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CHROMODYNAMIC THEORY OF HADRONS

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INTRODUCTION

During the last few years a new framework to describe strong interaction physics has emerged: quantum chromodynamics. My lectures will be dealing with this subject as it appears today: not yet a well-defined quantum field theory with known spectral properties as, for example, quantum electrodynamics, but rather a collection of many different ideas, all related to the concept of color. Some of these ideas are very phenomenological, for example the naive, non-relativistic concepts to describe bound states, some of them rather abstract, like the ideas about the structure of the renormalization group equations at small distances. There is no way to cover all the details, thus I shall concentrate on some aspects, which to me look important and interesting at the moment. I shall not speak about the more theoretical aspects of gauge theories in general, which will be discussed in the lectures of Prof. Pietschmann. The plan of the lectures is roughly as follows: As an introduction the basic ideas of color and chromodynamics are discussed. It follows a presentation of the more advanced fieldtheoretic aspects (renormalization, asymptotic freedom, confinement). Finally I shall discuss some of the perturbative aspects of QCD, which begin to play an important role in our understanding of jet phenomena in hadronic physics.

I. COLOR AND THE CHROMODYNAMIC FORCE

1.) Color - the secret of the Δ -resonance

The argument which is still the most convincing one in favor of the color quantum number is given by the non-relativistic naive quark model. It is well-known that the low-lying baryon states are symmetric in the quark and spin indices. For example the Δ^{++} -resonance, in the $J_3 = + 3/2$ state, is described by a quark wave function in which all three quark spins are aligned:

$$| \Delta^{++}, J_3 = + 3/2 \rangle = | u \uparrow u \uparrow u \uparrow \rangle. \quad 1.1$$

The structure of this wave function poses a problem: the wave function is symmetric in the quark and spin indices. On the other hand, one expects that the space part of the wave function is symmetric in the case of the Δ^{++} state, since the latter is the

ground state of the three quark system composed of three u-quarks. In this case the wave function is totally symmetric, which, on the other hand, cannot be, if quarks, like any other known spin 1/2 objects, obey Fermi-Dirac statistics. An answer to this spin-statistics puzzle (and meanwhile we are almost sure, it is the correct one) was given more than ten years ago:^{1,2)} each quark flavor comes in three different "colors", say red, green, and blue. In this case the wave functions of the ground states are symmetric in space, flavor and spin, but antisymmetric in color:

$$|qqq\rangle \rightarrow \frac{1}{\sqrt{6}} \epsilon^{ijk} |q_i q_j q_k\rangle \quad 1.2$$

(i,j,k: color index; ϵ^{ijk} antisymmetric tensor in three dimensions).

The world of hadrons discovered thus far can be described by twelve colored quarks: the six "light" quarks

$$\begin{pmatrix} u_r & u_g & u_b \\ d_r & d_g & d_b \end{pmatrix},$$

and the six "heavy quarks"

$$\begin{pmatrix} c_r & c_g & c_b \\ s_r & s_g & s_b \end{pmatrix}.$$

2.) The confinement dogma

We "observe" the Δ^{++} -wave function to be symmetric in spin and space, hence antisymmetric in color. Why is the color part of the wave-function antisymmetric? What is special about this antisymmetry? Quarks are triplets under the unitary group SU_3^C , acting on the color indices. Three quarks can form a singlet under SU_3^C : $3 \times 3 \times 3 = 1 + 8 + 8 + 10$, and the singlet state is just the one which is totally antisymmetric in the color index. Thus a consistent way to describe the baryon spectrum would be to suppose that all baryons are singlets under SU_3^C .²⁾ Furthermore, the same principle can be applied to mesons as well, since we can form SU_3^C singlets out of quarks and antiquarks: $3 \times \bar{3} = 1 + 8$. The SU_3^C singlet states are identified with the mesons whose wave functions would be

$$| \text{Meson} \rangle = \frac{1}{\sqrt{3}} (\bar{q}_r q_r + \bar{q}_g q_g + \bar{q}_b q_b). \quad 1.3$$

Note that two quarks cannot form a color singlet:

$3 \times 3 = \bar{3} + 6 \not\equiv 1$. Also a four-quark system cannot be in a SU_3^C -singlet state. Fortunately, two-quark or four-quark states have never been observed, and a consistent way to describe the whole spectrum of hadrons is to postulate that all hadrons and, moreover, everything which can be directly observed (currents,...) are color singlets.²⁾

We emphasize that the line of argument presented above leads necessarily to a three-color structure. Any other number of colors (2,4,...) is clearly ruled out (four colors, for example, would imply the simplest multiquark states to be of the form $|qqqq\rangle$, in disagreement with observation).

The confinement postulate rules out the existence of states like quarks (q), diquarks (qq), etc. The simplest color singlet configurations are mesons ($\bar{q}q$) and baryons (qqq). However, also more complicated configurations can exist as color singlets, e.g. $\bar{q}q\bar{q}q$ ("dimesons") or $qqq\bar{q}q$ ("mesobaryons"). Thus far such more complicated configurations have not been clearly identified in experiments. However, recently evidence is increasing that new states exist at ~ 2 GeV in the baryon-antibaryon channel.³⁾ It is quite possible that these baryonium states are of the type $\bar{q}q\bar{q}q$.⁴⁾

3.) Seeing color

Although we suppose that color is a hidden quantum number, permanently confined, there are a few places in nature where one can look behind the confining wall and see how the colors operate. One of these places is e^+e^- -annihilation into hadrons at high energies. In lowest order of QED this process is described by the annihilation of the electron-positron pair into a heavy virtual photon. The latter can disintegrate into anything which carries electric charge, especially into a quark-antiquark pair, which subsequently turns into hadrons (see Fig. (1)).

Fig. 1

Fig.(1): e^+e^- -annihilation into hadrons

If we suppose that this "turning into hadrons" of the $q\bar{q}$ -pair happens only after the quarks have moved away from each other a little such that the original pointlike vertex describing the coupling of the virtual photon to the $q\bar{q}$ -pair is not (or at least only very little) disturbed by the final state interaction, the total annihilation rate is given simply by the quark charges:

$$R = \sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-) = \sum_{\text{quarks}} e_i^2 \quad 1.4$$

(e_i : electric charge of quark i , normalized in units of e).

In the "low energy domain" of e^+e^- -annihilation, i.e. below 3.6 GeV (below charm threshold) only the three flavors u, d, s contribute to the sum in eq. (1.4). Thus we have

$$R = 3(e_u^2 + e_d^2 + e_s^2) = 3[(2/3)^2 + (-1/3)^2 + (-1/3)^2] = 2.$$

The present experimental data ⁵⁾ give $R \approx 2 - 2.5$ in the region between 2 and 3.6 GeV; there is good agreement with the color prediction 2. Note that in the absence of the color quantum number one would have $R = 2/3$, in total disagreement with experiment.

Another place in particle physics where we can see color indirectly is the decay $\pi_0 \rightarrow 2\gamma$. Applying PCAC one can describe this decay by relating it to the coupling of the axial vector current to two photons. The latter is described in the low energy limit ($q_\mu \rightarrow 0$) by the lowest order diagram which can contribute, namely

the triangle diagram (see Fig. (2)).

Fig.(2): The decay $\pi_0 \rightarrow 2\gamma$ and its relationship to the triangular coupling of the axialvector current to two photons. It can be shown that all higher order strong interaction corrections do not contribute in the zero frequency limit provided the strong interactions are described by a renormalizable field theory.⁶⁾

The decay width of the π_0 can be calculated, using PCAC, in terms of the pion decay constant F_π measured by the weak decay of $\pi^\pm$ and the number and charges of quarks which contribute to the triangle diagram.

One obtains

$$\Gamma_{\pi_0 \rightarrow 2\gamma} \approx \frac{M_\pi^3}{32\pi F_\pi^2} \left(\frac{\alpha}{\pi}\right)^2 \left(\frac{N}{3}\right)^2, \quad 1.5$$

where N is the number of colors. Using the experimental value $F_\pi = 0.96 M_\pi$, one finds

$$\Gamma_{\pi_0 \rightarrow 2\gamma} = 7.87 \cdot \left(\frac{N}{3}\right)^2 \text{ eV}. \quad 1.6$$

Excellent agreement with the observed width $7.95 \pm 0.55 \text{ eV}$ is obtained if $N = 3$.²⁾ In the case of one color the decay width comes out to be 0.87 eV , in total disagreement with observation.

4.) Chromoelectric Forces

Although the confinement hypothesis is thus far not in conflict with experiment, it goes much further than required by experiment.

From experiments we know that colored states (quarks, diquarks,...) must be much heavier than the conventional color singlet hadrons. It could well be that colored states are produced in high energy collisions above a certain threshold, which may be of the order of 10 GeV. Such a situation might be referred to as "partial confinement".

Certainly the case where the color threshold is at infinite energy would be the simplest one: in this case no new scale parameter like E_c (color threshold) would be present in strong interaction physics. But only experiments can decide here.

If the color quantum number is introduced, the nature of the strong interactions becomes rather well specified. No matter what kind of confinement one prefers, the strong force between quarks must act such that color singlets and color nonsinglets are treated in a different way, i.e. the interquark force must not be color blind. This requirement eliminates, for example, any theory of the strong interactions in which the colored quarks are coupled to a flavor and color singlet gluon field, which then has the internal quantum numbers of the baryonic current (conventional gluon-quark theory).

The simplest way to introduce a color-dependent potential between quarks is to use a potential which couples to the color charges themselves, i.e. a potential of the type $[\lambda_i^{(a)}/2] \cdot [\lambda_i^{(b)}/2] \cdot V(r)$, where $V(r)$ is a function of the relative distance only, $\lambda_i^{(a)}/2$ is the color SU_3 matrix acting on quark a, and $\lambda_i^{(b)}/2$ is the color matrix acting on quark b, (see Fig. (3)).

Fig.(3): The interaction of two quarks via a color-dependent potential.

In order to study details of such a system, let us introduce the SU_3^C -analogue of the isospin operator $\vec{T}$, the so-called color spin $\vec{C}$. The color spin is the color analogue of the SU_3 F-spin operator and is given by the eight-dimensional vector $\vec{C} = (C_1, C_2, \dots, C_8)$ where the C_i are the eight SU_3^C -charges. They can be written in terms of quark fields as follows:

$$C_i = \int q^* \frac{\lambda_i}{2} q d^3 x. \quad 1.7$$

For each SU_3^C representation the color spin $\vec{C}$ is represented by matrices which transform like the generators C_i . For example for the triplet representation one has $\vec{C} = (\frac{\lambda_1}{2}, \frac{\lambda_2}{2}, \dots, \frac{\lambda_8}{2})$. The color spin of a composite system is equal to the vector sum of the color spins of the constituents:

$$\vec{C} \text{ (total)} = \sum_{\text{constituents}} \vec{C}. \quad 1.8$$

The square of the color spin $\vec{C}^2 = \sum_{i=1}^8 C_i^2$ is a Casimir operator of SU_3^C , i.e. it is SU_3^C invariant, and its eigenvalues can be used to label the various color representations. Below we give the eigenvalues of $\vec{C}^2$ for the simplest SU_3^C representations:

Representation	1	$3, \bar{3}$	6	8	10	15	24	27
$\vec{C}^2$	0	4/3	10/3	3	6	28/3	25/3	8

Let us suppose that the interaction Hamiltonian for a two-quark system in a nonrelativistic approach is as mentioned above:

$$H = \frac{p_a^2}{2m_a} + \frac{p_b^2}{2m_b} + m_a + m_b + V(r) \sum_{i=1}^8 \frac{\lambda_i(a)}{2} \frac{\lambda_i(b)}{2} = \frac{p_a^2}{2m_a} + \frac{p_b^2}{2m_b} + m_a + m_b + V(r) \vec{C}(a) \cdot \vec{C}(b). \quad 1.9$$

Using the identity

$$\begin{aligned} (\vec{C}(a) + \vec{C}(b))^2 &= \vec{C}^2(a) + \vec{C}^2(b) + 2 \vec{C}(a) \cdot \vec{C}(b) \\ &= 8/3 + 2 \vec{C}(a) \cdot \vec{C}(b), \end{aligned} \quad 1.10$$

we find

$$H = \frac{p_a^2}{2m_a} + \frac{p_b^2}{2m_b} + m_a + m_b + 1/2 V(r) (\vec{C}^2(\text{total}) - 8/3). \quad 1.11$$

It follows that the mass eigenvalues of $\bar{q}q$ or qq systems can be written as

$$M = m_a + m_b + c_0 + c_1 \cdot \vec{C}^2(\text{total}) \quad 1.12$$

where c_0 and c_1 are constants, depending on the space part of the wave function. If $V(r)$ is an attractive potential, one has $c_1 > 0$.

