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WEAK DECAYS OF HEAVY VECTOR MESONS

TOPONIUM AND OPEN TOP

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ABSTRACT

We point out that for sufficiently heavy top quark masses, i.e., $m_t \geq 20$ GeV, weak interactions govern the decays of both the corresponding pseudoscalar and vector mesons, T and T^* . Furthermore they significantly modify toponium decays. We discuss in detail the implications for T_b^* decays and describe how especially its leptonic decays provide such spectacular signatures that they could serve to detect top production in hadron-hadron collisions. We also draw a scenario for toponium decays.

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1. INTRODUCTION

Recent experimental findings on charm decays¹⁾ have cast serious doubts on our previous understanding of weak decays. No clear and convincing new theoretical picture has yet emerged, although there is wide-spread hope that weak decays of even heavier flavour states, like bottom states, will clarify the situation.

In this note we wish to point out that novel features appear in the decays of the states containing the hypothetical top quark t ²⁾, the weak isospin partner of the b quark. The present experimental lower limit on its mass is 17 GeV. We will extend our studies up to masses of 40 GeV. New systems like $\bar{t}b$ which are not only heavy but also of size $\ll 1/\Lambda$, where Λ is the QCD scale parameter are expected to shed light on the dynamics of weak decays. Although it has been discussed before that toponium, $t\bar{t}$, made out of such massive quarks will have a sizeable branching ratio for weak transitions³⁾, nevertheless the treatment of these effects has been incomplete. Furthermore we will present the observation that for such heavy masses not only the pseudoscalar T , which is presumably the lowest lying meson with "open" top quantum number, but also the corresponding vector meson T^* will predominantly decay weakly with some spectacular experimental signatures.

The paper will be organized as follows: in Section 2 we will argue that the decays of the vector mesons T^* are dominated by weak forces and discuss the implications; in Section 3 meson decay constants will be discussed in the context of a non-relativistic potential model; in Section 4 we will draw a scenario of toponium decays and in Section 5 give some numerical estimates of the size of the corresponding effects which can be expected in $p\bar{p}$ or pp collisions at very high energies. Our conclusions will be presented in Section 6.

2. $T^* - T$ MASS SPLITTINGS AND T^* DECAYS

The argument leading to the conclusion that besides the pseudoscalar mesons also the vector mesons T^* undergo weak decays consists of three steps:

i) We use a non-relativistic potential model for bound states where one quark is very heavy and the other much lighter, e.g., $t\bar{u}$, $t\bar{s}$, $t\bar{c}$ etc. Adopting the QCD inspired picture that the spin-spin forces of the constituents are proportional to their colour magnetic moments, we find that the mass splitting between pseudoscalar states T and vector states T^* decreases rapidly with increasing quark masses. More quantitatively:

$$M(T_q^*) - M(T_q) \sim \frac{m_c}{m_t} \frac{m_u}{m_q} [M(D^{*0}) - M(D^0)] \quad (2.1)$$

$$T_q \equiv (t\bar{q})$$

For $17.5 \text{ GeV} \leq m_t \leq 40 \text{ GeV}$, $m_b \sim 5 \text{ GeV}$, $m_c \sim 1.5 \text{ GeV}$, $m_s \sim 0.54 \text{ GeV}$ and $m_u \sim m_d \sim 0.33 \text{ GeV}$ we obtain mass splittings of at most a few MeV. Therefore T^* cannot decay strongly into $T + \pi$.

ii) Nevertheless T^* can decay electromagnetically into T via an M1 transition. We apply the usual formula describing such transitions

$$\Gamma((Q\bar{q})_V \rightarrow (Q\bar{q})_P + \gamma) \sim \frac{1}{3} \alpha k^3 \left(\frac{e_q}{m_q} + \frac{e_Q}{m_Q} \right)^2 \delta^{rr'} \quad (2.2)$$

where e_q, e_Q stand for the quark charge, m_q, m_Q for the quark masses, k for the photon three momentum and $\delta^{rr'}$ for the spatial overlap of the vector and pseudoscalar wave functions. For radiative charm decays one obtains:

$$\Gamma(D^{*0} \rightarrow D^0 + \gamma) \sim 40 \text{ keV} \quad (2.3)$$

Experimentally⁴⁾

$$\Gamma(D^{*0}) < 5 \text{ MeV} \quad (2.4a)$$

$$BR(D^{*0} \rightarrow D^0 + \gamma) \sim 45 \pm 15\% \quad (2.4b)$$

(2.3) and (2.4b) together yield

$$\Gamma(D^{*0} \rightarrow D^0 + \pi^0) \sim 50 \text{ keV} \quad (2.5)$$

At first sight this width might appear too small for a strong decay but a crude comparison with $\rho \rightarrow \pi\pi$ shows consistency: for $g_{D^*D\pi} \sim g_{\rho\pi\pi}$ we estimate for this P wave mode:

$$\Gamma(D^* \rightarrow D\pi) \sim \Gamma(\rho \rightarrow \pi\pi) \left(\frac{m_\rho}{m_{D^*}} \right)^2 \left(\frac{k_{D^*}}{k_\rho} \right)^3 \sim 30 \text{ keV} \quad (2.6)$$

We extrapolate Eq. (2.2) up to $T^* \rightarrow T\gamma$ transitions, where we obtain widths of at most a few eV, as is shown in Fig. 1.

iii) For weak decays on the other hand we estimate using the cascade picture:

$$\Gamma_{\text{casc}}(T) \approx \Gamma_{\text{casc}}(T^*) \sim \frac{N_f}{192\pi^3} G_F^2 m_t^5 \quad (2.7)$$

$N_f \equiv$ number of effective weak doublets

which typically gives values of roughly the order of a keV, as exhibited in Fig. 1, i.e., much larger than the M1 widths.

