_j \right) T(R_i^H) \delta_{\alpha\beta} = T_{\{R\}}^i(R_H) \delta_{\alpha\beta} \quad (\text{A.6})$$

where n_j is the dimension of R_j^H . For example, if R_H is the 24 of $SU(5)$ [whose $(SU(3), SU(2))$ content is $(1,1) + (1,3) + (8,1) + (3,2) + (3^*,2)$] parametrizing H by¹⁷⁾

$$H = \begin{pmatrix} H_8 + \frac{2H_0}{\sqrt{30}} I & \tilde{H}_x & \tilde{H}_y \\ \tilde{H}_x^\dagger & \frac{H_8}{\sqrt{2}} - \frac{3}{\sqrt{30}} H_0 & H^+ \\ \tilde{H}_y^\dagger & H^- & -\frac{H_8}{\sqrt{2}} - \frac{3}{\sqrt{30}} H_0 \end{pmatrix} \quad (\text{A.7})$$

we have

$$\begin{aligned} T_{\{H_8\}}^{(3)}(R_H) &= 3 & T_{\{H_0\}}^{(2)}(R_H) &= 0 \\ T_{\{H^+, H_8, H\}}^{(3)}(R_H) &= 0 & T_{\{H^+, H_8, H\}}^{(2)}(R_H) &= 2 \\ T_{\{H_8\}}^{(3)}(R_H) &= 0 & T_{\{H_0\}}^{(3)}(R_H) &= 0 \end{aligned} \quad (\text{A.8})$$

Now, writing (A.5) with $a = \alpha$, $b = \beta$ and using (A.4) and (A.6) we obtain

$$T(R_H) = \sum_{\{R\}} T_{\{R\}}^i(R_H) + C_2(G) - C_2(G_i) \quad (\text{A.9})$$

In our example, obviously

$$T(\underline{24}) = 5 = T_{\{H_8\}}^{(3)}(R_H) + 5 - 3 = T_{\{H^+, H_8, H\}}^{(2)}(R_H) + 5 - 2 \quad (\text{A.10})$$

Finally, h being a given physical Higgs field, we define for a generator T^{α_0} of the group G_i (α_0 arbitrary but fixed)

$$T_R^i(R) = \left[(T^{\alpha_0})^2 + 2 \sum_{\beta \neq \alpha_0} T^{\alpha_0} T^\beta \right]_{RR} \quad (\text{A.11})$$

We choose $T^{\alpha_0} = T_3$ when G_i is $SU(2)$ [see Eq. (5.4)]. Of course, each $T_R^i(R)$ depends on α_0 but not the sum:

$$T_{\{R\}}^i(R_H) = \sum_{R \in \{R\}} T_R^i(R_H) \quad (\text{A.12})$$

iii) The left-handed fermion and antifermion fields transform according to the representation (R) of G in which the matrices of the group generators are t^a . The index $f = \{f_1, f_2\}$ will refer to general fermions, f_1 being the particle, f_2 the antiparticle; since we include in our computation the decoupling of heavy fermions, we use indices ψ for light fermions and F for heavy ones. We adopt the same notations as for Higgs. For example

$$T_{\frac{f}{f}}^i(R) = \left((t^{\alpha_0})^2 + 2 \sum_{\beta \neq \alpha_0} t^{\alpha_0} t^{\beta} \right)_{\frac{f}{f} \frac{f}{f}} \quad (A.13a)$$

$$T_{\frac{f}{f}}^i(R) = \frac{1}{2} (T_{\frac{f}{f}}^i(R) + T_{\frac{f}{f}}^i(R)) \quad (A.13b)$$

$$T(R) = 2 \sum_f T_{\frac{f}{f}}^i(R) \quad (A.13c)$$

If we take as an example the usual $10 + \bar{5}$ content of SU(5)

$$\begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ \nu_e \end{pmatrix}_L = \begin{pmatrix} 0 & u_3^c & -u_2^c & -u_1 & -d_1 \\ -u_3^c & 0 & u_1^c & -u_2 & -d_2 \\ u_2^c & -u_1^c & 0 & -u_3 & -d_3 \\ u_1 & u_2 & u_3 & 0 & -e^+ \\ d_1 & d_2 & d_3 & e^+ & 0 \end{pmatrix}_L \quad (A.14)$$