In the case of a n -quark system, eq. (1.9) is replaced by

$$H = \sum_{a=1}^n \left(\frac{p_a^2}{2m_a} + m_a \right) + \sum_{\substack{a < b \\ a, b=1}}^n \vec{C}(a) \cdot \vec{C}(b) V(r_{ab}), \quad 1.13$$

if we suppose that the interaction is solely due to two-body forces. In case of symmetric space configurations the mass eigenvalues are given by

$$M = \sum_{a=1}^n m_a + c_0 + c_1 \vec{C}^2(\text{total}). \quad 1.14$$

From eq. (1.12) and (1.14) we learn immediately that the color singlet states are the lowest energy eigenstates, since for them $\vec{C}^2(\text{total})$ vanishes. The energy "gap" between color singlet states ("ordinary hadrons") and the colored states is given by $\Delta E^C = c_1 \vec{C}^2(\text{total})$. The constant c_1 depends critically on the properties of the interquark force. All speculations about quark or color confinement center around the idea that c_1 may be infinite. Of course, in simple potential models this is the case if the potential between quarks increases sufficiently strong as $r \rightarrow \infty$ (e.g. linear potential, harmonic oscillator potential).

5.) Quantum Chromodynamics

The simplest fieldtheory which incorporates a color-dependent force among quarks is the non-Abelian gauge theory in color space: Quantum Chromodynamics (QCD).⁷⁾ Here the forces among the colored quarks are generated by the exchange of colored vector gluons coupled to quarks in a gauge-invariant manner. This theory, which by now has emerged as the most promising candidate for a field theory of the strong interactions is a close relative of the most

successful quantum fieldtheory in physics: QED. The only, but important difference between QED and QCD is the gauge group involved. QED is an Abelian gauge theory [gauge group $U(1)$], QCD is a non-Abelian gauge theory [gauge group SU_3^c]. The formal Lagrangian of QCD is given by

$$L(x) = -1/4 \sum G_{\mu\nu}^A G^{\mu\nu}_A + \bar{q}[i\gamma^\mu (\partial_\mu - ig B_\mu^A \frac{\lambda_A}{2}) - \mathcal{M}]q \quad 1.15$$

where q stands for $u, d, \dots$,

$$G_{\mu\nu}^A = \partial_\mu B_\nu^A - \partial_\nu B_\mu^A - g f_{ABC} B_\mu^B B_\nu^C,$$

and $\mathcal{M}$ is the quark mass matrix discussed below. We emphasize the universality of the coupling constant in QCD: All quark flavors as well as the gluons are coupled with the universal strength g .

An essential difference between QCD and QED lies in the behaviour of the renormalized coupling constant. It is well known that in electrodynamics the physical coupling e , defined by the large distance behaviour of the electric potential (Thomson limit), is smaller than the effective coupling constant e_{eff} , one would measure at small distances, due to the presence of vacuum polarization effects. Since a "bare" electron is surrounded by an infinitely extended sea of electron-positron pairs, it will attract the virtual positrons contained in the vacuum polarization cloud, and thus shield part of its electric charge.

The behaviour of the effective coupling constant as a function of the distance can easily be calculated in perturbation theory (see diagram Fig.(4)).

Fig.(4): Vacuum polarization in QED.

In momentum space one finds in lowest order of perturbation theory

$$\alpha(\vec{q}^2) = \alpha(\mu^2) \left[1 + \frac{\alpha(\mu^2)}{3\pi} \ln \left(\frac{\vec{q}^2}{\mu^2} \right) \right] \quad 1.16$$

where $\alpha(\mu^2)$ is the electromagnetic coupling constant $e^2(\mu)/4\pi$ corresponding to the ultraviolet cutoff μ . In higher orders a whole series of terms of type $[\alpha(\mu^2) \ln(\frac{\mu^2}{\vec{q}^2})]^n$ appears, and it is possible to sum the leading logarithms. ⁸⁾ One finds

$$\alpha(\vec{q}^2) = \frac{\alpha(\mu^2)}{1 - \frac{\alpha(\mu^2)}{3\pi} \ln \left(\frac{\vec{q}^2}{\mu^2} \right)} + O(\alpha^2). \quad 1.17$$

Thus the effective coupling constant tends to increase at small distances (large $\vec{q}^2$). If one takes eq. (1.17) at face value and disregards the higher order corrections, $\alpha(\vec{q}^2)$ develops a pole if $\ln(\vec{q}^2)$ is becoming large:

$$\alpha(\vec{q}^2) \rightarrow \infty, \text{ if } \vec{q}^2 \rightarrow \mu^2 e^{3\pi/\alpha(\mu^2)}. \quad 1.18$$

However, this pole in $\alpha(\vec{q}^2)$ cannot be taken seriously. There is no reason to expect that the higher order terms in eq. (1.17) can be neglected for very large values of $\vec{q}^2$. Unfortunately, it is not possible to calculate the high energy limit of $\alpha(\vec{q}^2)$ in QED with present techniques based on perturbation theory.

In QCD the behaviour of the effective coupling constant is strikingly different. The reason for this difference is the new feature of QCD, which is not shared by QED, namely the self-interaction of the vector fields. The interactions between gluons gives rise to gluonic contributions to the vacuum polarization.

It is useful to separate the contributions due to the transverse gluonic degrees of freedom and the "Coulomb" degrees of freedom by using the Coulomb gauge (see e.g. ref. (9)). The contributions of the transverse quanta to the vacuum polarization is exhibited in Fig. (5), while the contribution due to one transverse and one "Coulomb" quantum is exhibited in Fig. (6).

Fig.(5): Vacuum polarization in QCD due to transverse gluons (dashed lines).

Fig.(6): Vacuum polarization in QCD due to a transverse and a "Coulomb" gluon.

Explicit calculation shows that the contribution of the transverse gluons leads to "color charge" screening just like the fermion-antifermion contributions in QED. In perturbation theory one finds

$$\kappa(\vec{q}^2) = \kappa(\mu^2) \left[1 + 5 \frac{\kappa(\mu^2)}{4\pi} \ln\left(\frac{\vec{q}^2}{\mu^2}\right) \right] \quad 1.19$$

where $\kappa = g^2/4\pi$ is the strong interaction analog of α . Very interesting are the contributions shown in Fig. (6).

Fig.(7): Logarithmic flux line collimation in QCD.

They have the opposite sign:

$$\kappa(\vec{q}^2) = \kappa(\mu^2) \left[1 - 16 \cdot \frac{\kappa(\mu^2)}{4\pi} \ln\left(\frac{\vec{q}^2}{\mu^2}\right) \right], \quad 1.20$$

i.e. they lead to an "antiscreening" of the color charge, which is larger than the screening due to transverse gluons.

Furthermore, each quark flavor contributes to the color screening as follows:

$$\kappa(\vec{q}^2) = \kappa(\mu^2) \left(1 + 2/3 \frac{\kappa(\mu^2)}{4\pi} \ln\left(\frac{\vec{q}^2}{\mu^2}\right) \right). \quad 1.21$$

Adding all the different contributions, one obtains

$$\kappa(\vec{q}^2) = \kappa(\mu^2) \left(1 - (11 - 2/3 n) \frac{\kappa(\mu^2)}{4\pi} \ln\left(\frac{\vec{q}^2}{\mu^2}\right) \right) \quad 1.22$$

(n: number of quark flavors).

Summing up all powers of $\kappa \cdot \ln\left(\frac{\vec{q}^2}{\mu^2}\right)$ in the perturbation expansion, one finds

$$\kappa(\vec{q}^2) = \frac{\kappa(\mu^2)}{1 + B \frac{\kappa(\mu^2)}{4\pi} \ln\left(\frac{\vec{q}^2}{\mu^2}\right)} \quad 1.23$$

where $B = (11 - 2/3 n) / 4\pi$.

Thus in QCD the vacuum polarization leads to a logarithmic increase of the force between colored constituents, compared to the Coulombic force. The effective coupling constant decreases for large $\vec{q}^2$ (asymptotic freedom) and approaches zero for $\vec{q}^2 \rightarrow \infty$, provided $11 - 2/3 n > 0$, i.e. $n < 16$.¹⁰⁾

This means that the color flux lines in QCD between two heavy color sources (say heavy quarks) are more collimated than in a pure Coulombic field. Evidently this is a feature unique to Yang-Mills theories.

It is possible to interpret this logarithmic flux line collimation in QCD by the following simple picture. The diagram Fig.(6) corresponds to a situation in which the Coulomb type field lines are drawn

together by the exchange of a transverse gluon (see Fig.(7)). This attractive force between the field lines is similar to the attractive force between two parallel electric currents (law of Biot and Savart): the latter is generated by the exchange of transverse photons.

Eq. (1.21) is valid only for $|\vec{q}|^2 \gg m_q^2$ (m_q : effective quark mass). If one has $|\vec{q}|^2 \ll m_q^2$, the quark contributions to the vacuum polarization are suppressed by powers of $|\vec{q}|^2/m^2$. Consequently in the renormalization of κ for a particular value of $\vec{q}^2$ only those quark flavors are of importance for which $\vec{q}^2 > m_q^2$. All quark flavors with $m_q^2 > \vec{q}^2$ are "turned off". This implies in particular that the behaviour of $\kappa(\vec{q}^2)$ for large distances (small $\vec{q}^2$) is independent of the quark masses.

If κ is sufficiently small, it is allowed to neglect the higher order terms in eq. (1.23)., and we obtain

$$\kappa(\vec{q}^2) = \frac{\kappa(\mu^2)}{1 + B \frac{\kappa(\mu^2)}{4\pi} \ln\left(\frac{\vec{q}^2}{\mu^2}\right)} . \quad 1.24$$

This equation can also be written as follows, redefining the normalization point:

$$\kappa(\vec{q}^2) = \frac{1}{B \ln\left(\frac{\vec{q}^2}{\Lambda^2}\right)} . \quad 1.25$$

For four flavors one has especially

$$\kappa(\vec{q}^2) = \frac{12\pi}{25 \ln\left(\frac{\vec{q}^2}{\Lambda^2}\right)} . \quad 1.26$$

Various experimental data (e.g. scaling violations in deep inelastic reactions) indicate $\Lambda \approx 500 \text{ MeV}$.¹¹⁾

Of course, relation (1.26) can only be trusted as long as $\kappa/\pi \ll 1$. Nevertheless it is interesting to note that according to lowest order perturbation theory $\kappa(\vec{q}^2)$ becomes singular at $\vec{q}^2 = \Lambda^2$, i.e. at a distance Λ^{-1} , which is about 0.5 fm for $\Lambda = 500 \text{ MeV}$.

6.) Who confines the quarks?