Even weak annihilation (s or t channel charged currents) can lead to much larger rates than those given by Eq. (2.2) for M1 transitions: since T^* is a vector state there is no helicity suppression factor. Weak annihilation in the s channel for example gives in a parton picture:

$$\Gamma_{\text{ann},s}(T_q^*) \sim \frac{N_f}{12\pi} G_F^2 f_V^2 (m_t + m_q)^3 \quad (2.8)$$

With

$$f_V \sim \frac{30 \sqrt{\text{MeV}}}{\sqrt{m_t + m_q} \left(\frac{1}{m_t} + \frac{1}{m_q} \right)}$$

an ansatz which will be explained in Section 3, we obtain for T_b^* decays the values shown in Fig. 1. For weak annihilation in the t channel the result depends on whether one takes colour factors seriously or not⁵⁾:

$$\Gamma_{\text{ann},t}(T_q^*) \sim \frac{1}{N_c} \frac{1}{12\pi} G_F^2 f_V^2 (m_t + m_q)^3 \quad (2.9)$$

Two extreme choices are $N_c = 3$ or 1. For $f_V \sim 700 \text{ MeV}$ even these contributions are typically much larger than radiative M1 rates.

While weak t channel annihilation contributes to T_U^* and T_C^* decays (for $e_t = 2/3$) weak s channel annihilation in principle can influence T_d^* , T_s^* and T_b^* decays, depending on the mixing angles for $t \rightarrow d$, $t \rightarrow s$ and $t \rightarrow b$. Not only the theorist's bias for simplicity, but also already existing phenomenology seem to put some stringent bounds on their relative transition rates⁶⁾:

$$\frac{R(t \rightarrow s)}{R(t \rightarrow b)} \sim 0.04 - 0.3$$

$$\frac{R(t \rightarrow d)}{R(t \rightarrow b)} \sim 10^{-3} - 10^{-2}$$

Consequently we expect the largest effects due to s channel weak annihilation in T_b^* decays, which we will discuss in more detail. T channel weak annihilation might be significant in T_u^* and T_c^* decays, if colour suppression factors can be ignored as has been suggested in the literature; yet we will not describe them in detail, since we chose to adopt a conservative attitude in this paper and since one can easily generalize the following discussion of T_b^* decays to T_u^* and T_c^* decays.

T_b^* decays due to weak annihilation will exhibit some striking features:

- i) These decays show a two jet structure instead of the three jet structure of the cascade decays.
- ii) Purely leptonic decays are not suppressed for a vector state, allowing one to estimate in a naive parton picture:

$$\frac{\Gamma(T_b^* \rightarrow e\nu, \mu\nu, \tau\nu)}{\Gamma_{\text{ann}}(T_b^*)} \sim \frac{3}{N_f} = \frac{1}{3} \quad (2.10)$$

From Fig. 1 one can read off

$$BR(T_b^* \rightarrow e\nu, \mu\nu, \tau\nu) \sim 10 [4] \% \text{ for } m_t \sim 20[30] \text{ GeV} \quad (2.11)$$

Such decays will have a very striking and spectacular signature in the final state: one large $p_T (\sim M_{T_b^*}/2)$ lepton plus large missing p_T and energy. Of course a parton model picture is an oversimplification. One has to allow for gluon radiation from the annihilating quark lines. Usually one argues that gluon emission from such heavy quark lines can reliably be calculated in perturbation theory, i.e., that it is a small effect. When the hadrons thus produced carry large p_T and large invariant mass then such contributions should furthermore be damped strongly by propagator effects. Therefore the basic picture given above - large p_T lepton and large missing p_T - should not be changed very much by gluon effects.

iii) Nevertheless it is interesting to study also the hadronic states in the annihilation decays: $T_b^* \rightarrow (\text{large } p_T) \ell\nu + \text{few, soft hadrons}$. Since these hadrons are presumably generated mainly from gluons, one can expect to have a good chance of identifying them as coming from the intermediate decay of a glueball state⁷⁾. It is expected that the lowest lying of these states has a mass not exceeding two GeV and a width of roughly 20 MeV.

iv) Decays produced by annihilation should also differ markedly from the cascade decays in their flavour content. The idea behind this statement is the

following: in the annihilation reaction the decay proceeds via a W boson which fragments into the three colour doublets of $u\bar{d}$ and $c\bar{s}$ each and into the three lepton doublets $e\nu$, $\mu\nu$ and $\tau\nu$:

$$T_b^* \rightarrow u\bar{d}, c\bar{s}, l^+ \nu_l \quad (2.12)$$

The further development into the observed final state is presumably governed by more or less soft gluons, whose transition into $s\bar{s}$ is suppressed. Crude estimates give for the probability of a gluon turning into $s\bar{s}$:

$$\text{Prob}(g \rightarrow s\bar{s}) \sim \frac{1}{6} - \frac{1}{9} \quad (2.13)$$

The cascade picture on the other hand leads to the following chain:

$$T_b^* = (t\bar{b})^* \rightarrow b + \bar{b} + W^+ \rightarrow c + \bar{c} + 2W^+ + W^- \quad (2.14)$$

For energy transfers as they are involved in the transitions $t \rightarrow bW^+$ and $b \rightarrow cW^-$, the subsequent fragmentation $W^+ \rightarrow c\bar{s}$ should not be suppressed much. Simply counting the states yields for these "hard" W bosons:

$$\text{Prob}(W_{\text{hard}}^+ \rightarrow c\bar{s}) \sim \frac{1}{3} \quad (2.15)$$

The final step in the chain describes charm decays:

$$T_b^* \rightarrow \dots \rightarrow c + \bar{c} + 3W_{\text{hard}} \rightarrow s + \bar{s} + 2W + 3W_{\text{hard}} \quad (2.16)$$

The additional two W bosons with low virtual mass do not lead to further strangeness production (as far as the Cabibbo favoured decays are concerned).