we have

$$\begin{aligned} T_{\{u_i, i=1,2,3\}}^{(3)} &= T_{\{u_i^c\}}^{(3)} = T_{\{u_i, u_i^c\}}^{(3)} = T_{\{d_i\}}^{(3)} = T_{\{d_i^c\}}^{(3)} = T_{\{d_i, d_i^c\}}^{(3)} = \frac{1}{2} \\ T_{e^-}^{(3)} &= T_{e^+}^{(3)} = T_{\nu}^{(3)} = 0 \\ T_{\{u_i, i=1,2,3\}}^{(2)} &= T_{\{d_i\}}^{(2)} = 3 \times \frac{1}{4} \quad T_{\{u_i^c\}}^{(2)} = T_{\{d_i^c\}}^{(2)} = 0 \\ T_{\{u_i, u_i^c\}}^{(2)} &= \frac{3}{8} \quad T_{\{d_i, d_i^c\}}^{(2)} = \frac{3}{8} \\ T_{e^-}^{(2)} &= T_{\nu}^{(2)} = \frac{1}{4} \quad T_{e^+}^{(2)} = 0 \quad T_{\{e^+, e^-\}}^{(2)} = \frac{1}{8} \quad T_{\{\nu, \bar{\nu}\}}^{(2)} = \frac{1}{8} \\ T_{\{u_i, i=1,2,3\}}^{(1)} &= T_{\{d_i\}}^{(1)} = 3 \times \frac{1}{3C^2} \quad T_{\{u_i^c\}}^{(1)} = 3 \times \frac{1}{9C^2} \quad T_{\{d_i^c\}}^{(1)} = 3 \times \frac{1}{9C^2} \\ T_{\{u_i, u_i^c\}}^{(1)} &= \frac{17}{24C^2} \quad T_{\{d_i, d_i^c\}}^{(1)} = \frac{5}{24C^2} \\ T_{\{e^-\}}^{(1)} &= T_{\{\nu\}}^{(1)} = \frac{1}{4C^2} \quad T_{\{e^+\}}^{(1)} = \frac{1}{C^2} \quad T_{\{e^+, e^-\}}^{(1)} = \frac{5}{8C^2} \quad T_{\{\nu, \bar{\nu}\}}^{(1)} = \frac{1}{8C^2} \\ T(R) &= T(\underline{\bar{5}}) + T(\underline{10}) = \frac{1}{2} + \frac{3}{2} = 2 \end{aligned} \quad (A.15)$$

Finally, we note

$$\sum_a (t^a t^a)_{\psi\psi} = C_2(R), \quad \sum_A (t^A t^A)_{\psi\psi} = \tilde{C}_2(R), \quad \sum_\alpha (t^\alpha t^\alpha)_{\psi\psi} = C_2(R^i) \quad (A.16)$$

We are now interested in computing the diagrams of Fig. 2, that is the one-loop renormalization induced by heavy fields of a three-point Green's function consisting of two light fermion legs and one boson leg. Throughout the calculation, we use the Feynman gauge.

The renormalization of the fermion propagator reads (Fig. 2b)

$$\Sigma(p) = - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \tilde{C}_2(R) \not{p} \left[\frac{2}{\epsilon} - 2\ln \frac{\mu}{p} - \frac{1}{2} + o\left(\frac{p^2}{\mu^2}\right) \right] \quad (A.17a)$$

where $\frac{1}{\hat{\epsilon}} = \frac{1}{\epsilon} - \frac{\gamma}{2} + \frac{\ln 4\pi}{2}$

and γ is the Euler constant ($\gamma = 0.577 \dots$). This gives a fermion wave function renormalization constant induced by superheavy fields:

$$\tilde{Z}_2 = 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \tilde{C}_2(R) \left(\frac{2}{\hat{\epsilon}} - 2\ln \frac{\mu}{p} - \frac{1}{2} \right) \quad (A.18a)$$