The fact that in QCD the color flux lines between two heavy colored objects are collimated more strongly than in a pure Coulomb potential may be taken as a hint that perhaps in QCD the force between two colored constituents does not fall off sufficiently fast with distance in order to lead to a finite ionization energy. This idea is supported by the formulation of QCD on a lattice.¹²⁾ Here the attraction between the color flux lines becomes so strong that at large distances the field lines tend to be parallel, and the force between two colored constituents becomes constant (see Fig. (8))¹³⁾

Fig. (8): Hypothetical collimation of flux lines in QCD.

One expects in this case that the system breaks up into pieces as soon as there is enough energy available to produce a new $q\bar{q}$ -pair (meson). If this picture of chromodynamics at large distances is really correct, quarks and gluons would be fields without free particles associated to them. Color would be confined, and the spectrum of hadrons would emerge as the spectrum of color singlet states in QCD.

What are the parameters which enter in the QCD description of hadrons? Obviously the quark masses $m_u, m_d, m_s, m_c, \dots$ are free parameters, which must be introduced from outside. They have nothing to do directly with the dynamics of QCD; their role is similar to the one played by the lepton masses in QED. Furthermore, there exists another scale parameter in QCD, which however does not show up directly in the Lagrangian, namely the renormalization mass μ , at which the renormalized quantities of the theory, for example the vertices, coupling constants, etc. are defined.

The appearance of an intrinsic scale μ in QCD is an interesting phenomenon, since a mass scale is introduced purely as a quantum effect.¹⁴⁾ This mass scale is independent of the effective quark masses. If confinement is indeed true in QCD, one is tempted to associate this mass scale with the typical hadronic mass scale (e.g. given by the size of hadrons, slopes of Regge trajectories, meson decay constants, etc.). In particular, it would be natural to associate the critical length Λ with the typical size of a hadron ~ 1 fm. As we have seen above, such a picture is consistent with the measured value of Λ of the order of 500 MeV.

As we shall see below, the effective masses of the light quarks u, d, s are less than $\Lambda \approx 500$ MeV. On the other hand quarks which are confined within a region of extension Λ^{-1} have a typical momentum inside a hadron of the order of Λ . Thus the light quarks will certainly be moving relativistically inside the nucleon. On the other hand we have $m_c \gg \Lambda$, i.e. charm-anticharm bound states are (with sufficient accuracy) non-relativistic bound states.

We interpret this difference between "light" quarks and charmed quarks, indicated by the inequality $m_{u,d,s} \ll \Lambda \ll m_c$, as an important difference. Charm-anticharm states can indeed be described

relatively well by a nonrelativistic potential model, while such a discription can only be a very rough approximation in case of the light quarks.

The potential of QCD should behave like a $1/r$ Coulomb potential for distances small compared to Λ^{-1} , while for distances large compared to Λ^{-1} it should rise to provide for quark confinement. At small distances the potential is given by the lowest order "Coulomb" terms (one gluon exchange). For large distances the potential cannot be described simply by the one-gluon exchange. Almost certainly the rising part of the QCD potential at large distances cannot be obtained perturbatively. The effective potential which one might consider is a potential of the form

$$V(r) = - \frac{\kappa}{r} c_1 + c_2 + c_3 \cdot r. \quad 1.27$$

In order to fit the spectrum of $\bar{c}c$ states, one can use¹⁵⁾

$$V(r) = - \frac{0.27}{r} + 0.25 r - 0.76 \text{ [GeV]} \quad 1.28$$

where r is measured in units of GeV^{-1} . This potential is described in Fig. (9).

Fig.(9): Effective potential in QCD. The solid line describes the potential eq.(1.28), the dashed line the one in eq.(1.29).

The linear part of the potential is abstracted from the string-like picture discussed above. We must note, however, that the linear potential is not at all established as the correct effective potential between two heavy color sources at large distances. Other types of potentials are consistent with the confinement idea as well, for example a harmonic oscillator potential.

More recently a logarithmic potential has been proposed as the - potential between quarks.¹⁶⁾ We do not believe that such a potential is the correct potential, however it must be emphasized that for relatively small distances the funnel-like potential described by eq.(1.27) is indeed rather similar to a logarithmic potential. In order to see this we have displayed in Fig.(9) such a potential:

$$U(r) = 1.14 \ln (r/3.4) \text{ [GeV]}. \quad 1.29$$

It is evident that for r less than 10 the logarithmic and the "funnel" potential are very similar. Of course, at large distances this is not true anymore. For example for $r = 1000$ we have $V(1000) = 250$, but $U(1000) = 6.5$. Thus, if the logarithmic potential would be the ultimate truth, highly excited charmonium levels would be tremendously spread in space, e.g. as big as a hydrogen atom. We find this extremely unrealistic.

7.) Chromomagnetism

Within the QCD picture of hadrons many features observed in the hadron spectrum can be understood, at least qualitatively. Among those are the facts that the ground state $J^{PC} = 1^{--}$ vector mesons ($\rho, \omega, \phi, \psi, \dots$) are heavier than their pseudoscalar partners, that the $J = 3/2$ baryon resonances ($\Delta, \dots$) are heavier than the $J = 1/2$ baryons ($p, n, \dots$), and that $M(\Lambda) < M(\Sigma)$. Those mass differences are "hyperfine splittings" and can be understood as effects due to chromomagnetic forces.

Let us first consider the situation in QED. The interaction energy of two magnetic moments $\vec{\mu}_1$ and $\vec{\mu}_2$ is in a nonrelativistic approximation

$$H^{\text{magn.}} = -\frac{8\pi}{3} \vec{\mu}_1 \cdot \vec{\mu}_2 \delta(\vec{r}) + \frac{1}{r^3} [\vec{\mu}_1 \cdot \vec{\mu}_2 - 3(\vec{r} \cdot \vec{\mu}_1)(\vec{r} \cdot \vec{\mu}_2)] \quad 1.30$$

($\vec{r}$: relative coordinate, $|\mu| = \frac{e}{2m}$ for Dirac fermions).

The first term in eq. (1.30) is the Fermi hyperfine interaction, while the second term (dipole-dipole interaction) is zero for s-wave states. According to eq. (1.30) the magnetic moments (spins) of electrons in a multielectron system tend to be parallel. For example a three electron system in which the three electron spins are aligned in some direction has a lower energy than a system where the spins are not aligned, e.g.

$$E(e^{-\uparrow} e^{-\uparrow} e^{-\uparrow}) < E(e^{-\downarrow} e^{-\downarrow} e^{-\uparrow}). \quad 1.31$$

For an $e^{-}e^{+}$ system the configuration with the lowest energy is the one where the spins are antiparallel:

$$E(e^{-\uparrow} e^{+\downarrow}) < E(e^{-\uparrow} e^{+\uparrow}), \quad 1.32$$

i.e. parapositronium is lighter than orthopositronium.

Let us suppose that the chromomagnetic interaction between quarks is dominated by the one gluon exchange force,¹⁵⁾ i.e. that the strong, confining force is purely electrical in character (for a justification of this assumption within the lattice approach see e.g. ref. (12)). In this case the interaction (1.30) can be applied in order to study the chromomagnetic forces between quarks, if one makes the following substitution:

$$\begin{aligned} \alpha &= e^2/4\pi \rightarrow -4/3 \cdot \kappa & 1.33 \\ &(\text{in case of } \bar{q}q\text{-systems}) \\ \alpha &\rightarrow -2/3 \cdot \kappa \\ &(\text{in case of } qq\bar{q}\bar{q}\text{-systems}). \end{aligned}$$

The factors appearing in eq. (1.33) can be calculated as follows. For $\bar{q}q$ - systems one has to calculate the diagram Fig. (10), which is given by

Fig.(10): Color gluon exchange within a meson.

$$\begin{aligned}
 & \sum_{\alpha=1}^8 \sum_{a,b,d,d} \frac{1}{\sqrt{3}} \delta_{ab} \left(g \cdot \frac{\lambda_{ac}^{\alpha}}{2} \right) \left(-g \cdot \frac{\lambda_{db}^{\alpha}}{2} \right) \cdot \frac{1}{\sqrt{3}} \delta_{cd} \\
 & = - \frac{g^2}{12} \sum_{\alpha} \sum_{b,c} \lambda_{bc}^{\alpha} \lambda_{cb}^{\alpha} = - \frac{g^2}{12} \sum_{\alpha} \text{tr}(\lambda^{\alpha} \lambda^{\alpha}) \\
 & = - \frac{4}{3} g^2.
 \end{aligned}
 \tag{1.34}$$

(Note: the factors $1/\sqrt{3} \delta_{ab}$ appear, since the meson wave function is $1/\sqrt{3} \sum_{a=1}^3 | \bar{q}_a q_a \rangle$. The coupling strength of the quarks is given by $g \cdot \lambda^{\alpha}/2$, the coupling strength of the antiquarks by $-g \cdot \lambda^{\alpha*}/2$).

In case of qqg-systems one finds (see Fig.11)

Fig.(11): Color gluon exchange within a baryon.

$$\begin{aligned}
 & \sum_{\alpha=1}^8 \sum_{\substack{a,b,c \\ d,e,f}} \frac{\epsilon_{abc}}{\sqrt{6}} (g/2 \cdot \lambda_{ad}^{\alpha}) (g/2 \cdot \lambda_{be}^{\alpha}) \frac{\epsilon_{def}}{\sqrt{6}} \delta_{cf} \\
 &= \frac{g^2}{24} \sum_{\alpha} \sum_{\substack{ab \\ de}} \lambda_{ad}^{\alpha} \lambda_{be}^{\alpha} \sum_c \epsilon_{abc} \epsilon_{dec}.
 \end{aligned}
 \tag{1.35}$$

Using the identity

$$\sum_c \epsilon_{abc} \epsilon_{dec} = \delta_{ad} \delta_{be} - \delta_{ac} \delta_{bd}
 \tag{1.36}$$

one finds

$$\begin{aligned}
 & g^2/24 \sum_{\alpha} (\text{tr}(\lambda^{\alpha}) \cdot \text{tr}(\lambda^{\alpha}) - \text{tr}(\lambda^{\alpha} \cdot \lambda^{\alpha})) \\
 &= -2g^2/3.
 \end{aligned}
 \tag{1.37}$$

Equation (1.34) implies:

$$M(\bar{q}q, \uparrow\uparrow) > M(\bar{q}q, \uparrow\downarrow),
 \tag{1.38}$$

i.e. the spins like to be antiparallel. The situation is analogous to the situation in QED (see eq.(1.32)). Thus one finds:

$$M(\rho) > M(\pi), M(K^*) > M(K), M(D^*) > M(D), M(F^*) > M(F) \dots$$

Eq. (1.37) expresses an important fact: In QCD the sign of the magnetic hyperfine interaction in qqq systems is opposite to the sign one has in QED, i.e.

$$M(qqq, \uparrow\uparrow\uparrow) > M(qqq, \uparrow\downarrow\uparrow).
 \tag{1.39}$$

The nonabelian character of the chromomagnetic forces is the reason why the Δ is heavier than the proton.

Following eq. (1.30) we can apply the ideas discussed above to the lowest baryon states. Here only the chromomagnetic Fermi interaction contributes, and we can write for states composed of the same set of quarks

$$M = M_0 + \xi \left\langle \sum_{i>j} \lambda_i \lambda_j \vec{s}_i \cdot \vec{s}_j \right\rangle
 \tag{1.40}$$

where ξ is a positive parameter, depending on the space parts of the wave functions, and λ_i is the chromomagnetic moment of the quark (in the nonrelativistic limit $g/2m_i$). Isospin symmetry requires $\lambda_u = \lambda_d$.