Therefore by comparing two- and three-jet events in T_b^* decays one can obtain information on the difference in strangeness (or any heavy flavour) production due to "hard" processes, namely by W bosons, and due to gluon production, which will presumably be mainly soft.

To get a rough estimate of strange multiplicities in the two mechanisms one can compare

$$T_b^* \rightarrow \begin{matrix} u\bar{d} \\ c\bar{s} \end{matrix} + 5 \text{ gluons} \quad (\text{annihilation})$$

with

$$T_b^* \rightarrow s + \bar{s} + 3 W_{\text{hard}} + 2 W_{\text{soft}} \quad (\text{cascade})$$

Further development of the two configurations into the final state should then be roughly the same. Using the numbers given above we find:

$$\langle N_K \rangle = \langle N_{\bar{K}} \rangle \sim 0.7 - 0.9 \quad (2.17a)$$

for annihilation and

$$\langle N_K \rangle = \langle N_{\bar{K}} \rangle \sim 2 \quad (2.17b)$$

for the cascade. Thus the difference is slightly more than one unit per decay.

So far we have discussed weak decays only of the vector states T^* , ignoring the pseudoscalar mesons T . Yet looking at our estimates of the mass difference $M(T^*) - M(T)$, one sees immediately that it will be hard to separate the two, since they are almost degenerate in mass. Although 0^-0^- , 1^-0^- and 1^-1^- production in e^+e^- annihilation shows a somewhat different threshold behaviour and the angular dependence can be quite different⁸⁾, this is probably not sufficient to separate the various contributions. The situation improves considerably when polarized beams are available.

It has been argued that weak annihilation contributes to the decays of charmed pseudoscalar mesons despite the apparent occurrence of helicity factors⁹⁾. We think that such effects, should they exist, will be unimportant, at least for T_c and T_b decays. We base this expectation on the general argument that the probability of finding constituent gluons in T_q mesons should depend on a parameter like $\Lambda^2/(\Lambda^2 + m_q^2)$, where Λ normalizes the strong coupling; in that case it is strongly suppressed for $m_q \gg \Lambda$. Since spin counting yields $\sigma(T^*) \sim 3\sigma(T)$ the main impact of T production will be to dilute somewhat the emergence of typical weak annihilation effects by maybe $\sim 20\%$.

The sore point of our proposal presented so far is obvious: the most striking (and possibly only measurable) effects are expected for T_b^* decays. Yet the production rate T_b^* on e^+e^- annihilation is presumably quite small compared with T_u^* , T_d^* and T_s^* production. A crude estimate based on leading log calculations gives¹⁰⁾:

$$\sigma(T_b^*) \sim \frac{1}{10} \times \sigma(T_c^*) \sim \frac{1}{100} \times \sigma(T_{u,d,s}^*) \quad (2.18)$$

well above threshold energy. Furthermore we expect that T_b^* production, setting in around 10 GeV above $T\bar{T}$ threshold, will rise rather slowly since the second constituent, the b quark, is also very heavy. Nevertheless we believe that the annihilation decay $T_b^* \rightarrow (\text{large } p_T) \text{ lepton} + \text{missing } p_T$ will provide a clear signature even for a small production rate, as long as the total energy is below W production threshold.

Such an annihilation decay will lead to spectacular events such as

$$e^+e^- \rightarrow T_b^* \bar{T}_b^* \begin{cases} \rightarrow e^-(\mu^-) + \text{missing } p_T \\ \rightarrow e^+(\mu^+) + \text{missing } p_T \end{cases}$$

Yet we have to keep in mind that heavy lepton production will lead to the same final state with a larger rate:

$$e^+e^- \rightarrow \tau^+ \tau^- \begin{cases} \rightarrow e^-(\mu^-) + \nu's \\ \rightarrow e^+(\mu^+) + \nu's \end{cases}$$

It is therefore better to look for a reaction of the type

$$e^+e^- \rightarrow T_b^* \bar{T}_b^* \begin{cases} \rightarrow 2-3 \text{ hadronic jets} \\ \rightarrow e^+(\mu^+) + \text{missing } p_T \end{cases}$$

since the τ lepton with its mass ~ 1.8 GeV cannot generate hadronic jets.

Another possible background is charm production followed by semi-leptonic decay. Yet in such events one expects the lepton to be accompanied by a hadronic jet recoiling against another hadronic jet. In the case of $e^+e^- \rightarrow T_b^* \bar{T}_b^*$, on the other hand, the lepton is not associated with the hadronic jets; well above T_b^* threshold the hadronic jets are obviously not back-to-back and the lepton is even in the opposite hemisphere. Furthermore one has to note that the missing energy (or p_T) is of the order of half the beam energy.

A hypothetical fourth heavy lepton would produce very similar final states with presumably larger rates. Its production would already clearly manifest itself in global quantities like total cross-sections; if such a lepton were

produced it would bury the effects we have discussed above. We will come back to this point later. In section 4 we will discuss another reaction leading to T_b^* production.