The vertex renormalization is given by Fig. 2c:

$$\Gamma^P(p, q) = \gamma^P \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \left\{ [C_2(G) - C_2(G_i)] \left(\frac{2}{\epsilon} - 2\ln \frac{\mu}{p} \right) + \tilde{C}_2(R) \left(\frac{2}{\hat{\epsilon}} - 2\ln \frac{\mu}{p} - \frac{1}{2} \right) + o\left(\frac{p^2, q^2}{\mu^2}\right) \right\} \quad (A.17b)$$

which gives

$$\tilde{Z}_1 = 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \left\{ [C_2(G) - C_2(G_i)] \left(\frac{2}{\epsilon} - 2\ln \frac{\mu}{p} \right) + \tilde{C}_2(R) \left(\frac{2}{\hat{\epsilon}} - 2\ln \frac{\mu}{p} - \frac{1}{2} \right) \right\} \quad (A.18b)$$

Finally, the contribution of heavy gauge bosons, unphysical Higgs and Faddeev-Popov ghosts to the light gauge boson propagator is:

$$\omega_{\lambda\rho}^{(1)}(q) = - \frac{ig_0^2 \mu^{-\epsilon}}{16\pi^2} 3 [C_2(G) - C_2(G_i)] \left[\frac{2}{\epsilon} - 2\ln \frac{\mu}{p} + \frac{2}{3} + o\left(\frac{q^2}{\mu^2}\right) \right] (q^\lambda q^\rho - q^2 g^{\lambda\rho}) \quad (A.17c)$$

the contribution of physical Higgs is

$$\omega_{\lambda\rho}^{(2)}(q) = \frac{ig_0^2 \mu^{-\epsilon}}{16\pi^2} \frac{1}{3} \sum_R T_R^i(R) \left[\frac{2}{\epsilon} - 2\ln \frac{\mu}{p} + o\left(\frac{q^2}{\mu^2}\right) \right] (q^\lambda q^\rho - q^2 g^{\lambda\rho}) \quad (A.17d)$$

the contribution of heavy fermions is

$$\omega_{\text{FP}}^{(s)}(q) = \frac{ig_0^2 \mu^{-\epsilon}}{16\pi^2} \frac{1}{3} \sum_F T_F^i(R) \left[\frac{2}{\epsilon} - 2\ln \frac{M_F}{\mu} - \ln 2 + o\left(\frac{q^2}{M_F^2}\right) \right] (q^\lambda q^\rho - q^2 g^{\lambda\rho}) \quad (\text{A.17e})$$

The total of these contributions gives

$$\begin{aligned} \tilde{Z}_3 = 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \left\{ -\frac{3}{2} [C_2(G) - C_2(G_i)] \left[\frac{2}{\epsilon} - 2\ln \frac{M}{\mu} + \frac{2}{3} \right] + \frac{1}{6} \sum_R T_R^i(R_n) \times \right. \\ \left. \cdot \left[\frac{2}{\epsilon} - 2\ln \frac{M_R}{\mu} \right] + \frac{4}{3} \sum_F T_F^i(R) \left[\frac{2}{\epsilon} - 2\ln \frac{M_F}{\mu} - \ln 2 \right] \right\} \end{aligned} \quad (\text{A.18c})$$

On the other hand, the renormalization due to light particles is given as in the $\overline{\text{MS}}$ scheme by

$$(Z_1)_{\overline{\text{MS}}} = 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} C_2^i(R) \frac{2}{\epsilon} \quad (\text{A.19a})$$

$$(Z_1)_{\overline{\text{MS}}} = 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} [C_2^i(R) + C_2(G_i)] \frac{2}{\epsilon} \quad (\text{A.19b})$$

$$(Z_3)_{\overline{\text{MS}}} = 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \left[-\frac{5}{3} C_2(G_i) + \frac{4}{3} \sum_\Psi T_\Psi^i(R) \right] \frac{2}{\epsilon} \quad (\text{A.19c})$$