In case of the Σ^{*0} , Σ^0 and Λ states we have

$$M = M_O + \xi < \lambda_u^2 \vec{s}_u \vec{s}_d + \lambda_u \lambda_s (\vec{s}_u \vec{s}_s + \vec{s}_d \vec{s}_s) >. \quad 1.41$$

The wave function for the Σ^0 and Σ^{*0} state contains the (u,d) quark system in an $I = 1$ state. Consequently the spins are aligned, and we have $\vec{s}_u \cdot \vec{s}_d = +\frac{1}{4}$. Since we must have

$$\begin{aligned} \vec{S}^2 &= (\vec{s}_u + \vec{s}_d + \vec{s}_s)^2 = \frac{3}{4} \cdot 3 + 2 \vec{s}_u \vec{s}_d + 2 \vec{s}_s (\vec{s}_u + \vec{s}_d) \\ &= 3/4 (\Sigma^0) \text{ or } 15/4 (\Sigma^{*0}), \end{aligned} \quad 1.42$$

we find $\vec{s}_s (\vec{s}_u + \vec{s}_d) = -1 (\Sigma^0)$ or $(+\frac{1}{2}) (\Sigma^{*0})$, and obtain

$$M_{\Sigma^0} = M_O + \xi [\lambda_u^2 \cdot 1/4 + \lambda_u \lambda_s \cdot (-1)] = M_O + \xi \lambda_u^2 (\frac{1}{4} - \rho) \quad 1.43$$

$$M_{\Sigma^{*0}} = M_O + \xi [\lambda_u^2 \cdot 1/4 + \lambda_u \lambda_s \cdot \frac{1}{2}] = M_O + \xi \lambda_u^2 (\frac{1}{4} + \frac{1}{2}\rho),$$

where we have introduced the ratio $\rho = \frac{\lambda_s}{\lambda_u}$. In the Λ -state the u-d-system has $I = 0$, hence the spins are anti-parallel, and we find

$$\begin{aligned} \vec{s}_s (\vec{s}_u + \vec{s}_d) &= 0 \\ M_\Lambda &= M_O + \lambda_u^2 (-3/4). \end{aligned} \quad 1.44$$

Combining the relations (1.43) and (1.44), we find ¹⁷⁾

$$\frac{\lambda_u}{\lambda_s} = \rho^{-1} = 1 + \frac{3}{2} \frac{M_{\Sigma^0} - M_\Lambda}{(M_{\Sigma^{*0}} - M_\Sigma)}. \quad 1.45$$

The ratio ρ is the ratio of the chromomagnetic moments of the s and u quark. On the other hand we expect the chromomagnetic moments to be proportional to the electromagnetic moments, divided by the quark charge.¹⁸⁾ The electromagnetic moments of the baryon ground states are given by

$$\mu(\text{baryon}) = \sum_{\text{quarks}} \sigma_q \cdot M_q \quad 1.46$$

where M_q is the electromagnetic moment of the quark q .

Applying eq. (1.46) and assuming M_q to be proportional to the quark charge ($M_q = e_q \cdot \mu_q$, $\mu_u = \mu_d$), we find

$$\mu(p) = \mu(u), \quad \mu(n) = -2/3 \mu_u, \quad \mu(\Lambda) = -\frac{1}{3} \mu_s. \quad 1.47$$

The experimental values are

$$\mu(p) = 2.79 \frac{e}{2M_p}, \quad \mu(n) = -1.91 \frac{e}{2M_p} \quad 1.48$$

$$\mu(\Lambda) = (-0.67 \pm 0.06) \frac{e}{2M_p},$$

and we find

$$\frac{\mu_s}{\mu_u} = \frac{(-3) \mu(\Lambda)}{\mu(p)} = 0.72. \quad 1.49$$

The chromomagnetic moments are proportional to $\mu(q)$, hence we have

$$\rho = \frac{\lambda_s}{\lambda_u} = \frac{\mu_s}{\mu_u} = 0.72 \quad 1.50$$

From the experimental values $M_\Lambda = 1116 \text{ MeV}$, $M_\Sigma = 1193 \text{ MeV}$, $M_{\Sigma^*} = 1385 \text{ MeV}$ one finds

$$\rho^{-1} = \frac{\lambda_u}{\lambda_s} = 1.60 = 0.63^{-1}. \quad 1.51$$

The agreement with the result eq. (1.50) is satisfactory. We conclude that the Σ - Λ mass splitting can be understood in sign and magnitude as a consequence of SU_3 breaking in the chromomagnetic force. Especially it is due to the fact that the chromomagnetic moment of the s-quark is less than the chromomagnetic moment of the

u or d quark by about a factor 1.60.

We should like to mention that the slight discrepancy between eq.(1.50) and (1.51) is not surprising. The chromomagnetic moments of the quarks and the corresponding electromagnetic moments μ_q are proportional only if the quark-gluon coupling constant couples universally to u-and s-quarks. Since the effective mass of the s-quark is larger than the effective u-quark mass, we expect the coupling of s-quarks to gluons to be slightly reduced compared to the coupling of u or d-quarks to gluons. This makes the difference between eq. (1.50) and eq.(1.51) at least plausible.

In the case of mesons the analog of eq. (1.40) is

$$M(\bar{q}q) = M_0(\bar{q}q) + \eta \lambda_{\bar{q}} \cdot \lambda_q < \vec{s}_{\bar{q}} \cdot \vec{s}_q > \quad 1.52$$

where η is a positive parameter. Note that $\vec{s}_{\bar{q}} \cdot \vec{s}_q = \frac{1}{2}[(\vec{s}_{\bar{q}} + \vec{s}_q)^2 - (\vec{s}_{\bar{q}})^2 - (\vec{s}_q)^2]$ is 1/4 for s-wave vector states and - 3/4 for s-wave pseudoscalars. From eq. (1.52) we deduce the relation

$$\frac{M_\rho - M_\pi}{M_{K^*} - M_K} = \frac{\lambda_u}{\lambda_s} = \frac{M_{D^*} - M_D}{M_{F^*} - M_F}. \quad 1.53$$

Using the experimental values $M_\rho - M_\pi = 640$ MeV, $M_{K^*} - M_K = 400$ MeV, we find $\lambda_u/\lambda_s = 1.60$, in excellent agreement with eq. (1.51). Eq. (1.53) predicts

$$M_{F^*} - M_F = (M_{D^*} - M_D) \cdot \frac{\lambda_s}{\lambda_u} = (140 \text{ MeV}) \cdot (1.6)^{-1} \approx 90 \text{ MeV}. \quad 1.54$$

Furthermore one finds from eq. (1.52), assuming that the space wave function of noncharmed and charmed mesons are about equal:

$$\frac{M_{D^*} - M_D}{M_\rho - M_\pi} = \frac{\lambda_c}{\lambda_u}. \quad 1.55$$

Using the experimental data $M_{D^*} - M_D = 140$ MeV, one obtains $\lambda_c/\lambda_u = 0.22$.

Applying eq.(1.43) and (1.44) to charmed baryons one finds

$$\frac{M_{\Sigma_C} - M_{\Lambda_C}}{M_{\Sigma_C} - M_{\Lambda_C}} = \frac{\lambda_u^2 \left(\frac{1}{4} - \frac{\lambda_s}{\lambda_u} \right) - \lambda_u^2 (-3/4)}{\lambda_u^2 \left(\frac{1}{4} - \frac{\lambda_c}{\lambda_u} \right) - \lambda_u^2 (-3/4)} = \frac{1 - \frac{\lambda_s}{\lambda_u}}{1 - \frac{\lambda_c}{\lambda_u}} \quad 1.56$$

(note: Σ_C, Λ_C are obtained from Σ, Λ by making the substitution $s \rightarrow c$).

Combining eq. (1.54) and (1.55), the result is

$$M_{\Sigma_C} - M_{\Lambda_C} \approx 2.2 (M_{\Sigma} - M_{\Lambda}) = 170 \text{ MeV}. \quad 1.57$$

Summarizing our discussion about chromomagnetism, one can say that the idea of a chromomagnetic force among quarks analogous to the Fermi interaction in atomic physics leads to a satisfactory picture of all hyperfine splitting effects. The signs of all observed mass differences agree with the expectation from chromomagnetism. Quantitatively the agreement with naive calculations is good. We regard this agreement as a strong indication that the forces among quarks at relatively small distances are mediated by color octet vector quanta (gluons), coupled to quarks by the minimal γ_μ coupling.

II. SYMMETRY PROPERTIES OF QCD

1.) Quark masses and conservation of color

In QCD color is exactly conserved (globally and locally). The QCD gauge group SU_3^C is a so-called non-chiral gauge group ("vector gauge group"), i.e. the left-handed and righthanded parts of the fermion fields transform under SU_3^C as equivalent representations. This implies that the following theorem is true:

The quark mass matrix m , which in general contains γ_5 -terms and need not be hermitean, can be made hermitean and free of γ_5 terms by a unitary transformation of the righthanded or lefthanded quark fields only. Finally it can be diagonalized by a common unitary transformation of q_R and q_L (R: righthanded, L: lefthanded).

Proof:

The mass term in the Lagrangian can be written as

$$M = q_L^* \gamma_0 \mathcal{M} q_R + q_R^* \gamma_0 \mathcal{M}^+ q_L. \quad 2.1$$

($\mathcal{M}^+$: hermitean conjugate of $\mathcal{M}$).

The mass matrix is arbitrary in flavor space, but has to commute with the color generators, since otherwise color would not be conserved. Any matrix can be written as a product of a hermitean and a unitary matrix, and we can write:

$$\mathcal{M} = \mathcal{M}^h \cdot U \quad 2.2$$

($\mathcal{M}^h$: hermitean matrix, U : unitary matrix).

Thus eq. (2.1) takes the form:

$$M = q_L^* \gamma_0 \mathcal{M}^h U q_R + q_R^* \gamma_0 U^{-1} \mathcal{M}^h q_L. \quad 2.3$$

Let us redefine the righthanded quark fields:

$$U q_R \rightarrow q_R',$$

in which case we obtain

$$\begin{aligned} M &= q_L^* \gamma_0 \mathcal{M}^h q_R' + q_R'^* \gamma_0 \mathcal{M}^h q_L \\ &= \bar{q} \mathcal{M}^h q. \end{aligned} \quad 2.4$$

The hermitean matrix $\mathcal{M}^h$ can be transformed into a diagonal matrix by a unitary transformation V acting both on q_L and q_R :

$$V^{-1} \mathcal{M}^h V = \begin{pmatrix} m_u & & \\ & m_d & \\ & & \ddots \end{pmatrix}, \quad 2.5$$

and we find:

$$M = \sum_n m_n \bar{q}_n q_n = m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s + m_c \bar{c}c + \dots$$

(q_n : quark field of flavor n).

In QCD the symmetries of the strong interactions are determined by the symmetries of the quark mass matrix. The latter are determined by all unitary transformations U which leave the quark mass matrix invariant:

$$U^{-1} \gamma_0 \mathcal{M} U = \gamma_0 \mathcal{M} . \quad 2.6$$

It follows that parity, charge conjugation, and CP are left invariant by the strong interactions. This can be seen most easily using eq. (2.6). There each term $m_q \bar{q}q$ is, of course, left invariant by a parity, charge conjugation or CP transformation. Note, however, that this argument is based on formal considerations. It may be that nonperturbative effects which cannot be seen in the formal Lagrangian of QCD spoil eventually the P,C, or CP invariance (see our discussion below).