3. ESTIMATE OF THE DECAY CONSTANTS

In this section we argue for our choice of meson decay constants and discuss their possible range. Only in truly non-relativistic system can we reliably calculate

$$\begin{aligned} f^2(^3S_1(\bar{\mu}\mu) \rightarrow \bar{e}e) &= |\phi(0)|^2 \cdot 4 \cdot M_{\mu\bar{\mu}}^{-1} = \\ &= \frac{3 \pi \Gamma_{ee}}{2 \pi \alpha^2} \end{aligned} \quad (3.1)$$

where $\phi(0)$ is the Schrödinger wave function at the origin. When the states become more and more relativistic, perturbation theory quickly becomes unreliable and breaks down, as is probably the case for the light mesons π and K . A relativistic treatment of the bound state problem is only known in principle or for special cases, e.g., the Bethe-Salpeter equation in ladder approximation. In practice we have no tool for relativistic bound states but must rely on more fundamental principles, supplemented by guesses. Yet nature was kind enough to let us discover heavier resonances as higher energies became available, and some of these more and more resemble non-relativistic bound states. We seem to understand charmonium and hopefully bottomonium in terms of the same non-relativistic potential picture¹¹⁾. We even predict the toponium properties with some confidence. For charmonium and bottomonium we find, both phenomenologically and experimentally¹²⁾

$$\frac{f_v^2(\Upsilon \rightarrow e^+e^-)}{M_\Upsilon} = \frac{f_v^2(J/\psi \rightarrow e^+e^-)}{M_{J/\psi}} \quad (3.2)$$

We can easily extend (3.2) to systems composed of different quarks: the light quark mass ($\approx$ reduced mass μ of the system) governs the wave function and the standard potential model indicates $|\phi(0)|^2 \sim \mu^2$. This is tested by relation (3.2). Other dimensional parameters are provided by the total mass M of the system, so that we arrive at

$$f_v = c \mu M^{-\frac{1}{2}} \quad ; \quad c = 30 \sqrt{\text{MeV}} \quad (3.3)$$

where c is given by the potential model¹³⁾. Equation (3.3) is also valid for pseudoscalar mesons, provided one of the constituents is sufficiently heavy to make spin-spin effects negligible. The range of validity of (3.3) may be expanded considerably, in potential models up to a reduced mass of 10 to 15 GeV. Above that range c increases to approach its asymptotic behaviour $c \sim \sqrt{\mu}$ (for infinitely large μ 's QCD scales and $|\phi(0)|^2 \sim \mu^3$). One of us had argued¹³⁾ that one may even expand the range of validity of (3.3) down to the ρ meson. In any case, for mesons like T_c or T_b the reduced masses are approximately equal to the c or b mass, respectively, and we can safely apply (3.3) ($m_c = 1.5$ GeV, $m_b = 5$ GeV):

$$f(T_c, T_b) \approx (240, 690) \text{ MeV} \quad (3.4)$$

while the corresponding numbers for T_u and T_s are more uncertain. We remind the reader that decay constants used in the literature for D and F mesons are generally larger than what we would estimate by using (3.3). While we get $f_{PS}(D^0) \approx 150$ MeV one finds values up to 800 MeV used in the literature¹⁴⁾. Similar discrepancies apply to the T_u and the T_s where we estimate, by using (3.3):

$$f(T_u, T_s) \approx (60, 90) \text{ MeV} \quad (3.5)$$

($m_u = 330$ MeV, $m_s = 540$ MeV). For the toponium system we might well underestimate $f(t\bar{t})$ as discussed above:

$$f(t\bar{t}) \approx 1.5 [2.0] \text{ GeV} \quad \text{for } m_t = 20 [30] \text{ GeV} \quad (3.6)$$

Throughout this paper we will use (3.4 - 3.6) at face value, mentioning possible effects in those cases where one might guess larger decay constants.

4. A SCENARIO OF TOPONIUM DECAYS

Weak toponium decays have been studied before³⁾. Especially Goggi and Penso have presented a scenario of toponium decays below, on, and above the Z^0 pole and have studied the feasibility of a toponium search with LEP. Above the Z^0 pole toponium clearly decays weakly by charged current interactions, leading to events with very high sphericity/low thrust. In the Z^0 pole the neutral current in $(t\bar{t})$ annihilation dominates. This leads to events exactly like in the continuum: $q\bar{q}$ and $q\bar{q}g$ with low sphericity/high thrust. We also expect interference structures here in the total cross-section, arising by interference of the Z^0 with the $(t\bar{t})$ poles¹⁵⁾. Below the Z^0 pole we find the transition region where $(t\bar{t})$

decays partly by the conventional channels ($ggg, \bar{q}q$) and partly by charged weak current decays of the t quarks. This energy region from $\sqrt{s} > 35$ GeV as the present experimental limit on the $\bar{t}t$ mass, up to 80 GeV, which is close to the maximum energy of the possibly next e^+e^- machine, HERA¹⁶⁾, will be the most interesting one for the near future. We present the toponium decay scenario, confined to this energy region i) for completeness and ii) because this transition region deserves a more detailed discussion. All the basic formulae will be repeated again for completeness and the results are displayed as branching fractions in Figs 2 and 3. In contrast to Goggi and Penso³⁾ we find smaller branching fractions for the three gluon decay. We will omit the more exotic channels like γ Higgs but will include a new decay channel via an intermediate T_b^* : $(\bar{t}t) \rightarrow T_b^* + W \rightarrow \ell_1 \ell_2 \bar{\nu}_1 \bar{\nu}_2, \ell \bar{\nu} q \bar{q}$.