From Eq. (3.5), we obtain $(Z_1)_{\overline{\text{DS}}}$ and using Eqs (3.6), (A.18) and (A.19) we get

$$\begin{aligned} g_{\overline{\text{DS}}}^i = g_0 \mu^{-\epsilon/2} \left\{ 1 - \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \left[\frac{1}{\epsilon} \left[-\frac{11}{3} C_2(G) + \frac{1}{6} (C_2(G) - C_2(G_i)) + \frac{1}{6} \sum_R T_R^i(R_n) \right. \right. \right. \\ \left. \left. + \frac{4}{3} \sum_F T_F^i(R) \right] + \frac{7}{2} (C_2(G) - C_2(G_i)) \left(\ln \frac{M}{\mu} - \frac{1}{21} \right) - \frac{1}{6} \sum_R T_R^i(R) \ln \frac{M_R}{\mu} - \frac{4}{3} \sum_F T_F^i(R) \ln \frac{M_F \sqrt{2}}{\mu} \right] \right\} \end{aligned} \quad (\text{A.20})$$

Using Eqs (A.9) and (A.13c), we recognize, in the pole term, the renormalization of the unifying gauge group G coupling constant

$$g = \mu^{-\epsilon/2} g_0 \left\{ 1 + \frac{g_0^2 \mu^{-\epsilon}}{16\pi^2} \frac{1}{\epsilon} \left[\frac{11}{3} C_2(G) - \frac{4}{3} \frac{1}{2} T(R) - \frac{1}{6} T(R_n) \right] \right\} \quad (\text{A.21})$$

The unusual factor $1/2$ in front of $T(R)$ comes from the fact that we have put all the left-handed fermion fields in R (hence $1 - \gamma^5/2$ in the trace).

So finally

$$\begin{aligned} g_{\overline{\text{DS}}}^i = g \left\{ 1 - \frac{g^2}{16\pi^2} \left[\frac{7}{2} (C_2(G) - C_2(G_i)) \left(\ln \frac{M}{\mu} - \frac{1}{21} \right) - \frac{1}{6} \sum_R T_R^i(R_n) \ln \frac{M_R}{\mu} \right. \right. \\ \left. \left. - \frac{4}{3} \sum_F T_F^i(R) \ln \frac{M_F \sqrt{2}}{\mu} \right] \right\} \end{aligned} \quad (\text{A.22})$$

Therefore κ is $e^{-1/21}$ for heavy gauge bosons, 1 for heavy Higgs and $\sqrt{2}$ for heavy fermions.

Appendix B

Hagiwara and Nakazawa¹⁰⁾ have recently discussed a model where they found some trouble in applying the minimal subtraction at the two-loop level. We show in this Appendix that there is no such problem with the DS scheme. This also provides us with an explicit example of the method described in Section 4.

The Hagiwara and Nakazawa model can be summarized by the Lagrangian

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2 - \frac{\lambda}{4!} \phi^4 + \frac{1}{2} (\partial_\mu \chi)^2 - \frac{1}{2} M^2 \chi^2 - \frac{\kappa}{4!} \chi^4 - \frac{\kappa}{2!2!} \phi^2 \chi^2 \quad (\text{B.1})$$

They study the decoupling of the scalar particle χ ($m \ll M$). Following them, we have, at the two-loop level for λ (one-loop for κ)

$$\lambda_0 = \mu^\epsilon \left[\lambda_R + \frac{1}{(4\pi)^2} \frac{3\lambda_R^2 + 3\kappa_R^2}{\epsilon} + \frac{1}{(4\pi)^4} \frac{-17/6 \lambda_R^3 - 5/2 \lambda_R \kappa_R^2 - 6\kappa_R^3}{\epsilon} + \frac{1}{(4\pi)^4} \frac{9\lambda_R^3 + 12\lambda_R \kappa_R^2 + 12\kappa_R^3 + 3\kappa_R^2 \lambda_R}{\epsilon^2} \right] \quad (\text{B.2})$$

$$\kappa_0 = \mu^\epsilon \left[\kappa_R + \frac{1}{(4\pi)^2} \frac{\lambda_R \kappa_R + 4\kappa_R^2 + \kappa_R \lambda_R}{\epsilon} + \dots \right] \quad (\text{B.3})$$

where our $\epsilon(4 - N)$ is twice their $(2 - N/2)$. Without any difficulty, we can derive the corresponding β functions²⁾