Using the special form of $\mathcal{M}$ (eq. (2.6)), one observes that for each flavor exists a separate U_1 gauge invariance. The Lagrangian of QCD is invariant under the transformation

$$q_n \rightarrow e^{i\alpha_n} q_n, \quad 2.7$$

i.e. each quark flavor is separately conserved. We can define the flavor charges $U, D, \dots$ by

$$\begin{aligned} \tilde{U} &= \int u^* u d^3x \\ \tilde{D} &= \int d^* d d^3x, \text{ etc. } . \end{aligned} \quad 2.8$$

Conventionally one is using the linear combinations

$$B = 1/3 (\tilde{U} + \tilde{D} + \tilde{S} + \tilde{C} + \dots) \quad 2.9$$

$$Q = 2/3 \tilde{U} - 1/3 \tilde{D} - 1/3 \tilde{S} + 2/3 \tilde{C} + \dots$$

$$T_3 = 1/2 (\tilde{U} - \tilde{D})$$

$$S = - \tilde{S}$$

$$C = \tilde{C}, \dots .$$

The strong interactions are invariant under a wider symmetry if mass degeneracies exist. For example, if among the n quark flavors m happen to be degenerate in mass, the QCD Lagrangian is invariant under the unitary transformation

$$\begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{pmatrix} \rightarrow U \begin{pmatrix} q_1 \\ q_2 \\ \vdots \\ q_m \end{pmatrix} \quad 2.10$$

where U is an arbitrary SU_m matrix.

Isospin and the conventional SU_3 symmetry are, for example, symmetries of this type. We should like to mention the possibility that mass degeneracies exist for heavy quark flavors. For example the T -state ¹⁹⁾ might be a manifestation of two new quark flavors, say t and b . In case $m_t = m_b$ a new type of isospin would exist, generated by the quark doublet $\begin{pmatrix} t \\ b \end{pmatrix}$. Such possibilities should be kept in the mind of the theorists analyzing the spectra of new heavy particles.

2.) How to define quark masses?

In QCD the quark masses play the role of symmetry breaking parameters (breaking of flavor symmetries). In the Lagrangian or in the field equations of QCD the quark masses appear always as multiplied by the scalar or pseudoscalar densities $\bar{q} q$, $\bar{q} \gamma_5 q$. For example the divergence of an axial vector current (color singlet) in QCD is, according to the field equations,

$$\partial^\mu \mathcal{F}_{\mu}^5 = \partial^\mu (\bar{q} \gamma_\mu \gamma_5 q_2) = i(m_1 + m_2) \bar{q} \gamma_5 q \quad 2.11$$

(q_i : mass eigenstates with eigenvalues m_i).

On the other hand, the scalar or pseudoscalar densities $\bar{q} q$ or $\bar{q} \gamma_5 q$ are not invariant under renormalization, only the densities $m \bar{q} q$, $m \bar{q} \gamma_5 q$. As a consequence the quark mass parameters m are renormalization dependent quantities, i.e. they depend on the subtraction point μ .

A definition of the absolute values of the quark masses is as follows. In QCD as well as in the free quark theory there exist relations between the light cone commutator of two (pseudo) scalar densities and the light cone commutator of two vector or axial vector currents,²⁰⁾ for example:

$$\begin{aligned} [\partial^\mu \bar{q}(x) \gamma_\mu \gamma_5 q(x), \partial^\nu \bar{q}(y) \gamma_\nu \gamma_5 q(y)] &= [2im \bar{q}(x) \gamma_5 q(x), 2im q(y) \gamma_5 q(y)] \\ &\quad (x-y)^2 \approx 0 \quad (x-y)^2 \approx 0 \\ \hat{=} -2m^2 g^{\mu\nu} [\bar{q}(x) \gamma_\mu q(x), \bar{q}(y) \gamma_\nu q(y)] &\quad (x-y)^2 \approx 0 \end{aligned} \quad 2.12$$

where the symbol $\hat{=}$ is meant to indicate that in eq.(2.12) terms which are less leading near the light cone by powers of $(x-y)^2$ are neglected. Eq.(2.12) defines the quark masses in absolute scale as a function of $(x-y)^2$, in Fourier space it defines the quark masses as a function of $(-q^2)$.

Another possibility to define the quark masses in QCD is more formal and consists of assuming that the quark propagator $S(\not{p}, m)$, for a given spacelike μ^2 , agrees with free field theory²¹⁾

$$[S^{-1}(\not{p}, m)]_{p^2 = \mu^2} = (\not{p} - m). \quad 2.13$$

This defines m as a function of q^2 just like eq. (2.12). Both definitions are equivalent within perturbation theory.

The renormalization of the quark masses is given by the following renormalization group equation:

$$\frac{dm(\mu)}{d\mu} = \frac{m}{\mu} \cdot \gamma_m(g, \frac{m}{\mu}), \quad 2.14$$

where γ_m is the anomalous dimension of the quark two point function. In lowest order of the QCD perturbation expansion one finds.

$$\gamma_m = - \frac{g^2(\mu)}{2\pi^2} \quad 2.15$$

where we have neglected terms of order m^2/μ^2 . The negative sign in eq. (2.14) implies that the quark masses shrink if the

renormalization mass μ increases.

If we choose μ large enough such that the lowest loop approximation to the renormalization group equation is valid one finds

$$m(q^2) = \frac{m(\mu^2)}{1 + g^2/4\pi^2 \ln(q^2/\mu^2)} . \quad 2.16$$

This result is found by taking into account only the lowest order self energy diagram (see Fig. (12)).

Fig.(12): QCD self energy diagram of a quark

We emphasize that for $|q^2| \gg m_q^2$ the renormalization of m_q depends solely on g^2 . The ratio $m(q^2)/m(\mu^2)$ is flavor independent. For $|q^2| \rightarrow \infty$ the effective quark mass $m(q^2)$ goes to zero. We should like to mention that the same effect is true in QED, where the effective electron mass $m(q^2)$ slides to zero as $|q^2| \rightarrow \infty$.

In Fig. (13) we display the ratio $m(q^2)/m(\mu^2)$ as a function of $\sqrt{q^2}$ for the case $\mu = 10$ GeV, $\Lambda = 0.5$ GeV. The effective quark mass decreases by a factor 2, if $\sqrt{q^2}$ increases from 10 GeV to about 3 000 GeV.

Fig.(13): The shrinking of the quark mass in QCD.

According to eq. (2.14) the ratios of the effective quark masses

$$m(q_i) / m(q_j) \quad 2.17$$

approach finite values for $\mu \rightarrow \infty$:

$$\lim_{\mu \rightarrow \infty} \frac{m(q_i, \mu^2)}{m(q_j, \mu^2)} =: \frac{m_O(q_i)}{m_O(q_j)}. \quad 2.18$$

These ratios are defined to be the ratios of the bare quark masses. We emphasize that the approach of the quark mass ratios to their limiting values proceeds rather quickly. In the region of μ where eq. (2.16) can be applied, the quark mass ratios are already independent of μ .

3.) Quark masses and approximate chiral symmetries of the strong interactions

In modern gauge theories of the flavor interactions the masses of the elementary fermions (leptons, quarks) are generated via

spontaneous symmetry breaking.²²⁾ Specifically the quark masses are proportional to vacuum expectation values of scalar fields. These quark mass parameters are the "bare" quark masses, i.e. the masses of the quarks in the limit where the strong interactions are switched off.²³⁾ According to eq. (2.18) the ratios of the "bare" quark masses are equal to the ratios of the effective quark masses in the limit $\mu \rightarrow \infty$.

In the limit where all quark masses vanish ($m \rightarrow 0$) the strong interaction are invariant under the flavor symmetry group $SU_n^L \times SU_n^R$ (n : number of quark flavors). It is commonly assumed, in accordance with the ideas of current algebra and PCAC, that in QCD this symmetry is not fully realized in the particle spectrum, but realized via the Nambu-Goldstone mechanism. There exist (n^2-1) massless pseudoscalar Goldstone bosons, transforming like the adjoint representation of SU_n . All other particles are massive. In the real world the chiral $SU_n^L \times SU_n^R$ symmetry is broken by the quark mass terms. Correspondingly the pseudoscalar bosons acquire masses.

If some of the quark masses, say $m_1, m_2, \dots, m_k$ ($k < n$) are set to zero, the QCD Lagrangian is invariant under the $SU_k^L \times SU_k^R$ subgroup of $SU_n^L \times SU_n^R$. Correspondingly (k^2-1) of the (n^2-1) pseudoscalar mesons are massless, while the others acquire masses. It is useful to give a measure of the quality of the chiral symmetry. In case $m = 0$ the only scale parameter entering the strong interactions is Λ , which is measured to be about 500 MeV. Therefore, we expect that the chiral symmetry is very good if one has

$$m(\text{quark}, \mu = \Lambda) \ll \Lambda. \quad 2.19$$

On the other hand chiral symmetry is expected to be a bad approximation if

$$m(\text{quark}, \mu = \Lambda) \gg \Lambda. \quad 2.20$$

We shall define light quarks those quarks for which $m(\mu=\Lambda) < \Lambda$, and heavy quarks those for which $m(\mu=\Lambda) > \Lambda$.

In the real world one observes a rather accurate symmetry

$SU_2^L \times SU_2^R$, acting in the (u,d) flavor space, and a less accurate, but still useful symmetry $SU_3^L \times SU_3^R$, acting in the (u,d,s) flavor space. There is no evidence for an approximate $SU_4^L \times SU_4^R$ symmetry. Correspondingly one expects

$$m_u, m_d < m_s < \Lambda < m_c. \quad 2.21$$

Using lowest order perturbation theory in the symmetry breaking parameters m_q , one finds for the (mass)² of any hadron h composed of the light quarks u,d,s:

$$M^2 = M_0^2 + \langle h | \bar{q} m_q q | h \rangle, \quad 2.22$$

where M_0 is the mass of the state h in the limit $m_u=m_d=m_s=0$. In this limit the eight pseudoscalar mesons (pions, kaons, η) acquire zero mass. Thus we have especially for the pseudoscalar mesons:

$$M_{ps}^2 = \langle ps | m_u \bar{u}u + m_d \bar{d}d + m_s \bar{s}s | ps \rangle. \quad 2.23$$

One may expect the matrix elements of $\bar{q}q$ ($q = u, d, s$) to be SU_3 invariant in some approximation. U spin invariance gives especially

$$\langle \pi^+ | \bar{d}d | \pi^+ \rangle \approx \langle K^+ | \bar{s}s | K^+ \rangle. \quad 2.24$$

In the $SU_2^L \times SU_2^R$ limit ($m_u=m_d=0$) the pion mass must be zero, which implies according to eq. (2.23):

$$\langle \pi | \bar{s}s | \pi \rangle |_{m_u=m_d=0} = 0. \quad 2.25$$

Using eq.(2.25) as well as SU_3 symmetry for the matrix elements of $\bar{q}q$ one finds the well-known PCAC relations for the pseudoscalar mesons masses:²³⁾

$$\begin{aligned} M_\pi^2 &\approx (m_u+m_d)M_0 + \gamma_\pi 0 \\ M_{K^0}^2 &\approx (m_d+m_s)M_0 + \gamma_{K^0} \\ M_{K^+}^2 &\approx (m_u+m_s)M_0 + \gamma_{K^+} \\ M_\eta^2 &\approx \frac{4m_s+m_u+m_d}{6} M_0 + \gamma_\eta. \end{aligned} \quad 2.26$$

Here M_O is an SU_3 invariant mass parameter proportional to Λ , and $\gamma_\pi \dots$ denote the finite electromagnetic self-energies to the various mesons masses. It is possible to show using current algebra ²⁴⁾ that in the $SU_3^L \times SU_3^R$ limit the contributions to the electromagnetic self-energies of the pseudoscalar mesons are due to the lowest intermediate states only (only the pseudoscalar meson octet itself contributes) in which case one has:

$$\gamma(\pi^0) \approx \gamma(K^0) \approx \gamma(\eta) \approx 0, \quad 2.27$$

$$\gamma(\pi^+) \approx \gamma(K^+).$$

Using those relations, one obtains

$$\begin{aligned} M^2(\pi^0) &= (m_u + m_d) M_O \\ M^2(K^0) &= (m_d + m_s) M_O \end{aligned} \quad 2.28$$

$$M^2(K^+) - M^2(K^0) - (M^2(\pi^+) - M^2(\pi^0)) = (m_u - m_d) M_O$$

$$\frac{M^2(K^0)}{M^2(\pi^0)} = \frac{m_s + m_d}{m_u + m_d} \approx 14$$

$$\frac{M^2(K^0) - M^2(K^+) + M^2(\pi^+)}{2M^2(\pi^0) + M^2(K^+) - M^2(K^0) - M^2(\pi^+)} = \frac{m_d}{m_u} \approx 1.8.$$

Thus we obtain:

$$m_s/m_d \approx 20 \quad m_s/m_u \approx 36 \quad 2.29$$

$$\frac{m_s}{(\frac{m_u + m_d}{2})} \approx 26 \quad \frac{m_d - m_u}{(\frac{m_d + m_u}{2})} \approx 0.57.$$

The mass ratios denoted in eq. (2.29) are the ratios of the bare quark masses, which are equal to a good approximation to the effective quark masses $m(\mu)$ as long as $\mu \gg \Lambda$.

