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TOWARDS A QUANTITATIVE DESCRIPTION OF HADRONIC WEAK DECAYS :

EVOLUTION EQUATIONS FOR HEAVY FLAVOUR MESONS

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A B S T R A C T

We develop a formalism for describing hadronic weak annihilation decays in analogy to the treatment of deep inelastic lepton-nucleon scattering : we write down evolution equations for colour singlet and octet ($Q\bar{q}$) systems inside mesons of increasing mass. Using D decays as input we can predict weak annihilation decay rates of heavier mesons in a semi-quantitative fashion despite our ignorance on bound state dynamics.

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1. - INTRODUCTION

Recent experimental findings on charm decays ¹⁾ have cast serious doubts on our previous understanding of weak decays, which was based on the notion that charm and other heavy flavour decays are a short-distance phenomenon. In this note we will assume that these preliminary data represent a real effect.

It has been argued that conventional strong final state interactions, e.g., in the form of resonances, can create the observed effects ²⁾. At the moment one cannot rule out this possibility and it seems likely that strong final state interactions contribute, yet we think it unlikely that they represent the whole story. We state two reasons for our belief :

- i) Although we do not find it impossible that strong final state interactions could enhance even totally inclusive rates by a factor of up to two, we consider it unlikely that a lifetime ratio $\tau(D^+)/\tau(D^0) \sim 5-10$ could be produced by an (isospinor) resonance.
- ii) From emulsion experiments there exists some evidence now that the lifetime of F^+ mesons is closer to that of D^0 than that of D^+ mesons. It requires an additional ad hoc assumption that there exists another resonance at the F^+ mass, this time with isospin one, giving rise to a similar enhancement. Furthermore there are indications that the Cabibbo disfavoured modes are enhanced in D^+ decays. Attributing it to final state interactions requires the existence of an isovector resonance at the D mass.

Therefore such a conventional ansatz is both ad hoc and uneconomical. In any case it is interesting to explore how to accommodate the observed effects in a QCD picture.

The mildest departure from the QCD ansatz previously used consists in assuming that the values of the renormalization coefficients in the effective weak Lagrangian are much larger than derived in the leading log M_W approach ³⁾. This could be due to higher loop contributions or it could be faked by soft gluon radiation ⁴⁾, as will be described in Section 3. However, for reasons discussed elsewhere ⁵⁾ we do not believe that the whole phenomenon of hadronic weak decays can be explained in such a way.

Also a more radical modification of the QCD picture involved has been suggested : reactions in which the heavy quark Q and the antiquark $\bar{q}$ annihilate in the t or s channel should enhance certain decays. The contributions

had previously been discarded for decays of pseudoscalar mesons : if the $Q\bar{q}$ system carries spin zero like the meson, then helicity arguments suppress such annihilation processes.

Two arguments have been put forward to evade this conclusions :

- i) charm mesons contain not only $c\bar{q}$, but also gluons ⁶⁾. In that case $c\bar{q}$ can carry spin one and annihilation will proceed unimpeded by helicity factors ;
- ii) a perturbative variant of this approach considers single gluon emission from the quark lines before annihilation occurs ⁷⁾. Since gluons have spin one, the annihilating $Q\bar{q}$ system will also carry spin one.

We want to stress, however, that although both schemes lead to a rich and interesting phenomenology, which can be checked ⁸⁾, they are far from being quantitative or even semi-quantitative. As long as bound states cannot be treated quantitatively, one does not know how to estimate the gluonic component in the hadronic wave function. Also although the second variant sketched above appears to be based on perturbation theory, this point of view is fallacious. A numerically large contribution is obtained only if one assumes that the $\bar{u}$ propagator (for D^0 decays) is soft. In Ref. 7) a non-relativistic picture was used, i.e., the $\bar{u}$ was put on-shell with a mass ~ 300 MeV. At such scales one is not dealing with a short-distance phenomenon and one cannot apply simple perturbation theory.

In addition both schemes are hard pressed to reproduce the size of the lifetime ratio $(\tau(D^+)/\tau(D^0)) \gtrsim 5$. Even assuming the $(c\bar{q})$ system to be in a spin one state with 100% probability and then calculating a vector meson decay in the usual quark model way, i.e., using just vacuum insertions into the current-current product, one is forced to postulate very large form factors to reproduce a large lifetime ratio. Of course, one can argue that inserting just vacuum states into the current-current matrix element does not saturate it. Nevertheless the fact remains that this argument does not make the whole ansatz more definite. It is obviously very difficult to make absolute predictions.

In this note we will develop a formalism in close analogy to evolution equations in deep inelastic lepton-nucleon scattering, by which one uses D decays as input to make predictions for the decays of heavier states.

We will not claim that such a formalism produces a precision test of QCD in a straightforward way. As in deep inelastic scattering the question of the relationship between leading and non-leading terms and higher twist contributions will

arise, but we will not discuss this problem in much detail. In addition also new problems will appear, not encountered in deep inelastic scattering : the evolution equations are more complex and contain more terms that depend on the bound state dynamics which cannot be handled quantitatively in a rigorous fashion at the moment. It is also not obvious which input values to use since our equations will apply to the annihilation width Γ_{ann} only, while the observed total width Γ_{tot} can contain contributions from strong final state interactions, etc. Nevertheless our approach yields a clear pattern for the behaviour of Γ_{ann} with gluons playing a prominent rôle and one can refine the formalism step by step.

The paper will be organized as follows. In Section 2 we will set up the evolution equations for $(Q\bar{Q})$ mesons and solve them ; in Section 3 we will discuss briefly various choices of meson form factors and more extensively the impact of colour quantum numbers and of hard and soft gluon radiation on decay amplitudes ; in Section 4 we will present a semi-quantitative scenario and finally in Section 5 we will give our conclusions.

2. - EVOLUTION EQUATIONS FOR HEAVY MESONS

In deep inelastic scattering the following picture is invoked : when a probe with resolution $1/Q^2$ "sees" a quark then another probe with $Q^2 > Q_0^2$ resolves this object into a quark plus a gluon.