$$\beta(\lambda_R) = \mu \frac{d\lambda_R}{d\mu} \Big|_{n=4} = \frac{3}{(4\pi)^2} (\lambda_R^2 + \kappa_R^2) + \frac{2}{(4\pi)^4} \left(-\frac{17}{6} \lambda_R^3 - \frac{5}{2} \lambda_R \kappa_R^2 - 6\kappa_R^3 \right) \quad (\text{B.4})$$

$$\beta(\kappa_R) = \mu \frac{d\kappa_R}{d\mu} \Big|_{n=4} = \frac{1}{(4\pi)^2} (\kappa_R \lambda_R + 4\kappa_R^2 + \kappa_R \lambda_R) \quad (\text{B.5})$$

Our decoupling subtraction reads

$$\lambda_0 = \mu^\epsilon \left[\lambda'_R + \frac{1}{(4\pi)^2} \frac{3\lambda'^2_R + \left(\frac{M}{\mu}\right)^{-\epsilon} 3\kappa'^2_R}{\epsilon} + \frac{1}{(4\pi)^4} \frac{-17/6 \lambda'^3_R + \left(\frac{M}{\mu}\right)^{-2\epsilon} \left(-\frac{5}{2} \lambda'_R \kappa'^2_R - 6\kappa'^3_R \right)}{\epsilon} + \frac{1}{(4\pi)^4} \frac{9\lambda'^3_R + \left(\frac{M}{\mu}\right)^{-2\epsilon} (12\lambda'_R \kappa'^2_R + 12\kappa'^3_R + 3\kappa'^2_R \lambda'_R)}{\epsilon^2} \right] \quad (\text{B.6})$$

$$\lambda_0 = \mu^\epsilon \left[\lambda_R' + \frac{1}{(4\pi)^2} \left(\frac{H}{\mu} \right)^{-\epsilon} \frac{\lambda_R' \kappa_R' + 4\kappa_R'^2 + \kappa_R' \Lambda_R'}{\epsilon} + \dots \right] \quad (B.7)$$

We expand the $(M/\mu)^{-k\epsilon}$ as a series in ϵ . We define

$$\begin{aligned} \hat{\lambda}_R &= \lambda_R' - \frac{3}{(4\pi)^2} \ln \frac{H}{\mu} \kappa_R'^2 + \frac{3\epsilon}{2(4\pi)^2} \ln^2 \frac{H}{\mu} \kappa_R'^2 + \frac{5}{(4\pi)^4} \ln \frac{H}{\mu} \lambda_R' \kappa_R'^2 \\ &+ \frac{12}{(4\pi)^4} \ln^2 \frac{H}{\mu} \lambda_R' \kappa_R'^2 + \frac{12}{(4\pi)^4} \ln \frac{H}{\mu} \kappa_R'^3 + \frac{12}{(4\pi)^4} \ln^2 \frac{H}{\mu} \kappa_R'^3 + \frac{3}{(4\pi)^4} \ln^2 \frac{H}{\mu} \kappa_R'^2 \Lambda_R' + O(\epsilon \lambda_R'^3, \epsilon^2) \end{aligned} \quad (B.8)$$

$$\begin{aligned} \hat{\kappa}_R &= \kappa_R' - \frac{1}{(4\pi)^2} \ln \frac{H}{\mu} \lambda_R' \kappa_R' + \frac{1}{2(4\pi)^2} \epsilon \ln^2 \frac{H}{\mu} \lambda_R' \kappa_R' - \frac{4}{(4\pi)^2} \ln \frac{H}{\mu} \kappa_R'^2 \\ &+ \frac{2}{(4\pi)^2} \epsilon \ln^2 \frac{H}{\mu} \kappa_R'^2 - \frac{1}{(4\pi)^2} \ln \frac{H}{\mu} \kappa_R' \Lambda_R' + \frac{1}{2(4\pi)^2} \epsilon \ln^2 \frac{H}{\mu} \kappa_R' \Lambda_R' + O(\epsilon \kappa_R'^3, \epsilon^2) \end{aligned} \quad (B.9)$$