We emphasize that we have used SU_3 symmetry for the meson matrix elements of the quark bilinears $\bar{q}q$. It is not clear how good this approximation is in the real world. There may exist a relatively large violation of SU_3 symmetry for these matrix elements, in which

case the relations (2.29) would have to be modified.²⁵⁾

The u and d quark masses are much smaller than m_s . One interesting feature of eq. (2.29) is the large isospin breaking exhibited by the quark masses. The mass difference $m_d - m_u$ is of the same order as the quark masses m_d, m_u themselves. Consequently isospin symmetry has nothing to do with a degeneracy $m_u = m_d$, but is rather a consequence of the fact that m_u and m_d are very small: $m_u, m_d \ll \Lambda$.

The u - d mass difference contributes to the mass difference within isospin multiplets besides the electromagnetic self energy. It is, for example, the reason why the neutron is heavier than the proton. It is well-known that this two-component picture of the isospin mass splittings is very successful in predicting relations between mass differences.

Finally, we should like to emphasize that relation (2.29) is incompatible with the case $m_u = 0$, $m_d \neq 0$, since in this case one would have $m_d - m_u / \frac{(m_d + m_u)}{2} = 2$, and not 0,57, as given in eq. (2.29). SU_3 breaking effects such as the ones discussed above will, of course, change this ratio slightly, but it seems unlikely that the symmetry breaking could account for such a large deviation to allow $m_u = 0$.

4.) Absolute quark masses

The relations (2.12) or (2.13) define the absolute values of the effective quark masses. What are those values in the real world? Unfortunately, the definitions (2.12) or (2.13) have no practical use, and we must consider a different method of extracting the absolute values of the quark masses from experimental numbers. One possibility is the following one. It is well-known that the mass of a strange particle depends in a simple way on the number of strange quarks in the corresponding hadrons. The mass difference between a strange and a nonstrange hadron in the same SU_3 multiplet is roughly proportional to $-S$. Furthermore, it depends very little on which SU_3 representation is involved. Typically the mass shift is $|\Delta S| \times 150$ MeV. According to eq. (2.23) the hadron matrix elements of $\bar{s}s$ must be approximately proportional

to the s quark counting operator s^+s and we can define a quark mass $\tilde{m}$ as follows: ²³⁾

$$\Delta M \approx \langle m_s \bar{s}s \rangle \approx \langle \tilde{m}_s s^+s \rangle. \quad 2.30$$

Numerically one has

$$\tilde{m}_s \approx 150 \text{ MeV}. \quad 2.31$$

What is the relation between $m_s(\mu)$ and $\tilde{m}_s$?

This question cannot be answered precisely, without knowing dynamical details about the bound state structure of the particles and the confinement mechanism. However, it seems reasonable to suppose that $\tilde{m}_s$ is equal to the effective s quark mass at $\mu = \text{typical mass of strange hadrons} \approx 1 \text{ GeV}$, i.e.

$$\tilde{m}_s = 150 \text{ MeV} = m_s(\mu \approx 1 \text{ GeV}). \quad 2.32$$

In this case we find using eqs. (2.29): ^{26,23)}

$$m_d(\mu \approx 1 \text{ GeV}) \approx 7.5 \text{ MeV} \quad 2.33$$

$$m_u(\mu \approx 1 \text{ GeV}) \approx 4 \text{ MeV}.$$

We should like to add a comment about the heavy charmed quarks. For charmed hadrons nonrelativistic potential models have been rather successful and give $\tilde{m}_c \approx 1.2 \text{ GeV}$. Since $\tilde{m}_c > \Lambda$ we do not expect chiral $SU_4 \times SU_4$ symmetry to be a good symmetry for charmed particles. Thus relations such as e.g. eq. (2.28) cannot be generalized to charmed hadrons.

The mass parameters discussed above are not to be confused with the mass parameters one uses in naive constituent models of hadrons. If a very light quark, for example a u quark, is bound inside a "bag" of extension $\approx 1 \text{ fm}$, it acquires a dynamical mass, which might be taken as 300 - 400 MeV, as most models suggest. This mass is due to nonperturbative effects involving the strong interaction; no trace of the chiral $SU_2^L \times SU_2^R$ symmetry is left. There is no connection between these masses and the bare quark masses (for an alternative point of view see ref. (21)).

5.) The U(1) - Problem

One of the outstanding problems in the modern QCD approach to hadron physics is the U(1) - Problem (see e.g. ref. (27)). This problem will be discussed later in the lecture of Crewther, thus I shall make only a few remarks about it.

One observes in the hadron spectrum that neutral mesons whose quark composition is flavor diagonal, e.e. $\rho_0, \omega, \phi, \psi, f, f'$, etc., tend to segregate according to quark flavor, e.g. the ϕ -meson is almost purely $\bar{s}s$. The mixing among the flavor neutral mesons which in QCD is provided by the annihilation of the flavor neutral $\bar{q}q$ system into gluons (see e.g. ref. (28)) is observed to be small. The only exception to this rule are the pseudoscalar mesons, which show a strong flavor mixing. The mass eigenstates are

$$\begin{aligned}\pi_0 &= \frac{1}{\sqrt{2}} (\bar{u}u - \bar{d}d) \\ \eta &\approx \frac{1}{\sqrt{6}} (\bar{u}u + \bar{d}d - 2\bar{s}s) \\ \eta' &\approx \frac{1}{\sqrt{3}} (\bar{u}u + \bar{d}d + \bar{s}s).\end{aligned}\tag{2.34}$$

Especially the η' -state violates the PCAC mass formula (see eq. (2.26)), which would give

$$M_{\eta'}^2 \approx \frac{1}{3}(m_u + m_d + m_s)M_0 \approx \frac{m_s}{3} M_0 \approx \frac{1}{3} M_K^2 \approx 290 \text{ MeV}^2.$$

Furthermore in the limit $m_u = m_d = m_s = 0$ one has a complete $U_3^L \times U_3^R$ symmetry. The axial SU_3 singlet current is formally conserved:

$$\partial^\mu (\bar{u}\gamma_\mu \gamma_5 u + \bar{d}\gamma_\mu \gamma_5 d + \bar{s}\gamma_\mu \gamma_5 s) = 0.\tag{2.35}$$

However there exists no pseudoscalar Goldstone boson, corresponding to this current; the lowest state with the quantum numbers of the axial singlet current is the η' -meson (we neglect SU_3 breaking effects) which, however, does not follow the mass formula dictated from PCAC.

In the limit $m_u = m_d = 0$ the QCD Lagrangian exhibits a $U_2^L \times U_2^R$ symmetry, i.e. there should exist a massless Goldstone boson with

the quark composition $\frac{1}{\sqrt{2}} (\bar{u}u + \bar{d}d)$. According to the PCAC mass formula one expects in the real world this state to be essentially degenerate with the π_0 . Such a state is not observed.

Summarizing those observations, one can say that there must exist a special mechanism which causes the Goldstone theorem not to be applicable in the flavor singlet channel. It is now generally agreed among theorists that such a mechanism has been found. It is provided by the two gluon anomaly: In the limit $m_q = 0$ the divergence of the flavor neutral axialvector current $q\gamma_\mu\gamma_5 q$ is not zero, but one has

$$\partial^\mu (\bar{q}\gamma_\mu\gamma_5 q) \sim g^2 \epsilon_{\mu\nu\rho\sigma} \text{tr} G^{\mu\nu} G^{\rho\sigma} \quad 2.36$$

It has been shown recently ²⁹⁾ that in QCD the anomalous divergence described above causes a violation of the Nambu-Goldstone mechanism, due to the existence of special gluon field configurations ("instanton"). These configurations lead to the existence of infinitely many degenerate vacua, distinguished from each other by an angle parameter θ .

Although the presence of instanton configurations in the QCD vacuum state is very useful (it abolishes the necessity of having a massless flavor singlet Goldstone boson) it is still mysterious why the mixing and mass pattern of the pseudoscalar mesons is really so strange as observed in nature. Somehow instanton configurations must be very important for the generation of mass (in particular the mass of the η'). How this mass generation really proceeds remains a mystery. ³⁰⁾

III. PERTURBATIVE ASPECTS OF QCD

In this part of the lectures I am going to discuss some of the perturbative aspects of QCD, in particular those which have attracted recently the attention of experimentalists and theorists alike. By the term "perturbative aspects" it is implied that one is dealing with quarks and gluons. But quarks and gluons are supposed to be confined by forces, which cannot be obtained in

perturbation theory. Thus the question arises: What sense does it make to use perturbation theory at all? Until recently the standard answer to this question was: Perturbation theory can be used for all phenomena, where short or nearly lightlike distances are involved. This limits the possibilities essentially to phenomena involving electromagnetic or weak currents, e.g. e^+e^- -annihilation, deep inelastic scattering.

More recently the points of view of many theorists towards the applicability of perturbation theory seems to have changed considerably. It has been suggested especially by Feynman³¹⁾ that perturbation theory involving quarks and gluons should be applied to many phenomena provided one makes an important substitution: Whenever one uses quarks or gluons in a perturbative calculation, one has to replace them in the real world by a quark jet or gluon jet of real hadrons. In order to do specific calculations within QCD, a specific suggestion has been made recently by Weinberg and Sterman:³²⁾ Define all observables (cross sections, structure functions,...) such that they are free of infrared singularities in QCD. It is hoped, although not really shown explicitly, that by using this principle one avoids the unsolved problems of confinement. We shall use the principle of infrared finiteness subsequently for the study of several processes. A more detailed discussion of the questions of jets in QCD can be found in the parallel lectures of Llewellyn Smith.

1.) Scaling violations in deep inelastic scattering.

A discussion of perturbative aspects in QCD would be incomplete without mentioning those problems which can be studied exclusively by short distance or light cone operator product expansions, for example the violation of scaling in deep inelastic scattering. The simplest way to look at those is to use perturbation theory for the quark-lepton scattering amplitudes. In lowest order of g only the Born terms contribute, e.g. one finds the scaling behaviour, expected in the free quark theory (exact scaling). To order g one obtains radiative QCD corrections to the free quark amplitudes, which involve logarithms of q^2 (q : current four momentum in deep inelastic scattering) which destroy the naive scaling law. The quarks obtain a fine structure. In Fig. (14) we exhibit diagrams which contribute

to the fine structure of quarks.

Fig. (14): QCD fine structure of quarks.

Furthermore through higher orders the photon can couple indirectly to gluons; the scaling violations in deep inelastic scattering are sensitive to the gluon content of the nucleon. In lowest order the corresponding diagram is given in Fig.(15).

Fig.(15): Production of new flavors by γ -gluon fusion.

We emphasize that the process described in Fig. (15) involves the production of a quark-antiquark pair. Since gluons couple universally to all flavors, the production of new flavors (s-quark, c-quarks,...) in deep inelastic scattering depends only on the effective quark mass. For $-q^2 \rightarrow \infty$, $x = \frac{-q^2}{2M\nu}$ (ν : energy transfer) fixed, where mass effects disappear, all production cross sections become equal.