By crossing two lines one obtains a picture for annihilation decays : a $(Q\bar{Q})$ system seen by a probe with resolution $\sim 1/Q_0^2$ turns into $(Q\bar{Q}) + \text{gluon}$ when probed with $Q^2 > Q_0^2$ (Fig. 1). The relevance of this statement rests on the observation that under emission of a vector gluon the $(Q\bar{Q})$ compound goes from a spin-zero to a spin-one state (for s wave configurations) ; in this spin-one state it can annihilate into light quarks without helicity suppression.

The analogy with deep inelastic scattering is, however, not complete, since one cannot vary Q^2 in annihilation decays in a straightforward way; semi-leptonic decays would offer the closest analogy were it not for the fact that neutrinos are not observable. In totally inclusive decays there exists only one fixed scale, namely the heavy mass : $Q^2 \sim M^2$ when one neglects the light quark masses.

We suggest therefore - and this is the basic ingredient in our analysis - to vary the scale by comparing the decays of different mesons (or baryons) keeping in mind that gluons are "flavour-blind". This procedure can be realized in the following way (besides many other possibilities) : in the $Q\bar{q}$ system we fix the mass of the antiquark - $m_{\bar{q}}$ - and let the quark mass - M_Q - go through a sequence of heavier and heavier masses, like charm, bottom, top, etc. The picture which emerges is then the following : although the gluonic component in D mesons cannot be calculated in perturbation theory since it is a long-distance phenomenon, one can calculate how it changes when one studies bottom, top, etc., mesons. Combining spin and colour quantum numbers one finds four different $(Q\bar{q})$ systems $(i, \underline{j})$ ⁹⁾, where the first entry denotes the spin and the second the SU(3) colour representation of $(Q\bar{q})$: $(1, \underline{1}) ; (0, \underline{8}) ; (1, \underline{1}) ; (1, \underline{8})$. In the constituent quark model one usually imposes the condition that $(Q\bar{q})$ possesses the quantum numbers of the meson while the other combinations are absent. But there is no general reason why this has to be so and we will therefore not assume it.

As said above :

$$(Q\bar{q})_{(0, \underline{1})} \rightarrow (Q\bar{q})_{(1, \underline{8})} + g$$

The situation is more complex for the $(0, \underline{8})$, $(1, \underline{1})$ and $(1, \underline{8})$ transitions : under gluon emission where the gluon carries quantum numbers $(1, \underline{8})$ they can go into more than one final state. E.g., both $(0, \underline{8}) \rightarrow (1, \underline{1}) + g$ and $(0, \underline{8}) \rightarrow (1, \underline{8}) + g$ are possible, and a priori one does not know how to weight them. We will use a simple statistical recipe and merely count states, as shown in the Table.

Probab.	$(0, \underline{1})$	$(0, \underline{8})$	$(1, \underline{1})$	$(1, \underline{8})$
$(0, \underline{1}) \rightarrow$	0	0	0	1
$(0, \underline{8}) \rightarrow$	0	0	$\frac{1}{9}$	$\frac{8}{9}$
$(1, \underline{1}) \rightarrow$	0	$\frac{1}{4}$	0	$\frac{3}{4}$
$(1, \underline{8}) \rightarrow$	$\frac{1}{36}$	$\frac{2}{9}$	$\frac{1}{12}$	$\frac{2}{3}$

Relative probabilities for the various $(Q\bar{q})$ transitions.

Next we have to determine the absolute transition rates, which we do by using perturbation theory. Gluon emission occurs in principle from both the Q and $\bar{q}$ line. But since the resolution of our probe is given by the energy release in the decay which is of order M_Q (assuming $M_Q \gg m_q$ for the moment), one cannot probe the structure of the Q quark, but only of the lighter $\bar{q}$ quark. Therefore gluon emission only off its line will be included.

To say it differently : fixing m_q while letting M_Q take larger and larger values - for example comparing D with B and T mesons - allows to probe the light quark $\bar{q}$ deeper and deeper. The structure of the heavy quark Q on the other hand cannot be resolved in the decay since it provides the scale for the whole process. The relevant variable is thus $t = \log(M_Q^2/m_q^2)$ with $M_Q \rightarrow \infty$ driving the evolution of the $Q\bar{q}$ compound.

Now we can write down the evolution equations in terms of gain and loss contributions ¹⁰⁾ :

$$\frac{df(0, \underline{1}; t, x)}{dt} \approx \frac{\alpha_s}{2\pi} \left\{ \int_x^1 \frac{dy}{y} \frac{1}{36} P\left(\frac{x}{y}\right) f(1, \underline{8}; t, y) - f(0, \underline{1}; t, x) \int_0^x \frac{dy}{x} P\left(\frac{y}{x}\right) \right\} \quad (2.1a)$$

$$\frac{df(1, \underline{1}; t, x)}{dt} \approx \frac{\alpha_s}{2\pi} \left\{ \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \left[\frac{1}{9} f(0, \underline{8}; t, y) + \frac{1}{12} f(1, \underline{8}; t, y) \right] - f(1, \underline{1}; t, x) \int_0^x \frac{dy}{x} P\left(\frac{y}{x}\right) \right\} \quad (2.1b)$$

$$\frac{df(0, \underline{8}; t, x)}{dt} \approx \frac{\alpha_s}{2\pi} \left\{ \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \left[\frac{1}{4} f(1, \underline{1}; t, y) + \frac{2}{9} f(1, \underline{8}; t, y) \right] - f(0, \underline{8}; t, x) \int_0^x \frac{dy}{x} P\left(\frac{y}{x}\right) \right\} \quad (2.1c)$$

$$\begin{aligned} \frac{df(1, \underline{8}; t, x)}{dt} \approx \frac{\alpha_s}{2\pi} \left\{ \int_x^1 \frac{dy}{y} P\left(\frac{x}{y}\right) \left[f(0, \underline{1}; t, y) + \frac{8}{9} f(0, \underline{8}; t, y) + \frac{3}{4} f(1, \underline{1}; t, y) + \frac{2}{3} f(1, \underline{8}; t, y) \right] \right. \\ \left. - f(1, \underline{8}; t, x) \int_0^x \frac{dy}{x} P\left(\frac{y}{x}\right) \right\} \quad (2.1d) \end{aligned}$$

with $t = \log(M_Q^2/m_q^2)$, x, y = fraction of the hadron energy carried by $Q\bar{q}$. $f(i, \underline{j}; t, x)$ stands for the probability to find $(Q\bar{q})$ inside the meson, with spin i , belonging to the $SU(3)$ colour representation $\underline{j}$, carrying a fraction of energy x when probed with resolution $1/Q^2$. $P(z)$ stands for the probability that $(Q\bar{q})$ emits a gluon with fraction of momentum $1-z$.