We can then write (B.6) and (B.7) in terms of $\hat{\lambda}_R, \hat{\kappa}_R, \hat{\Lambda}_R (= \Lambda_R = \Lambda_R')$

$$\begin{aligned} \lambda_0 &= \mu^\epsilon \left[\hat{\lambda}_R + \frac{1}{(4\pi)^2} \frac{3\hat{\lambda}_R^2 + 3\hat{\kappa}_R^2}{\epsilon} + \frac{1}{(4\pi)^4} \frac{-17/6 \hat{\lambda}_R^3 - 5/2 \hat{\lambda}_R \hat{\kappa}_R^2 - 6\hat{\kappa}_R^3}{\epsilon} \right. \\ &\quad \left. + \frac{1}{(4\pi)^4} \frac{9\hat{\lambda}_R^3 + 12\hat{\lambda}_R \hat{\kappa}_R^2 + 12\hat{\kappa}_R^3 + 3\hat{\kappa}_R^2 \hat{\Lambda}_R}{\epsilon^2} \right] \end{aligned} \quad (B.10)$$

$$\kappa_0 = \mu^\epsilon \left[\hat{\kappa}_R + \frac{1}{(4\pi)^2} \frac{\hat{\lambda}_R \hat{\kappa}_R + 4\hat{\kappa}_R^2 + \hat{\kappa}_R \hat{\Lambda}_R}{\epsilon} \right] \quad (B.11)$$

As noticed by 't Hooft, such an expansion (no $\epsilon^k, k > 0$ term) is unique.

Therefore

$$\hat{\lambda}_R = \lambda_R \quad \hat{\kappa}_R = \kappa_R$$

and going to $\epsilon = 0$, we can express λ_R', κ_R' in terms of λ_R, κ_R thanks to (B.8) and (B.9):

$$\begin{aligned} \lambda_R' &= \lambda_R + \frac{3}{(4\pi)^2} \ln \frac{H}{\mu} \kappa_R^2 - \frac{5}{(4\pi)^4} \ln \frac{H}{\mu} \lambda_R \kappa_R^2 - \frac{12}{(4\pi)^4} \ln \frac{H}{\mu} \kappa_R^3 \\ &- \frac{6}{(4\pi)^4} \ln^2 \frac{H}{\mu} \lambda_R \kappa_R^2 + \frac{12}{(4\pi)^4} \ln^2 \frac{H}{\mu} \kappa_R^3 + \frac{3}{(4\pi)^4} \ln^2 \frac{H}{\mu} \kappa_R^2 \Lambda_R \end{aligned} \quad (B.12)$$

$$\kappa'_R = \kappa_R + \frac{1}{(4\pi)^2} \ln \frac{M}{\mu} \lambda_R \kappa_R + \frac{4}{(4\pi)^2} \ln \frac{M}{\mu} \kappa_R^2 + \frac{1}{(4\pi)^2} \ln \frac{M}{\mu} \kappa_R \lambda_R \quad (\text{B.13})$$

One then checks that

$$\mu \frac{d\lambda'_R}{d\mu} = \frac{3}{(4\pi)^2} \lambda_R^2 - \frac{17}{3(4\pi)^4} \lambda_R^3 \quad (\text{B.14})$$

$$\mu \frac{d\kappa'_R}{d\mu} = 0 \quad (\text{B.15})$$

Therefore κ is frozen at the scale M and completely decouples from the β function of λ .

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FIGURE CAPTIONS

- Fig. 1 : One-loop diagrams (1.b) involved in the renormalization within the group G_i of the three-point Green's function (1.a): fermion-fermion-gauge boson (of G_i).
- Fig. 2 : (a) Renormalization induced by heavy fields of the three-point Green's function given in Fig. 1.a.
- (b) Fermion propagator renormalized by a superheavy gauge boson ( )
- (c) Fermion-fermion-light gauge boson ( ) vertex renormalized by a superheavy gauge boson.
- (d) Light gauge boson propagator renormalized by superheavy gauge bosons, Higgs fields (physical or non-physical: ---) and heavy fermions (————).

Fig.1

a)

b)

c)

d)

Fig. 2