We begin with a description of the basic formulae. Estimating Γ_{ee} is the easiest part. From our experience with potential models, subsumed in Eq. (3.3), we expect $\Gamma_{ee}(\bar{t}t) \approx 4$ keV for $M(\bar{t}t) \approx 40$ GeV. This rises up to $\Gamma_{ee}(\bar{t}t) \approx 8$ keV¹⁷⁾ for $M(\bar{t}t) \approx 80$ GeV, Fig. 4, for two reasons: first, $f_v(\bar{t}t)$ increases slightly over the value given in Eq. (3.3), this being the onset of the asymptotic behaviour of f_v . Secondly above 70 GeV the Z^0 pole starts contributing to Γ_{ee} of $(\bar{t}t)$, which amounts to $\approx 35\%$ at 10 GeV below the Z^0 pole. We will relate some decays to Γ_{ee} without the Z^0 contribution which we will denote as Γ_{ee}^* .

But beforehand we compare the non-resonant quark pair production with the resonance production. A measure for the signal is the peak value of R from resonance production alone,

$$R_{\text{peak}} = \frac{9\sqrt{\pi \ln 2} \Gamma_{ee}}{\alpha^2 \text{FWHM}}$$

for a Gaussian beam spread. This R_{peak} decreases with the c.m. energy because of the increasing machine beam spread. It has to be compared with the continuum R , Fig. 5. There we have also plotted the ratio $R_{\text{peak}}/R_{\text{cont}}$ assuming a full width at half maximum (FWHM) of the machine width of

$$\text{FWHM} = \frac{2}{3} \times 10^{-3} \sqrt{s}$$

or 20 MeV at $\sqrt{s} = 30$ GeV. The ratio $R_{\text{peak}}/R_{\text{cont}}$ decreases faster than R_{peak} . This is due to the increase of the non-resonant (continuum) cross-section caused by the Z^0 .

On resonance u, d, s, c, b quark pairs are produced via the electromagnetic (γ^*) and weak neutral (Z^0) current:

$$\Gamma_{(f\bar{f})} = \Gamma_{e\bar{e}}^{\gamma^*} \cdot 3 \sum_q e_q^2 \left[1 + \left(\frac{s}{s-M_Z^2} \right)^2 \frac{v_f^2 (v_q^2 + a_q^2)}{e_t^2 e_q^2 (4 \sin^2 2\theta_w)^2} + \frac{2s}{s-M_Z^2} \frac{v_t v_q}{e_t e_q 4 \sin^2 2\theta_w} \right] \quad (4.1)$$

where $v_f = a_f - 4 \sin^2 \theta_w \cdot e_f$, $a_f = e_f / |e_f|$ (but $a_\nu = 1$) in the standard $SU(2) \times U(1)$ model. For leptons the interference is negligible because of the smallness of v_e . For quarks the maximum of the negative interference is -30% for d, s, b quarks at $\sqrt{s} \approx 60$ GeV, while it is only -9% for u, c quarks here. Correspondingly the rate for d, s, b increases much faster above 70 GeV than that of u, c . QCD adds $\alpha_s / \pi \approx 5\%$ ¹⁸⁾ to the rate which partly cancels the effects of the negative interference in the total rate; we have included these QCD corrections in Figs 2 and 3. The chromodynamic annihilation is represented by the three gluon annihilation in Born approximation:

$$\Gamma_{3g} = \frac{\alpha_s^3}{\alpha^2 e_q^2} \frac{10(\pi^2 - 9)}{81\pi} \Gamma_{e\bar{e}}^{\gamma^*} = 1444 \alpha_s^3 \Gamma_{e\bar{e}}^{\gamma^*} \quad (4.2)$$

The appearance of gluon jets (already in direct T decays) strengthens our belief that the probability for the gluons to convert into hadrons is really one, for toponium as well as for T decays. The magnitude of the bottom and top quark masses should on the other hand justify the Born approximation. Both these statements are certainly not true for charmonium decays which explains that $\Gamma_{\text{direct hadrons}} / \Gamma_{ee}$ does not scale according to (4.2) from J/ψ to T . It should, however, from T to $(\bar{t}t)$ and we use $\alpha_s(m_b = 5 \text{ GeV}) = 0.18$ ¹⁹⁾ as input to arrive at $\alpha_s(m_t = 17.5 \text{ to } 40 \text{ GeV}) = 0.14 \text{ to } 0.12$. These values of α_s now determine Γ_{3g} via (4.2). The annihilation in one photon and two gluons is related to that into three gluons by

$$\Gamma_{\gamma g g} = \frac{36}{5} e_t^2 \frac{\alpha}{\alpha_s} \Gamma_{3g} \approx 0.18 \Gamma_{3g} \quad (4.3)$$

which is almost constant over the whole energy range. We therefore did not display this channel in the figures. In the mass range of interest we also witness the onset of charged currents. One mechanism leads to a mixing of $\bar{t}t$ with $\bar{b}b$: it is an annihilation of $\bar{t}t$ via a t channel charged current. With the colour factors from the parton model Born graph we find

$$\Gamma_{\text{weak ann.}} = \frac{1}{48\pi^2} \frac{G_F^2 M_{\bar{t}t}^4}{\alpha^2 e_t^2} \Gamma_{e\bar{e}}^{\gamma^*} \quad (4.4)$$

One might, however, take a different attitude to the colour counting, as discussed in Section 1 and multiply (4.4) and the corresponding curve in Figs 2 and 3 by a factor three.