As usual we form moments to disentangle these equations. We obtain for the N^{th} moment, written in matrix form

$$\frac{d}{dt} \begin{pmatrix} f^{(N)}(0,1;t) \\ f^{(N)}(1,1;t) \\ f^{(N)}(0,8;t) \\ f^{(N)}(1,8;t) \end{pmatrix} \approx \frac{\alpha_s}{2\pi} \begin{pmatrix} -A' & 0 & 0 & \frac{1}{36} A^N \\ 0 & -A' & \frac{1}{9} A^N & \frac{1}{12} A^N \\ 0 & \frac{1}{4} A^N & -A' & \frac{2}{9} A^N \\ A^N & \frac{3}{4} A^N & \frac{8}{9} A^N & \frac{2}{3} A^N - A' \end{pmatrix} \begin{pmatrix} f^{(N)}(0,1;t) \\ f^{(N)}(1,1;t) \\ f^{(N)}(0,8;t) \\ f^{(N)}(1,8;t) \end{pmatrix} \quad (2.2)$$

with $A^N \equiv \int_0^1 dz z^{N-1} P(z)$.

From Eq. (2.2) one can immediately read off two features familiar from the case of deep inelastic scattering :

$$(i) \quad \frac{d}{dt} \sum_{i,j} f^{(N)}(i,j;t) = 0$$

reflecting conservation of probability, and

$$(ii) \quad \frac{d}{dt} \sum_{i,j} f^{(N)}(i,j;t) = \frac{\alpha_s}{2\pi} (A^N - A') \sum_{i,j} f^{(N)}(i,j;t) < 0$$

for $N \geq 2$, since $A^N > A^{N+1}$, i.e., the distributions exhibit a shrinkage towards smaller values of x .

Following the spirit expressed at the end of the Introduction, we have assumed as the simplest choice a universal function $P(z)$ describing gluon emission off the various $Q\bar{q}$ systems. In principle it is an unknown function as long as one does not know how to describe quark compounds quantitatively in QCD. This, however, does not render our ansatz useless, since we are interested in inclusive rates, for which we need only $f^{(1)}(i,j;t)$. Then the unknown coefficient A^1 will just renormalize the coupling α_s by a factor which is presumably of order one, since it comes from an integrated probability density ^{*}). One has to keep in mind that the appropriate α_s is not known to a much higher precision anyhow, and therefore we will drop reference to A^1 . Later we will come back to this point.

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^{*}) Using for $P(z)$ the single quark vertex function $P_{q \rightarrow qg}(z) = \frac{4}{3} (1+z^2)/(1-z)$ one might argue that by integrating z up to its maximum $z_{\text{max}} \sim 1 - (f_M^2/Q^2)$ one generates another large logarithm instead of a constant. But one has to keep in mind that one has then to deal with a double logarithmic expansion, the other logarithm coming from the angular integration. In that case, as shown by Parisi and Petronzio ¹¹⁾ for an analogous problem, one can sum up the double logarithmic expansion to obtain a Sudakov-type form factor ; i.e., such infra-red contributions are strongly suppressed.

For the first moment one has to include also conservation of probability :

$$f^{(n)}(0,1;t) + f^{(n)}(1,1;t) + f^{(n)}(0,8;t) + f^{(n)}(1,8;t) = 1$$

and finds :

$$\frac{d}{dt} \begin{pmatrix} f^{(n)}(1,1;t) \\ f^{(n)}(0,8;t) \\ f^{(n)}(1,8;t) \end{pmatrix} \simeq \frac{\alpha_s}{2\pi} \begin{pmatrix} -1 & \frac{1}{9} & \frac{1}{12} \\ \frac{1}{4} & -1 & \frac{2}{9} \\ -\frac{1}{4} & -\frac{1}{9} & -\frac{4}{3} \end{pmatrix} \begin{pmatrix} f^{(n)}(1,1;t) \\ f^{(n)}(0,8;t) \\ f^{(n)}(1,8;t) \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (2.3)$$

Before we solve these equations we have to discuss at which scale the coupling α_s should be taken. Naively one might think that $\alpha_s = \alpha_s(Q^2)$ is the appropriate choice, Q^2 being the only scale in the process as in deep inelastic scattering. But this is not quite true, since the extension of the $Q\bar{q}$ object provides another scale, namely the form factor or decay constant. Again this question cannot be answered rigorously without a real understanding of the bound state dynamics. We then propose to use a fixed or "frozen" effective coupling for two reasons :

- i) annihilation requires the co-operation of both Q and $\bar{q}$. Therefore one is reminded of the occurrence of higher twist terms in deep inelastic scattering, which also lead to powerlike scale breaking effects ;
- ii) there is not too much difference between powerlike and logarithmic scale breaking in the functions $f(i,j;t)$, at least as long as one compares charm with bottom decays. As discussed at the end of the Introduction we want to take a pragmatic attitude in the semi-quantitative analysis presented here. It will be obvious how a "running" coupling would modify the results.

We want to add a further remark : we include diagrams as shown in Fig. 2a while ignoring the one of Fig. 2b although that could also lead to weak annihilation (Fig. 3). We expect such contributions to be unimportant since only for soft gluon propagators can one hope to obtain a non-negligible result. In that case, however, the gluon travels a long distance before it fragments into $q\bar{q}$ and we consider it to be very unlikely that such a $\bar{q}$ or q has sufficient overlap with the Q quark to undergo weak annihilation which at these energies is still local.