We add the following remarks about deep inelastic scattering:

- a) In Fig. (14a,b) the quark is struck by the virtual photon after it has emitted a gluon, i.e. it is less energetic.

This implies that the scaling functions "shrink" to smaller values of x . This effect is observed in the experiments (see e.g. ref.(11)).

- b) The transverse momentum of the gluon relative to the incoming (Fig. (14 a,b) or outgoing (Fig.14 c) quark increases in proportion to $\sqrt{-q^2}$. Thus the scaling violations in deep inelastic scattering are intimately related to the appearance of large transverse momenta (transverse with respect to the direction of the hadron or photon momentum).

2.) Jets in Chromodynamics: Angular Jets

In e^+e^- -annihilation into hadrons one observes jet-like configurations of the outgoing hadrons. It is generally believed by theorists that the appearance of jets in e^+e^- -annihilation is the manifestation of a $\bar{q}q$ -pair produced by the virtual photon which disintegrates into the final hadronic system. Such an interpretation is especially convincing since the distribution of the jets in the angle θ (angle between jet axis and e^+e^- beam) is consistent with a form $(1 + \cos^2\theta)$ which is expected for the production of a pair of spin 1/2 pointlike quarks.

The appearance of the jet phenomenon in e^+e^- -annihilation at high energies was a specific prediction of the parton approach. It is typical that jets could not be predicted within the short distance or light cone approach³³⁾ to deep inelastic scattering or e^+e^- -annihilation. As a consequence it was important to look for jets in e^+e^- -annihilation since they might tell us something about quarks which cannot be derived strictly in QCD by looking at operator products at small distances. After the discovery of jets at SPEAR in 1975³⁴⁾ it was a challenge for theorists to find a framework or an idea in order to show that jets are expected within the QCD approach.

Recently it has been argued forcefully by Weinberg and Sterman³²⁾ that this can be done in QCD provided one defines jets and the production cross sections for jets appropriately. This method is described below.

In QCD the total cross sections for e^+e^- -annihilation into hadrons or leptonproduction are (apart from logarithms) scale invariant, i.e. depend only on dimensionless ratios like the Bjorken scaling parameter x :

$$\sigma \sim \frac{1}{s} F(x, \dots). \quad 3.1$$

However, the cross section for producing a particular state depends in perturbation theory on mass parameters m_i , e.g. quark masses:

$$\sigma \sim \frac{1}{s} F(x, \dots; m_i/\sqrt{s}), \quad 3.2$$

and in general is singular in the limit $m_i/\sqrt{s} \rightarrow 0$. Such singularities are avoided if one sums suitably over intermediate states, for example by considering the total cross section.

The origin of the singularities in $m/\sqrt{s}$ is the fact that it is impossible in the limit $m \rightarrow 0$ to tell the difference between a single quark state or a quark accompanied by one or several gluons moving in the same direction. The situation is very similar to the corresponding situation in electrodynamics. Let us consider the scattering of an electron off a charged source (e.g. a nucleus). In lowest order of e this process is described by the diagram Fig.(16), the

Fig.(16): Electron scattering off a heavy charge.

cross section is given by the Mott formula, indicated by $d\sigma_0$. Now we consider the emission of a photon γ with energy k off the electrons (see diagram Fig. (17)).

Fig. (17): Inelastic electron scattering off a heavy charge.

Summing over the polarizations of the photons, one finds in the limit $k \ll p_0, p_0'$ (p_0, p_0' : energies of the incoming or outgoing photons):

$$d\sigma = \alpha \left[\frac{\vec{v}' \cdot \vec{n}}{1 - \vec{v}' \cdot \vec{n}} - \frac{\vec{v} \cdot \vec{n}}{1 - \vec{v} \cdot \vec{n}} \right] \frac{dk d\Omega(\vec{n})}{4\pi^2 k} d\sigma_0 \quad 3.3$$

($\vec{v}$ ($\vec{v}'$) velocity of incoming (outgoing) electron, $\vec{n}$: direction of emitted photon).

In the relativistic case $v, v' \rightarrow 1$ the emission of the photons proceeds mainly in the direction of $\vec{v}$ or $\vec{v}'$. Suppose we consider a photon emitted in the direction of the outgoing electron. In this case we find using eq. (3.3)

$$d\sigma \approx \frac{\alpha}{(1 - \cos \theta)^2} \frac{dk}{k} d(\cos \theta) \sim \frac{\alpha}{k \sin \theta} dk d\theta d\sigma_0, \quad 3.4$$

where θ is the angle between the outgoing electron and the photon.

In eq.(3.4) the well-known infrared singularity of electrodynamics appears. The probability for emitting a soft photon diverges as $k \rightarrow 0$. The usual way of dealing with this infrared singularity is to note that very soft photons accompanying an electron cannot be measured with a counter which has a finite energy resolution. Such a counter will not be able to distinguish the elastically scattered electrons from those electrons which are scattered inelastically by emitting a soft photon with $k < \Delta E$ (ΔE : energy resolution). On the other hand the contributions of order e^2 to the elastic scattering

(see Fig.(18))

Fig.(18): Radiative corrections to the elastic electron scattering off a heavy charge.

are divergent in the infrared limit. It is well-known that this divergence just cancels the divergence encountered in eq.(3.4) (see e.g. ref. (35)). The probability to find an electron plus an arbitrary number of soft photons scattered at a certain angle is finite. Such an electron accompanied by soft photons can be regarded as an electron jet characterized by its quantum number (electric charge, energy).

The same principle is now applied to QCD, replacing suitably electrons by quarks and photons by gluons. Since we do not observe quarks and gluons, but hadrons, we have to be clear from the beginning about the definition of a jet. One definition of a jet of hadrons (the thus far conventional definition) is a collection of hadrons with bounded $p_{\perp}$ (see Fig. (19)).

Fig.(19): Particle jet with bounded $p_{\perp}$.

The transverse momenta of the hadrons are bounded, $\langle p_{\perp} \rangle$ is independent of the total energy of the jet.

A second definition would be to define a jet as a collection of hadrons emitted within a certain (small) solid angle δ (see Fig.(2o)).

Fig. (2o): Particle jet in a fixed angular region.

While the first definition forces $\langle p_{\perp} \rangle$ to be energy independent, the second definition allows $\langle p_{\perp} \rangle$ to diverge as the energy approaches infinity. Which of the two definitions given above is the more sensible one in QCD? The answer to this question is not difficult, if one uses the principle put forward by Sterman and Weinberg:³²⁾

Jets must be defined such that the cross sections for producing jets are finite in QCD perturbation theory.

According to eq.(3.4) the cross section for producing a jet consisting of a quark and a gluon behaves like $(\kappa/k \sin\theta) dk d\theta d\sigma_0$. If we apply the first definition given above, one finds: for $k \cdot \sin\theta < p_{\perp}$ only a single jet is observed, while for $k \sin\theta > p_{\perp}$ two jets are observed. The probability to have two jets is

$$P(\text{two jets}) \sim \kappa \iint_{k \sin\theta > p_{\perp}} \frac{dk d\theta}{k \sin\theta} \xrightarrow{E \rightarrow \infty} \ln(E/p_{\perp}). \quad 3.5$$

Thus the distinction between a two jet or a one jet configuration cannot be made independent of E ; the probability to have two jets diverges logarithmically as $E \rightarrow \infty$.

If we apply the second definition, the two jet configurations appear if

$$\begin{aligned} \theta &> \delta \\ k &> \epsilon \cdot E \quad (\epsilon < 1, \text{ finite}). \end{aligned}$$

The probability to have two jets is

$$P(\text{two jets}) \approx \kappa \ln \epsilon \ln \delta, \quad 3.6$$

which remains finite for $E \rightarrow \infty$. It follows that the second definition ("angular jets") is the one which is most suitable for QCD. In our subsequent discussion we shall use the angular definition of jets exclusively.

3.) Angular jets in e^+e^- -annihilation

In order to apply the ideas discussed above to e^+e^- -annihilation into hadrons, we consider the cross section $\sigma(E, \theta, \epsilon, \delta)$ (E : total e^+e^- -energy, $\epsilon \ll 1, \delta \ll 1$) which is defined as the cross section for all annihilation events, in which $(1-\epsilon)$ of the total energy E is emitted within a pair of oppositely directed cones of opening angle 2δ . (see Fig. (21)).

Fig. (21): Two δ -cones at an angle θ with respect to the $e^+ e^-$ beam line.

To order $g^2(E)$ the cross section is

$$\frac{1}{\pi\delta^2} \sigma(E, \theta, \epsilon, \delta) = \frac{3\alpha^2}{4E^2} (1+\cos^2\theta) \sum_{\substack{\text{quark} \\ \text{flavors}}} e_i^2 \left[1 - \frac{4\kappa}{3\pi} (3\ln\delta + 4\ln\delta \ln 2\epsilon + \frac{\pi^2}{3} - \frac{5}{2}) \right]. \quad 3.7$$

For $\kappa = 0$ (free quark case) we can take $\delta \rightarrow 0$, in which case eq.(3.7) turns into the free quark production formula:

$$\frac{1}{\pi\delta^2} \sigma(E, \theta, \epsilon, \delta) \xrightarrow[\epsilon=1]{\kappa=0} \left(\frac{d\sigma}{d\Omega} \right) = \frac{3\alpha^2}{4E^2} (1+\cos^2\theta) \sum_{\substack{\text{quark} \\ \text{flavors}}} e_i^2.$$

The total cross section for e^+e^- -annihilation is

$$\sigma = \left(1 + \frac{\kappa}{\pi}\right) \sum_{\substack{\text{quark} \\ \text{flavors}}} e_i^2 \cdot \sigma(e^+e^- \rightarrow \mu^+\mu^-). \quad 3.8$$

Comparing eq.(3.7) and (3.8), one can calculate the fraction $F(\delta, \epsilon)$ of all events which are such that the energy $(1-\epsilon)E$ is emitted in a pair of opposite cones of half-angle δ :

$$F(\delta, \epsilon) = \frac{\left(1 - \frac{4\kappa}{3\pi} (3\ln\delta + 4\ln\delta \ln 2\epsilon + \frac{\pi^2}{3} - \frac{5}{2})\right)}{1 + \frac{\kappa}{\pi}} \quad 3.9$$

$$\approx 1 - \frac{4\kappa}{3\pi} (3\ln\delta + 4\ln\delta \ln 2\epsilon + \frac{\pi^2}{3} - \frac{7}{4}).$$

Example: Require 70% of all events such that at least 80% of E is emitted in two opposite cones with half-angle $\delta(E)$. Using $F=0.7$ and $\epsilon \approx 0.2$, we display the solution in Fig.(22).

Fig. (22): At $E = 5$ GeV one finds $\delta(5) \approx 22^\circ$; at $E = 30$ GeV we obtain $\delta(30) \approx 2^\circ$ (we use eq. (1.26) for $\kappa(E)$ with $\Lambda = 0.5$ GeV).

It is a consequence of these considerations that the angular jets expected within QCD are less narrow than the jets one might expect for a theory incorporating a fixed transverse momentum cut-off. Nevertheless, for energies above 10 GeV the jet structure of the e^+e^- -annihilation events becomes rather distinct.

Since the cross section eq.(3.7) is multiplied by $(1 + \cos^2\theta)$, the angular distribution of the jets is proportional to $(1 + \cos^2\theta)$. Note that this is valid only for $E \gg m_q$. Just above the threshold for producing a new flavor the angular distribution is expected to be proportional to $1 + (1 - \left(\frac{2m_q}{E}\right)^2) \cos^2\theta$.