The more important contribution of the weak interactions are the cascade decays. With the analog of the muon decay formula we find for $\bar{t}t$ decays

$$\Gamma_{\text{casc}} = \frac{2N_f}{192\pi^3} G_F^2 m_t^5 \int_0^{(m_t - m_b)^2} dq^2 \frac{M_w^4}{(m_t - m_b)^2 (q^2 - M_w^2)^2} \quad (4.5)$$

where N_f is the number of light weak doublets, three lepton and three times two quark doublets. In general the decay of one of the t quarks leads to the emission of a relatively fast b quark and a quark or lepton pair, originating from the W . But there is a finite chance that the b quark remains bound to the other t quark and shares its momentum with it, thus leading to a (tb) meson recoiling against the quark or lepton pair. We find only a small deviation from naïve spin counting: the (tb) meson is a pseudoscalar T_b 26% of the time and a vector T_b^* 74% of the time. The probability for the b quark to remain bound to the t quark can be calculated as an overlap integral of the initial $(\bar{t}t)$ or $(\bar{t}t)'$ state and the T_b or T_b^* state boosted by the recoil momentum of the b quark against the virtual W . We find this probability as displayed in the Table for T_b^* . The overlaps for a T_b in the final state are only slightly ($\approx 10\%$) larger. The radial node of the $(\bar{t}t)'$ wave function is responsible for the smallness of the overlap in $(\bar{t}t)' \rightarrow T_b^* + W$. The T_b^* thus created can now annihilate into another virtual W , leading to a unique intermediate $2W$ state. This annihilation can be conventionally calculated and we find

$$\Gamma_{\text{ann}}(T_b^*) = \frac{N_f}{12\pi} f^2(T_b^*) G_F^2 [M(T_b^*)]^3 \quad (4.6a)$$

where $f(T_b^*)$ can be taken from (3.3). In the spirit of Ref. 13) we would similarly find

$$\Gamma_{\text{ann}}(T_b) < \frac{N_f}{8\pi} f^2(T_b) G_F^2 [M(T_b)]^3 \quad (4.6b)$$

But following our conservative attitude we will neglect this process and only consider the annihilation of T_b^* . The $2W$ intermediate state leads to $\bar{q}q \bar{q}q$, $\bar{q}q \ell\nu$ and $\ell\nu \ell\nu$ final states. The rate for $(\bar{t}t)$ or $(\bar{t}t)'$ into these final states is simply a product of i) $\Gamma_{\text{casc}}(\bar{t}t)$, ii) the overlap v^2 of the Table,

iii) the probability to find T_b^* (74%); iv) the annihilation branching fraction of T_b^* as given in the Table and finally the branching fractions of the two W into the final states.

At this place a warning is in order: the wave function overlap employs Schrödinger wave functions at very high momentum (or short distances respectively), of roughly half the decay momentum. There relativistic corrections and higher order contributions become relevant and important. Unfortunately a calculation of higher orders in v/c and α_s becomes very complicated and is burdened with many still open theoretical questions, too. The numbers in the Table which are calculated using the Schrödinger wave functions^{*)} of our preferred potential model can therefore be not more than a guide. One can imagine that - once the theoretical calculation of this overlap can be performed with a high precision - an independent way opens to measure the wave functions of both $(\bar{t}t)$ and T_b^* at very short distances.

For the $(\bar{t}t)'$ state we have to consider radiative decays, too¹¹⁾. We scale the electromagnetic (dipole) transitions, $(\bar{t}t)' \rightarrow {}^3P_j(\bar{t}t)$, like k^2/m_Q to obtain 2 keV for $2m_t = 35$ GeV for all such transitions and less for higher m_t respectively. For the chromodynamic radiation, $(\bar{t}t)' \rightarrow \pi\pi(\bar{t}t)$, a $1/m_Q^2$ (times phase space) scaling has been proposed which seems to be operating between the $\bar{c}c$ and $\bar{b}b$ systems. We thus arrive at 0.8 keV for this transition for $2m_t = 35$ GeV and less at higher m_t respectively.

We now turn to a discussion of the emerging picture. The first striking observation is that below $2m_t \sim 50$ to 60 GeV the $(\bar{t}t)'$ state looks like a copy of the $(\bar{t}t)$ ground state: with just half the electric ($\propto \int_{res} dM \sigma$) and half the total width. The reason is simply that also $(\bar{t}t)'$ decays are dominated by annihilation. Above $2m_t \sim 50$ to 60 GeV we see the onset of an independent decay mechanism: the weak decay of a t in the bound state. This makes the $(\bar{t}t)'$ width rise faster with m_t than that of $(\bar{t}t)$ and its average $\langle 1 - T \rangle$ larger (Fig. 6). Above 70 to 75 GeV for $2m_t$ the average $\langle 1 - T \rangle$ drops again due to the increasing dominance of annihilation via Z^0 . But a close look at the event shape tells more than the average $\langle 1 - T \rangle$. Note that for $2m_t < 50$ to 60 GeV half to all hadronic events (without $\bar{\tau}\tau$) has a low $\langle 1 - T \rangle$, identical to the off resonance and background signal. A cut on these low $\langle 1 - T \rangle$ events would therefore remove 50% of the signal, too! Around $2m_t = 70$ to 75 GeV it still cuts more than 1/3 of a $(\bar{t}t)$ signal and a little less of a $(\bar{t}t)'$ signal.

^{*)} We have checked our calculations with harmonic oscillator wave functions which lead to slightly smaller overlap, as one expects.