Equation (2.3) can most easily be solved by choosing appropriate linear combinations :

$$\frac{d}{dt} \sum_i(t) = - \frac{\alpha_s}{2\pi} \frac{5}{4} \sum_i(t) \quad (2.5a)$$

$$\frac{d}{dt} \Sigma_8(t) = -\frac{\alpha_s}{2\pi} \frac{10}{9} \Sigma_8(t) \quad (2.5b)$$

$$\frac{d}{dt} \Sigma_R(t) = -\frac{\alpha_s}{2\pi} \frac{35}{36} \Sigma_R(t) \quad (2.5c)$$

with

$$\Sigma_1(t) \equiv f^{(n)}(1, \underline{1}; t) + f^{(n)}(1, \underline{8}; t) - \frac{4}{5}$$

$$\Sigma_8(t) \equiv f^{(n)}(1, \underline{8}; t) + f^{(n)}(0, \underline{8}; t) - \frac{9}{10}$$

$$\Sigma_R(t) \equiv f^{(n)}(1, \underline{1}; t) + \frac{8}{9} f^{(n)}(0, \underline{8}; t) + \frac{7}{9} f^{(n)}(1, \underline{8}; t) - \frac{4}{5}$$

Solving Eq. (2.5a-c) we find

$$\Sigma_1(t) = A \exp\left(-\frac{5\alpha_s}{8\pi} (t-t_0)\right) \quad (2.6a)$$

$$\Sigma_8(t) = B \exp\left(-\frac{5\alpha_s}{9\pi} (t-t_0)\right) \quad (2.6b)$$

$$\Sigma_R(t) = C \exp\left(-\frac{35\alpha_s}{72\pi} (t-t_0)\right) \quad (2.6c)$$

with three integration constants A , B and C . In principle we could use three different annihilation decay widths as inputs to fix these constants, analogous to what is done in inelastic scattering. Unfortunately, as we will see below, there are presumably not that many different hadronic weak decays which one could clearly compare with each other. Therefore in order not to jeopardize predictive power we suggest an intrinsic argument for fixing A , B and C : so far we have discussed the situation where $M_Q^2 \gg m_q^2$. Now we let M_Q decrease towards m_q assuming our ansatz still to make sense. Then we expect for $M_Q = m_q$, i.e., onia states that they are pure quark-antiquark states. Thus for $t_0 = (\log(M_Q^2/m_q^2)) = 0$ we demand

$$f(1, \underline{1}, 0) \approx f(0, \underline{8}, 0) \approx f(1, \underline{8}, 0) \approx 0 \quad \text{for pseudoscalar}$$

and

$$f(0, \underline{1}, 0) \approx f(0, \underline{8}, 0) \approx f(1, \underline{8}, 0) \approx 0 \quad \text{for vector states}$$

We state three reasons for these boundary conditions :

- i) if vector onia were not very pure ($Q\bar{Q}$) configurations, one would have no understanding of the OZI rule, i.e., why the width of ϕ , ψ , T , etc., is so small, since these decays should then be $\sim \alpha_s$ effects (Fig. 4) ;
- ii) furthermore in that case one would expect the decay widths of vector and pseudoscalar onia to be of roughly the same order of magnitude ; yet one knows experimentally that whereas $\Gamma(\psi/J) = 67 \text{ keV}$ one found for the η_c candidate $\Gamma(\eta_c) \sim 20\text{-}40 \text{ MeV}$;
- iii) ($Q\bar{Q}$) systems possess only one scale, namely M_Q ; thus one cannot resolve their inner structure in their totally inclusive decays.

Therefore we put for pseudoscalar states

$$A_{PS} = -\frac{4}{5} ; B_{PS} = -\frac{9}{10} ; C_{PS} = -\frac{4}{5}$$

and obtain after some trivial algebra :

$$f_{PS}^{(1)}(1, \underline{1}, M_Q^2) = -\frac{2}{25} \left(\frac{m_q^2}{M_Q^2} \right)^{\frac{5\alpha_s}{8\pi}} + \frac{18}{25} \left(\frac{m_q^2}{M_Q^2} \right)^{\frac{5\alpha_s}{9\pi}} - \frac{18}{25} \left(\frac{m_q^2}{M_Q^2} \right)^{\frac{35\alpha_s}{72\pi}} + \frac{2}{25} \quad (2.7a)$$

$$f_{PS}^{(1)}(1, \underline{8}, M_Q^2) = -\frac{18}{25} \left(\frac{m_q^2}{M_Q^2} \right)^{\frac{5\alpha_s}{8\pi}} - \frac{18}{25} \left(\frac{m_q^2}{M_Q^2} \right)^{\frac{5\alpha_s}{9\pi}} + \frac{18}{25} \left(\frac{m_q^2}{M_Q^2} \right)^{\frac{35\alpha_s}{72\pi}} + \frac{18}{25} \quad (2.7b)$$

For vector states, one follows the analogous procedure.

The picture which emerges from our treatment of the evolution equations for $(Q\bar{q}...)$ mesons is the following : for $M_Q = m_{\bar{q}}$, the meson is still described as a pure quark-antiquark system. For M_Q increasing with $m_{\bar{q}}$ fixed, i.e., when one "hops" from D to B and then to T mesons, etc., one finds the gluonic component to be more and more important.

3. - DISCUSSION OF DECAY CONSTANTS AND COLOUR FACTORS

What we have calculated so far is just the first moment of the various distribution functions. Since we are really interested in the annihilation contributions, we had in principle to fold in the annihilation amplitude before integrating over x . But it is presumably a reasonable approximation just to multiply the integrated distribution by the annihilation cross-section, since the $Q\bar{q}$ system will very likely carry most of the mass of the meson.

We need three ingredients to calculate annihilation contributions :

- i) The $Q\bar{q}$ overlap wave function, which depends on the bound state dynamics.

Therefore one does not know how to calculate it rigorously. Usually one calculates it in a non-relativistic $Q\bar{q}$ bound state model. Yet it is not clear whether such an ansatz can be applied when $m_{\bar{q}}$ is very light, say < 1 GeV ; indeed it is impossible to explain the large lifetime ratio $\tau(D^+)/\tau(D^0)$ as due to annihilation when one uses wave functions calculated in such a fashion ^{12),13)}. Furthermore we need the overlap for $Q\bar{q}$ in the presence of gluons, where one has to deal, in principle at least, with a three-body problem and where $Q\bar{q}$ is in a colour singlet or octet state. Therefore we will make two different choices which we consider typical :

$$f_M^2 \propto \frac{\mu^2 \text{ MeV}}{M} \quad (3.1a)$$

$$f_M^2 \propto \mu^2 \quad (3.1b)$$

with μ = reduced mass of the meson. We will fix the constants of proportionality, assumed to be flavour-independent in D decays and extrapolate them to other decays.

ii) The second ingredient is the annihilation amplitude for $(Q\bar{q})$ in a $(1, \underline{1})$ and $(1, \underline{8})$ configuration computed in a quark parton model before strong radiative corrections are included. The real issue involved here is the occurrence of colour factors. If there were no colour, i.e., $N_c = 1$, one would obtain :

$$\Gamma_{ann.}^{N_c=1}(Spin=1) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3$$

where it does not matter whether the annihilation occurs in the s or t channel. When one includes colour it does matter and we obtain for colour singlet configurations

$$\Gamma_{ann.,s}(1, \underline{1}) = \frac{N_f}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \quad (3.2a)$$

$$\Gamma_{ann.,t}(1, \underline{1}) = \frac{1}{N_c} \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \quad (3.2b)$$

with $N_f = N_c + \text{number of weak lepton doublets}$. Equation (3.2a,b) just reflects a Fierz transformation on the product of quark currents, e.g. :

$$\begin{aligned} \bar{s}_L \gamma_\mu c_L \bar{u}_L \gamma_\mu d_L &\xrightarrow{\text{Fierz}} \frac{1}{3} \bar{u}_L \gamma_\mu c_L \bar{s}_L \gamma_\mu d_L + \\ &+ 2 \bar{u}_L \gamma_\mu \frac{\lambda_i}{2} c_L \bar{s}_L \gamma_\mu \frac{\lambda_i}{2} d_L \end{aligned}$$

Therefore we obtain for colour octet configurations in charmed meson decays :

$$\Gamma_{ann.,s}(1, \underline{8}) = 0 \quad (3.3a)$$

$$\Gamma_{ann.,t}(1, \underline{8}) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \quad (3.3b)$$

iii) In the above expressions, i.e. (3.1)-(3.3), colour was introduced as a quantum number derived from a global symmetry. Yet it is generally believed to stem from a gauge symmetry with the accompanying gauge bosons, the gluons. Their exchange further modifies the effective coupling, which usually goes under

the heading of "short-distance effects". According to the conventional wisdom the effects of strong interactions on weak non-leptonic decays can be included by using an effective coupling ¹⁴⁾

$$\mathcal{L}_{\text{eff.}} = \frac{G_F}{\sqrt{2}} \left[\frac{c_-}{2} L_- + \frac{c_+}{2} L_+ \right] \quad (3.4)$$

where, for simplicity, we have dropped all references to mixing angles. $L_- [L_+]$ are products of quark currents which are odd [even] under Fierz transformations, e.g., for charm decays

$$L_- \equiv \bar{s}_L \gamma_\mu c_L \bar{u}_L \gamma_\mu d_L + \bar{u}_L \gamma_\mu c_L \bar{s}_L \gamma_\mu d_L$$

As soon as strong radiative corrections are included, i.e., $c_- > 1 > c_+$, the notion of a t channel or s channel annihilation becomes ambiguous. We will nevertheless still use it and call those decays t or s channel annihilation where this is true in the context of the quark model, i.e., for $c_- = c_+ = 1$. Thus s channel annihilation contributes to F^+ and the Cabibbo disfavoured D^+ decays, whereas t channel annihilation contributes to D^0 decays. With this notation we obtain :

$$\Gamma_{\text{ann},s}(1,1) = \frac{N_f}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \left(\frac{2}{3} c_+ + \frac{1}{3} c_- \right)^2 \quad (3.5a)$$

$$\Gamma_{\text{ann},t}(1,1) = \frac{N_c}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \left(\frac{2}{3} c_+ - \frac{1}{3} c_- \right)^2 \quad (3.5b)$$

$$\Gamma_{\text{ann},s}(1,8) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \frac{1}{4} (c_+ - c_-)^2 \quad (3.5c)$$

$$\Gamma_{\text{ann},t}(1,8) = \frac{1}{12\pi} G_F^2 f_V^2 (m_Q + m_{\bar{q}})^3 \frac{1}{4} (c_+ + c_-)^2 \quad (3.5d)$$

The coefficients $c_\mp$ were calculated perturbatively as a short-distance effect and one obtained ¹⁴⁾ :

$$c_{\pm} = \left(\frac{\alpha_s(m_c^2)}{\alpha_s(M_w^2)} \right)^{\gamma_{\pm}} + O\left(\frac{1}{\log \frac{M_w^2}{\Lambda^2}} \right) \quad (3.6)$$

$$\gamma_- = -\gamma_+ = \frac{12}{33-2N_f} \quad (3.7)$$

Thus

$$\begin{aligned} c_+(m_c^2) &= 0.68 \\ c_-(m_c^2) &= 2.15 \end{aligned} \quad (3.8)$$

Since the original analysis based on the dominance of cascade decays and on the short-distance renormalizations, Eq. (3.8), led to predictions which were not borne out by experiment, it was suggested ³⁾, as a minimal modification, to change the ratio c_-/c_+ drastically from the value three as derived from Eq. (3.6) and Eq. (3.7). We do not believe that the whole puzzle could be "explained" by such an (ad hoc) assumption ⁵⁾. Nevertheless it is quite possible that something like it occurs and contributes to the decay phenomena. One can envision it to happen in two ways, either as a genuine short-distance effect due to higher loops and/or non-leading contributions or it could be faked by soft-gluon exchanges as proposed by Deshpande et al. ⁴⁾.