Higher radially excited ($\bar{t}t$) resonances have much more decay channels, but should not exhibit other event shapes, because the additional channels are mainly hadronic cascades to low lying ($\bar{t}t$) states. This leads to the event shapes typical for low lying ($\bar{t}t$) states plus a few, soft, hadrons, which do not alter $\langle 1 - T \rangle$ significantly.

Going more into the detail now, we find that one quark flavour is produced not only via an intermediate γ^* or Z^0 : there is a substantial contribution to $\bar{b}b$ final states at least, in the mass range $2m_t = 70$ to 75 GeV, generated by the charged current annihilation of ($\bar{t}t$). This signal might even be larger by a factor three than drawn in Figs 2 and 3 due to a possibly different colour matching as discussed in Section 2. At least if one tries to measure angular asymmetries of d, s, b jets on resonance, this is a serious background. We

We have started before, that a $2W$ intermediate state may be produced by a cascade decay of one t quark, while the other t quark remains bound to the generated b in a T_b^* state, which can in turn also annihilate to a virtual W . While the $\bar{q}q$ signal from this source will be masked by higher order α_s contributions to $\bar{q}q$ or ggg final states, and the dilepton signal by the dilepton signal from $\bar{t}t$, the $\bar{q}q$ lepton signal appears as a novel feature. The $\bar{q}q$ pair will have an average invariant mass larger than $(m_t - m_b)/\sqrt{2}$ and the lepton will have an average energy of slightly more than $\frac{1}{2} m_t$, with another $\frac{1}{2} m_t$ of energy missing.

This signature is so unique that one may have a chance of seeing it although there are only 1 - 2 events of this kind per 1000 muon pairs. Of course this requires the absence of a new heavy lepton.

5. T_b^* PRODUCTION IN $p\bar{p}$ AND pp COLLISIONS

If $M_T > 20$ GeV then top states can be detected only at $p\bar{p}$ and pp colliders in the near future. This will not be an easy task due to the large background and the presumably large decay multiplicities and therefore a clear signature is required. If T_b^* mesons are produced with a small, yet finite rate, then their purely (or almost purely) leptonic decays $T_b^* \rightarrow e\nu, \mu\nu, \tau\nu$ would provide spectacular signatures. But in hadron-hadron collisions at sufficiently high energies one expects another and much more interesting reaction to occur which leads to basically the same final state, namely real W production:

$$p\bar{p} \rightarrow W + X \quad (5.1)$$

$$\quad \quad \quad \hookrightarrow e\nu, \mu\nu, \tau\nu$$

At first one might think that reaction (5.1) should easily dominate, since one cannot exclude it kinematically, unlike in e^+e^- annihilation.

But this might not be so, because T_b^* , being a hadron, can be produced strongly - in contrast to W bosons. It has been estimated²⁰⁾ that

$$\sigma_{p\bar{p}}(W^\pm) \sim \text{few} \times 10^{-33} \text{ cm}^2 \quad (5.2)$$

for $\sqrt{s} \sim 500 \dots 800$ GeV with $M_W \sim 80$ GeV. The production rate of T_b^* is even more uncertain. Total top production can be estimated in a simple QCD "inspired" approach by just calculating $p\bar{p} \rightarrow t + \bar{t} + X$ in lowest order using Q^2 dependent structure functions. For $m_t \sim 20$ [30] GeV one finds $\sigma(t\bar{t}) \sim 2 \times 10^{-31} [2 \times 10^{-32}] \text{ cm}^2$ at $\sqrt{s} \sim 800$ GeV²¹⁾. Then one can very crudely "guestimate" as in (2.18)

$$\sigma(T_b^*) \sim 10^{-33} [10^{-34}] \text{ cm}^2 \quad (5.3)$$

at $\sqrt{s} \sim 800$ GeV. Thus T_b^* production is not necessarily a small effect relative to real W production. One should also keep in mind that similar "QCD" calculations for hadronic charm production yielded numbers which are one to two orders of magnitude below the apparent experimental numbers²²⁾. Finally the leptonic branching ratios are quite comparable, to:

$$BR(W \rightarrow e\nu, \mu\nu, \tau\nu) \lesssim 25\% \quad (5.4)$$

$$BR(T_b^* \rightarrow e\nu, \mu\nu, \tau\nu) \sim 10 [4]\% \quad (5.5)$$

for $m_t \sim 20$ [30] GeV [Eq. (2.11)].

We also expect the lepton spectra for both cases to show (qualitatively at least) a similar behaviour; the main difference will be the position of the characteristic fast drop in p_T reflecting two body decay kinematics and the mass of the decaying particle.

As for e^+e^- annihilation, T_b^* will not only be produced strongly and pairwise, but also singly in the weak decays of toponium states. Using a scaling law for onia production in hadronic collisions as proposed by Gaisser, Halzen and Paschos²³⁾:

$$\sigma(t\bar{t}) \sim \frac{\Gamma_{t\bar{t}}}{M_{t\bar{t}}^3} F\left(\frac{s}{M_{t\bar{t}}^2}\right) \quad (5.6)$$

where F is a universal dimensionless function, one derives from ψ production:

$$\sigma(t\bar{t}) \sim \text{few} \times 10^{-34} \text{ cm}^2 \quad (5.7)$$

at $\sqrt{s} \sim 540$ GeV and for $m_t \sim 30$ GeV. Therefore using the overlap integrals given above, we estimate

$$\sigma(T_b^*) \sim 10^{-35} \text{ cm}^2 \quad (5.8)$$

at $\sqrt{s} \sim 540$ GeV and for $m_t \sim 30$ GeV. All these rates are obviously vague and certainly small, but probably not too small for observation, if other background sources can be controlled as well. One obvious background source is $\tau^+\tau^-$ production via the Drell-Yan mechanism:

$$\begin{array}{l} p\bar{p} \rightarrow \tau^+\tau^- + X \\ \quad \quad \quad \downarrow \\ \quad \quad \quad \begin{array}{l} \hookrightarrow e^-(\mu^-) + \bar{\nu}'s \\ \hookrightarrow e^+(\mu^+) + \nu's \end{array} \end{array}$$