For our numerical analysis given below we have used two different values for the ratio c_-/c_+ - namely $c_-/c_+ = 1$ or 3 - to see which quantities are sensitive to this ratio and which are not.

4. - SEMI-QUANTITATIVE SCENARIO OF HADRONIC WEAK DECAYS

As was said above the bound state characteristics enter the problem of decay widths via the decay constant f_M . Since a potential model approach has so far been very successful in the ψ and T family ¹⁵⁾ we feel confident to apply the same concepts to other non-relativistic states, like $(b\bar{c})$, $(t\bar{c})$ and

($t\bar{b}$) mesons. There we will use for f_M the expression given in (3.1a) ¹³⁾. The situation is different when one quark is light - u, d or s quark : then a non-relativistic ansatz might not be a useful approximation. Nevertheless we still believe that the bound state character of such systems should be the same as those having the same reduced mass. Thus

$$f_{c\bar{s}} \approx f_{b\bar{s}} \approx f_{t\bar{s}} \quad (4.1a)$$

$$f_{c\bar{u}} \approx f_{b\bar{u}} \approx f_{t\bar{u}} \quad (4.1b)$$

Of course, it is tempting to compare, say, also F^+ and D^0 decay widths, although this requires more knowledge on the decay constants.

If one uses reduced mass arguments, then

$$f_{F^+} \sim 1.5 f_D$$

If, on the other hand, one argues that for such light quarks another scale determines the bound state, maybe $\Lambda \sim 0.5$ GeV, used in the definition of α_s , then one would expect

$$f_{F^+} \sim f_D$$

A similar problem arises in the evolution equations with the terms $\log(M^2/m_q^2)$. For ($b\bar{c}$), ($t\bar{c}$), ($t\bar{b}$) mesons one puts $m_q^2 = m_c^2$ or $m_q^2 = m_b^2$. For s, u, d quarks it is not so clear which mass scale to insert : constituent masses or even current masses or something of the order of Λ . A leading log ansatz as ours does not allow an answer to this question. But, for the same reason why we argued above that for mesons with the same reduced mass the decay constants should be the same, we assume that for them m_q should be inserted with the same value. Thus we have the u/d class of mesons - $D^0, D^+, B_u^-, B_d^0, T_u, T_d$ - the s class - F^+, B_s^0, T_s - etc.

Of course, annihilation will contribute to the different decays with a strongly varying strength depending on mixing angles. For example D^+ decays receive contributions from annihilation only for the Cabibbo disfavoured modes, while in D^0 decays both Cabibbo favoured and disfavoured modes are affected.

Therefore to make full use of the machinery developed so far, we have to consider not only global rates like lifetimes, but also inclusive Cabibbo (etc.) suppressed rates. Thus we will compare

$$\frac{\Gamma(D^0)}{\Gamma(D^+)} , \frac{\Gamma(D^0 \rightarrow S=0)}{\Gamma(D^0 \rightarrow S=-1)} \quad \text{vs.} \quad \frac{\Gamma(D^+ \rightarrow S=0)}{\Gamma(D^+ \rightarrow S=-1)} , \frac{\Gamma(B^0)}{\Gamma(B^+)} , \dots$$

For our discussion we will mainly use a "reduced" annihilation width, i.e., one where the obvious M^3 dependence is already taken out and which is normalized to unity for D^0 decay :

$$R(M) \equiv \frac{\Gamma_{\text{ann}}(M)}{\Gamma_{\text{ann}}(D^0)} \left(\frac{M_D}{M} \right)^3 \quad (4.2)$$

Thus we can write down

$$R(B^0) = f_B^2 \left[3 \left(\frac{2}{3} c_+ - \frac{1}{3} c_- \right)^2 f^{(1)}(1, \underline{1}; M_B^2) + \frac{1}{4} (c_+ + c_-)^2 f^{(1)}(1, \underline{8}; M_B^2) \right] / K_{D^0} \quad (4.3a)$$

$$R(T^0) = f_T^2 \left[3 \left(\frac{2}{3} c_+ - \frac{1}{3} c_- \right)^2 f^{(1)}(1, \underline{1}; M_T^2) + \frac{1}{4} (c_+ + c_-)^2 f^{(1)}(1, \underline{8}; M_T^2) \right] / K_{D^0} \quad (4.3b)$$

$$R(D^+ \rightarrow S=0) = f_D^2 \left[N_f \left(\frac{2}{3} c_+ + \frac{1}{3} c_- \right)^2 f^{(1)}(1, \underline{1}; M_D^2) + \frac{1}{4} (c_+ - c_-)^2 f^{(1)}(1, \underline{8}; M_D^2) \right] (t_q^2 \theta_c) / K_{D^0} \quad (4.3c)$$

$$K_{D^0} \equiv f_D^2 \left[3 \left(\frac{2}{3} c_+ - \frac{1}{3} c_- \right)^2 f^{(1)}(1, \underline{1}; M_D^2) + \frac{1}{4} (c_+ + c_-)^2 f^{(1)}(1, \underline{8}; M_D^2) \right] \quad (4.3d)$$

In Fig. 5, we have plotted $f^{(1)}(1, \underline{1}; M^2/m_q^2)$ and $f^{(1)}(1, \underline{8}; M^2/m_q^2)$ versus the heavy mass M for $m_q = 0.33$ GeV and $\alpha_s = 0.5$. We see that in this mass range the ratio octet over singlet - $f^{(1)}(1, \underline{8})/f^{(1)}(1, \underline{1})$ - is still much larger than the asymptotic value nine.

In Fig. 6 we have plotted the mass dependence of $R(M)$ for t channel annihilation, i.e., D^0 , B^0 , T^0 , etc. decays, using the two different choices for f_M , namely $f_M \propto \mu^2/M$ (Fig. 6a) or $f_M \propto \mu^2$ (Fig. 6b) (μ is the reduced mass). One can make the obvious observation that with our choice of the strong

coupling α_s , i.e., $\alpha_s = 0.5$, the functions $f(1,1;M^2)$ and $f(1,8;M^2)$ rise more slowly than linearly in M . Therefore $R(M)$ reflects mainly the mass dependence of f_M , which, as said above, is not well known at the moment. This ignorance will hopefully diminish in the future and then a quantity like $R(M)$ will be more useful. Attributing $\tau(D^+)/\tau(D^0) \sim 5$ to annihilation effects one can obtain as a ball park estimate

$$\frac{\tau(B^-)}{\tau(B^0)} \sim 1.2 - 1.7$$

with $M_b \sim 4.5$ GeV.

Nevertheless even with our present knowledge we can derive interesting results from our evolution equations by comparing t and s channel annihilations at (almost) the same mass, like $\Gamma(D^0)$ and $\Gamma(D^+ \rightarrow S=0)/\text{tg}^2\theta_c$ or, cum grano salis, $\Gamma(D^0)$ and $\Gamma(F^+)$. It has been claimed that $\Gamma_{\text{ann}}(F^+) \lesssim \Gamma_{\text{ann}}(D^0)$ for two reasons mainly ⁶⁾ :

- i) the wave function overlap or the decay constant f_F is supposedly larger than f_D , since $m_s > m_u$;
- ii) the amplitude for s channel annihilation contains N_F as a factor, i.e., the number of effective weak doublets whereas the amplitude for the t channel process does not [Eqs (3.2a,b)].

We believe that these arguments are incomplete since two other points are not included :

- iii) as we have argued above the probability to find $Q\bar{q}$ in a $J=1$ state should decrease as m_q increases ; we expect this effect to compensate at least partially for $f_F > f_D$;
- iv) since due to our statistical weighting of transition rates $(Q\bar{q})$ is much more frequent in a colour octet than in a colour singlet state, the t channel process (in the language of the parton model) $\Gamma_{\text{ann}}(D^0)$ is enhanced relative to $\Gamma_{\text{ann}}(F^+)$; for the same reason

$$\frac{\Gamma_{\text{ann.}}(D^+ \rightarrow S=0)}{\Gamma_{\text{ann.}}(D^0) \times \text{tg}^2\theta_c} < 1$$

To illustrate this point more clearly, we write down what one expects for asymptotic masses [Eqs. (2.7a,b)] :

$$f_{PS}^{(1)}(1, \underline{1}; \infty) = \frac{1}{9} f_{PS}^{(1)}(1, \underline{8}; \infty) \quad (4.4)$$

Then

$$\left. \frac{\Gamma_{ann,s}(Q\bar{q})}{\Gamma_{ann,t}(Q\bar{q})} \right|_{M \rightarrow \infty} = \frac{\frac{N_F}{10} \left(\frac{2}{3} c_+ + \frac{1}{3} c_- \right)^2 + \frac{9}{40} (c_+ - c_-)^2}{\frac{N_C}{10} \left(\frac{2}{3} c_+ - \frac{1}{3} c_- \right)^2 + \frac{9}{40} (c_+ + c_-)^2} \approx \begin{cases} 0.53 & , \quad c_-/c_+ = 1 \\ 0.63 & , \quad c_-/c_+ = 3 \\ 0.9 & , \quad c_-/c_+ = 10 \end{cases} \quad (4.5)$$

i.e., the t channel process possesses a larger transition rate than the s channel reaction if one takes colour singlet and octet $Q\bar{q}$ configurations into account.

For sub-asymptotic masses the ratio of Eq. (4.5) is even smaller as can be read off from Fig. 7, where we have plotted $(\Gamma_{ann,s}(M)/\Gamma_{ann,t}(M))$ for two choices of c_-/c_+ , namely $(c_-/c_+) = 1$ and 3. We want to make three observations :

- i) $(\Gamma_{ann,s}(M)/\Gamma_{ann,t}(M))$ is much smaller than unity ;
- ii) it depends significantly on c_-/c_+ , and
- iii) it rises with an increasing mass.

As discussed above,

$$\left. \frac{\Gamma_{ann,s}}{\Gamma_{ann,t}} \right|_{M=M_D} = \frac{\Gamma_{ann}(D^+ \rightarrow S=0)}{(t_g^2 \theta_c) \Gamma_{ann}(D^0)} \quad (4.6a)$$

$$\left. \frac{\Gamma_{ann,s}}{\Gamma_{ann,t}} \right|_{M \sim M_D} \lesssim \frac{\Gamma_{ann}(F^+)}{\Gamma_{ann}(D^0)} \quad (4.6b)$$

where the ">" sign in Eq. (4.6b) allows for $f_F > f_D$ and $M_F > M_D$. Since for the "canonical" value $c_-/c_+ = 3$ we read off

$$\left. \frac{\Gamma_{ann,s}}{\Gamma_{ann,t}} \right|_{M=M_D} \sim \frac{1}{3} \quad (4.7)$$

we conclude that Cabibbo disfavoured D^+ decays are enhanced and that the lifetime of the F^+ is shorter than that of the D^+ , but not by as much as one expects naively ⁸⁾:

$$\begin{aligned} BR(D^+ \rightarrow S=0) &\sim (t_9^2 \theta_c) \frac{\tau(D^+)}{\tau(D^0)} \frac{\Gamma_{ann,s}}{\Gamma_{ann,t}} \\ &\sim 0.09 \div 0.18 \end{aligned} \quad (4.8a)$$

$$\frac{\tau(F^+)}{\tau(D^0)} \lesssim 3 \quad (4.8b)$$

where we have used $\tau(D^+)/\tau(D^0) \sim 5-10$. A similar test can be performed by comparing $\Gamma_{ann}(B^0)$ with B^- decays into final states without charm:

$$\left. \frac{\Gamma_{ann,s}}{\Gamma_{ann,t}} \right|_{M=M_B} = \frac{\Gamma_{ann}(B^- \rightarrow C=0)}{\Gamma_{ann}(B^0) \times (\text{mixing angles})^2} \sim \frac{1}{3} \quad (4.9)$$