Requiring large momenta for the observed leptons - e or μ - would certainly decrease drastically the size of this contribution. Nevertheless we do not believe that the two processes - T_b^* and τ production - could be disentangled in

$$p\bar{p} \rightarrow e^\pm \mu^\pm + (\text{large missing } p_T) + X$$

Analogous to the case in e^+e^- annihilation a cleaner signal is provided by

$$p\bar{p} \rightarrow (\text{large } p_T) \begin{array}{c} e \\ \mu \end{array} + \text{large missing } p_T$$

+ 2-3 opposite side jets + (low p_T) hadrons

Although of course, pairwise production of a hypothetical fourth lepton would produce very similar final states, the p_T spectrum of the observed lepton (e or μ) would look quite different since it came from a three body instead of a two body process: $\Delta \rightarrow \ell \nu_\ell \nu_\Delta$ vs. $T_b^* \rightarrow \ell \nu_\ell$. To summarize: it can well happen that large p_T electron or muon spectra in $p\bar{p}$ collisions will show two peaks or sharp shoulders. One of these, pointing to a decay mass of ~ 80 GeV, will be connected with real W boson production; yet the other one is not necessarily connected with the production of a second type of charged weak bosons, but can come from T_b^* production.

6. CONCLUSIONS

When (or if) toponium and top production is observed, novel features, not encountered in the lighter systems, will appear. For $m_t \gtrsim 20$ GeV weak effects will become sufficiently strong to determine the decays not only of the pseudo-scalar mesons, but also of the vector mesons; furthermore they will generate a sizeable fraction of toponium decays. While studies of the strong interaction in toponium decays (the three gluon intermediate state) will be hampered by the relative strength of electroweak interactions, we will in return obtain an excellent opportunity to study weak decays of hadrons with a sufficiently large phase space available: cascade versus annihilation decays, three jet versus two jet decays of top mesons and maybe more information on the short distance quark-anti-quark potential from $(\bar{t}t) \rightarrow T_b^* W$. We expect decay channels like $T_b^* \rightarrow e\nu, \mu\nu, \tau\nu$ which give spectacular signatures such that they might well lead to the detection of top production in hadron-hadron collisions in the first place.

We feel that the numbers we have given are rather conservative. Therefore the "unorthodox" effects we have described might turn out to be even larger than anticipated by us.

Table

Overlap for $(\bar{t}t) \rightarrow T_b^* + W$ and the annihilation branching fraction of T_b^* .

$2m_t$ [GeV]	35	40	50	60	70	80
$V^2[(\bar{t}t) \rightarrow T_b^* + W]$	0.18	0.13	0.08	0.05	0.04	0.03
$V^2[(\bar{t}t)' \rightarrow T_b^* + W]$	0.029	0.025	0.019	0.015	0.013	0.011
$\Gamma_{\text{ann}}/\Gamma_{\text{casc}}(T_b^*)$	0.66	0.44	0.23	0.13	0.083	0.056

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FIGURE CAPTIONS

- Fig. 1 : The weak cascade decay width $\Gamma_{\text{casc.}}$ of open top mesons, the weak s channel annihilation width Γ_{ann} of T^* and electromagnetic M1 widths of T_u^* and T_b^* , all as a function of the top mass.
- Fig. 2 : Toponium ground state branching fractions as a function of $2m_t$. $2m_t$ differs from $M(t\bar{t})$ only by the $(t\bar{t})$ binding energy, compare M. Krammer et al., Ref. 16).
- Fig. 3 : Branching fractions for the first radial toponium excitation.
- Fig. 4 : The electronic (Γ_{ee}) and total width (Γ_{tot}) of the toponium ground state $(t\bar{t})$ and its first radial excitation $(t\bar{t})'$. Γ_{ee} serves as a measure of the integrated resonance cross-section. The shaded areas are due to the Z^0 .
- Fig. 5 : Z^0 corrections to the continuum cross-section for up type ($e_q = 2/3$) and down type ($e_q = -1/3$) quarks and the sum of d, u, s, c, b quarks. $R_{\text{peak}}/R_{\text{cont}}$ is the relative increase of the cross-section on top of the resonance. The curve shown assumes a beam spread with FWHM = 20 MeV at $\sqrt{s} = 30$ GeV, compare text. This curve includes the (tiny) QCD corrections to the off resonance cross-section. The curves are calculated using $M(Z^0) = 90$ GeV. Due to the uncertainty of the Z^0 mass, this value might change by 1 to 3 GeV. This can be taken into account by reading the abscissa scale as $2m_t - 90 \text{ GeV} + M(Z^0)$.
- Fig. 6 : "Jettiness" on and off $(t\bar{t})$ and $(t\bar{t})'$ as a function of m_t . We have plotted the perturbative values of $\langle 1 - T \rangle$ as determined from the hadronic decay channels. The $\langle 1 - T \rangle$ values for those may be found in Refs 24) to 26). Non-perturbative effects should be small in the lower and negligible in the upper range of m_t 's.

FIG. 4

FIG. 3

FIG. 1

FIG. 2

FIG. 5

FIG.6