Mesons where both constituent quarks are heavy, e.g., B_c , T_c , etc., can be treated also in a different fashion. It seems to be a reasonable ansatz to treat them as non-relativistic bound states, for which one can more reliably calculate the decay constant. B_c will probably decay into $B_s + \pi$ most of the time due to the presumably small $b \rightarrow c$ mixing angle. For T_c , T_b , etc. decays, one relies on a non-relativistic potential model ¹⁵⁾ to obtain

$$f_M^2 \sim \frac{900 \text{ MeV} \mu^2}{M}$$

Thus, apart from T_b^* decays, one expects very small annihilation contributions ¹⁶⁾. Alternatively one can apply the non-relativistic analysis of Bander et al. to such a system to obtain

$$\frac{\Gamma(T^+)}{\Gamma(T_c)} \sim 1 + \left(\frac{m_{T_c}}{m_t} \right)^5 \left(\frac{f_{T_c}}{m_c} \right)^2 \frac{16 \pi \alpha_s}{27} \frac{(c_+ + c_-)^2}{4(2 + 2c_+^2 + c_-^2)} \quad (4.10)$$

For $m_t \sim 20$ GeV one finds $f_{T_c} \sim 240$ MeV and thus for $m_c \sim 1.5$ GeV

$$\frac{\Gamma(T^+)}{\Gamma(T_c)} \approx 1 \quad (4.11)$$

Therefore we conclude that although the same analysis, based on evolution equations as discussed above, can be applied to non-relativistic systems, we expect all annihilation effects to be very small for non-relativistic pseudoscalar mesons. At the same time we also expect non-relativistic vector mesons T_c^* , etc., to decay as suggested in a parton model approach¹³⁾.

5. - CONCLUSIONS

We have stated in the Introduction that our aim is not to propose a precision test of QCD. We wanted to show that :

- i) a semi-quantitative description can be formulated despite our still considerable ignorance regarding bound state dynamics, meson form factors, the impact of short-distance phenomena and their separation from long-distance phenomena, etc., and
- ii) that this description can be refined step by step.

We want to summarize the important lessons which can come out from such an analysis : if the salient features of the proposed description were confirmed by experiment, one would learn :

- i) gluons play a very important rôle in $(Q\bar{Q})$, etc., hadrons, unlike the situation for onia states $(Q\bar{Q})$;
- ii) although the assumed non-Abelian gauge nature of gluons (and QCD as a whole) will very likely not become apparent in studies as discussed here, their spin and colour quantum numbers affect the analysis in a very significant way as we have discussed in the comparison between s and t channel annihilations.

Therefore the issues involved are well worth the experimental effort which is required and on which we want to make a few comments. In this paper we have discussed the annihilation decay width Γ_{ann} , but it is not obvious how to extract it from the observed width Γ_{obs} , since it is quite likely that Γ_{obs}

receives further contributions not only from cascade decays, but also from strong final state interactions, say resonances. This problem could presumably be avoided by comparing decays of bottom and top mesons. Yet since the annihilation width increases presumably only like $\mathcal{M}^3$ (or even less), while cascade decays rise like $\mathcal{M}^5$ we expect the latter to be dominant in top decays, assuming $m(\text{top}) > 17.5$ GeV. Therefore it will be very difficult to pin down annihilation effects in top decays, with the exception of T_b^* decays, as has been discussed in the literature ¹⁶⁾.

In charm decays one can disentangle the contributions from different mechanisms by studying exclusive channels and assuming them to reflect the same pattern as inclusive decays. Since D^+ decays into an exotic final state it is affected neither by annihilation nor by resonances. Therefore one can study the ratio c_-/c_+ from the effective weak Lagrangian by comparing $D^+ \rightarrow \bar{K}^0 \rho^+$ and $D^+ \rightarrow (\bar{K}^0)^* \pi^+$. The annihilation effects on the other hand can be studied most clearly in a comparison of $D^0 \rightarrow \bar{K}^0 \phi$ with $D^0 \rightarrow \bar{K}^0 \rho^0$ and $D^0 \rightarrow (\bar{K}^0)^* \pi^0$ ^{4),17)}, of $D^0 \rightarrow \bar{K}^0 \eta$ with $D^0 \rightarrow \bar{K}^0 \pi^0$ ¹⁷⁾ and of $F^+ \rightarrow \rho^+ \pi^0, \omega \pi^+$ with $F^+ \rightarrow \eta \pi^+$ ^{4),17),18)}.

It was noted before that annihilation enhances the $\Delta I = \frac{1}{2}$ mode in K decays ¹⁹⁾; it was even claimed that it could explain the $\Delta I = \frac{1}{2}$ rule altogether ⁶⁾. We are not convinced by this latter claim. It involves an extrapolation from charm decays down to K and hyperon decays which depend crucially on the parameters used: one has to use a current quark mass $m_s \lesssim 100$ MeV instead of the constituent quark mass $m_s \sim 500$ MeV, which seems more reasonable to us. We share the belief of Finjord ⁹⁾ that other mechanisms like Penguin diagrams are still needed to explain the $\Delta I = \frac{1}{2}$ rule.

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FIGURE CAPTIONS

- Figure 1 $(Q\bar{Q})$ seen by a W boson at two different scales.
- Figure 2a   Diagrams included in the evolution equations.
- Figure 2b   Diagrams not included in the evolution equations.
- Figure 3   Annihilation diagram for contributions as shown in Fig. 2b.
- Figure 4 T decays in the presence of spectator gluons
- Figure 5   Singlet and octet moments $f^{(1)}(1, \underline{1}; (M^2/m_q^2))$ and $f^{(1)}(1, \underline{8}; (M^2/m_q^2))$ as a function of the heavy mass M for $m_q = 330$ MeV and $\alpha_s = 0.5$.
- Figure 6   Normalized t channel annihilation $R(M)$ as function of the heavy mass M with $f_v^2 \propto (\mu^2/M)$ (Fig. 6a) and $f_v^2 \propto \mu^2$ (Fig. 6b).
- Figure 7 $\Gamma_{\text{ann},s}(M)/\Gamma_{\text{ann},t}(M)$ as a function of M for $c_-/c_+ = 3$ (solid curve) and $c_-/c_+ = 1$ (broken curve).

Fig. 1

Fig. 2a

Fig. 2b

Fig. 3

Fig. 4

Fig. 5

Fig. 6a

Fig. 6b

Fig. 7