Thus far we have concentrated on two jet structures. However, also three particle jets can be obtained in e^+e^- -annihilation, namely if a hard gluon is radiated away from the quarks at a sizable angle. (see e.g. ref. (36)). The final state consists of two jets originating from the emitted quarks ("quark jet") and a jet

originating from the gluon radiated away ("gluon jet"). It is expected that a relatively large fraction ($\sim 10\%$) of all e^+e^- -annihilation events are of this nature.³⁶⁾ We emphasize that the three jet axes are coplanar if the annihilations proceed at rest.

Electron-positron annihilation is one of the few places where gluon jets could be observed. Another possibility is to observe them in γ -nucleon scattering ("QCD Compton process").³⁷⁾

4.) Heavy Quarkonium Decay - a Source of Gluon Jets.

Finally I shall discuss shortly the decay of heavy quarkonium states (e.g. $T(9.4)$) into gluon jets, as emphasized recently by several authors.^{38,39,40,41)}

Within the QCD approach to hadrodynamics the hadronic decay of heavy quark-antiquark $C_n = -1$ vectormesons (e.g. $\psi(J), T$) proceeds via the annihilation of the $Q\bar{Q}$ -system (Q : heavy quark) into gluons which in turn proceed to materialize into ordinary hadrons. The simplest hypothesis is to suppose that the total annihilation rate is given by the annihilation of the $Q\bar{Q}$ -system into three massless color octet gluons, which turn with probability one into the final hadronic system.⁴²⁾ It is well-known that the annihilation rate calculated this way in lowest order QCD perturbation theory according to the QCD analog of the Ore-Powell formula for ortho-positronium decay agrees with experiment ($\psi(J)$ decay) if $\kappa = g_{st}^2/4\pi$ (g_{st} : QCD quark-gluon coupling constant) is chosen to be about 0.2.

If the three gluon hypothesis is correct, one might suppose that not only the total annihilation rate but more detailed properties of the final hadronic state (jets, energy distributions, angular distributions) can be abstracted from the lowest order QCD formula. If this conjecture is correct, the study of heavy quarkonium decay in e^+e^- -annihilation experiments will provide us in the near future with rather detailed tests of QCD and its relevance for hadrodynamics.

Let us first recall that the total hadronic annihilation rate is given by

$$\frac{\Gamma^{\text{hadr.}}(\bar{Q}Q \rightarrow \text{hadrons})}{\Gamma(\bar{Q}Q \rightarrow e^+e^-)} = \frac{5}{18} \cdot \frac{\pi^{2-9}}{\pi} \frac{\kappa^3}{\alpha^2} \cdot \frac{4}{9e_Q^2} \quad 3.10$$

($\bar{Q}Q$: 1^{--} state).

We suppose that besides the total hadronic annihilation rate also detailed properties of the final hadronic state can be abstracted from the lowest order QCD formula. In particular we conjecture that in case of very heavy $\bar{Q}Q$ vectormesons the final hadronic state consists of three jets, which originate from the initial production of three "massless" gluons (see Fig. 23)).

Fig. (23): The decay of a heavy vectormeson into three gluons leading to a three jet structure of the final state.

Momentum conservation requires that the three particle jets are coplanar in the restsystm of the $\bar{Q}Q$ -state.

We denote the energies of the various "gluon jets" by E_i ($i=1,2,3$)

and introduce the scaling variables $\xi_i = 2E_i/M_{\bar{Q}Q}$. The jet configuration (energies, angles) is determined uniquely if two of the energies E_i are given. Energy and momentum conservation require $E_i < M_V/2$ (or $\xi_i < 1$). The energy distributions are identical to the ones obtained in ortho-positronium decay (see e.g. ref. (43)). One finds for the hadronic decay of a $(\bar{Q}Q) 1^{--}$ groundstate:

$$\frac{d^2 r^h / d\xi_1 d\xi_2}{r^h} = \left[\left(\frac{1-\xi_3}{\xi_1 \xi_2} \right) + \left(\frac{1-\xi_2}{\xi_1 \xi_3} \right) + \left(\frac{1-\xi_1}{\xi_2 \xi_3} \right) \right] \cdot \frac{1}{(\pi^2 - 9)}. \quad 3.11$$

Integration over ξ_2 leads to the one jet energy distribution function

$$\frac{dr^h/d\xi}{r^h} = \frac{2}{(\pi^2 - 9)} F(\xi) \quad 3.12$$

where $F(\xi)$ is given by

$$F(\xi) = \frac{\xi(1-\xi)}{(2-\xi)^2} + \frac{2-\xi}{\xi} + 2 \left(\frac{1-\xi}{\xi^2} - \frac{(1-\xi)^2}{(2-\xi)^3} \right) \ln(1-\xi). \quad 3.13$$

The function $F(\xi)$, which is well-known from the analysis of positronium decay, increases monotonically from 0 at $\xi=0$ to 1 at $\xi=1$ (see Fig. (24)).

Fig.(24): The energy distribution of the gluon jets in heavy quarkonium decay.

The energy distribution of gluon jets can in principle be measured directly, for example by a calorimeter experiment. We emphasize that the most probable case is $\xi = 1$, i.e. the case where a gluon jet carries the energie $M_V/2$.

The average energy of a gluon jet is given by

$$E_{av} = 1/2 M_V \cdot \xi_{av} = 1/2 M_V \cdot \int F(\xi) \cdot \xi d\xi / \int F(\xi) d\xi = \frac{1}{3} M_V. \quad 3.14$$

We can describe the three gluon decay within a Dalitz plot which is symmetric with respect to the variables ξ_1, ξ_2, ξ_3 ($\xi_1 + \xi_2 + \xi_3 = 2$). The variables ξ_i vary within the triangle, shown in Fig. (25),

Fig.(25): Dalitz plot for the three gluon decay. The $|\text{decay amplitude}|^2$ is shown. The distance from each point inside the triangle to the three sides of the triangle are a measure of the scaling parameters ξ_1, ξ_2, ξ_3 . The normalization is such that one has $\xi_1 + \xi_2 + \xi_3 = 2$.

where also the ratio

$$\frac{d^2 \Gamma^h / dS}{\Gamma^h} = |\text{decay amplitude}|^2$$

(dS : area element in triangle) is described. Note that the amplitude reaches its maximum at the three sides of the triangle, where one of the jet energies reaches its maximum $M_{QQ}^-/2$. The absolute value of the amplitude is uniform ($=\sqrt{2}$) for these configurations.

It is useful to consider the angles between the three jets, which we define as displayed in Fig. (26). The configuration is described uniquely by the two angles θ, ϕ ; one finds:

$$\begin{aligned} \xi_1 &= \frac{2 \sin(\theta - \phi)}{A} \\ \xi_2 &= \frac{2 \sin \phi}{A} \\ \xi_3 &= \frac{-2 \sin \phi}{A} \end{aligned} \quad A = \sin \phi - \sin \theta + \sin(\theta - \phi).$$

Fig.(26): Definition of the angles θ and ϕ .

Note the condition $0 < \theta < \pi$, $\pi < \phi < \pi + \theta$.

Probably it will be difficult in case of T (9.5) to determine the various jet axes with good accuracy in the e^+e^- -annihilation experiments. Therefore we divide all events into two classes:

Events of type A:

We require that each jet axis can be arranged to ly within the shaded area of Fig.(27), which describes all configurations, where the various jet axes deviate from the symmetric configuration $\xi_1 = 2/3$, $\theta = 120^\circ$, $\phi = 240^\circ$ by less than 20° ("Mercedes event"). These configurations obey the constraint that all angles between two neighbouring jets are more than 80° and less than 160° .

Events of type B: All other configurations.

Fig.(27): Type A events.

In Fig. (28) we have indicated the areas of A and B type events. Events of type A are such that perhaps the genuine three jet structure could be observed in $T(9.5)$ decay, while the events of type B will resemble a two jet event. We have calculated the probability to obtain a type A event; 38 % of all events are expected to be of type A. Correspondingly 62 % of all events are of type B, where it will be difficult to observe the three jet structure.

Fig. (28): The distribution of type A and type B events in the "three gluon" Dalitz plot (shaded area: type A).

Since the $\bar{Q}Q$ vectormeson is produced in the $J_3 = \pm 1$ state relative to the beam line, the angular distribution of the decay plane is given by

$$\frac{d\Gamma^h}{d(\cos\theta^*)} = \frac{3}{16} (2 + \sin^2\theta^*) \quad 3.15$$

where θ^* is the angle between the normal of the plane and the beam line. Note that this angular distribution is independent of the momentum partition of the gluon jets.

Very interesting is the decay channel $\bar{Q}Q \rightarrow \gamma + x$ (direct photons in $\bar{Q}Q$ decay),^{40,41)} If we substitute a gluon by a photon (Fig.(29)) we obtain from eq.(3.10)⁴⁴⁾

$$\frac{\Gamma(\bar{Q}Q \rightarrow \gamma + x)}{\Gamma(\bar{Q}Q \rightarrow \text{hadrons})} = \frac{36}{5} \cdot e_Q^2 \cdot \frac{\alpha}{\kappa}. \quad 3.16$$

Fig. (29): The decay of quarkonium into a direct photon (γ) and two gluon jets.

The inclusive γ -rate is surprisingly large for $e_Q = 2/3$ (12% for $\kappa = 0.2$).

The energy spectrum of the direct γ rays is given by the function F (see Fig.(24)). In the decay of $T(9.4)$ one expects to see γ rays with an energy up to 4.7 GeV. The average energy of the γ rays is $M_{\bar{Q}Q}/3$.

In the limit $\xi_\gamma = 2E_\gamma/M_{\bar{Q}Q} \rightarrow 1$ the final state consists of a γ ray with energy $M_{\bar{Q}Q}/2$ and two gluon jets emitted in the opposite direction. In this case the angular distribution is identical to the angular distribution of quark jets in e^+e^- -annihilation:⁴¹⁾

$$\frac{d\Gamma}{d(\cos\theta_\gamma)} = \frac{3}{8} (1 + \cos^2\theta_\gamma), \quad 3.17$$

where θ_γ is the angle between the emitted photon and the beam line.

We should like to mention the hadronic decay of ψ^{-+} , ψ^{++} and 2^{++} quarkonium states. In case of the T spectroscopy such states will be produced in e^+e^- -annihilation by the radiative transitions $T \rightarrow (\psi^{-+}) + \gamma$ or $T'(") \rightarrow (\psi^{++}, 2^{++}) + \gamma$. Their hadronic decays will proceed via the disintegration into a two gluon jet configuration.⁴⁵⁾ Provided the radiative transitions can be identified, e.g. by the detection of the emitted photon, it seems possible to study the two-jet structures in these decays in e^+e^- -annihilation experiments. It might be easier to see the two-jet structures in these decays than the three jet structures in case of 1^{--} -decay, since in the two-gluon decay both jets will carry an energy of nearly 5 GeV.

The study of heavy quarkonium decay will provide us with interesting information about gluons and about the structure of gluon jets. Unlike quark jets gluon jets will be flavorless. The leading particles are neutral in the mean, and it is expected that the fragmentation of a gluon into hadrons is rather soft. Since gluons are color octets one may expect that in processes involving gluons the multiplicity is about twice as high as in a process involving quarks (color triplets).⁴¹⁾

Finally I would like to mention an interesting test of the QCD picture for quarkonium decay. If this decay proceeds via gluons one expects that part of the time one of the gluons is far off shell and disintegrates into a pair of heavy quarks (e.g. $\bar{c}c$ in case of T -decay). Of course, this process is suppressed by an additional factor of the order of κ/π , but it is expected that 5 - 10 % of all hadronic decays of the T -state contain a pair of charmed particles or a $\bar{c}c$ -state. A rough calculation gives⁴⁶⁾

$$\frac{\Gamma(T \rightarrow \psi(J) + x)}{\Gamma(T \rightarrow \text{all})} \approx \text{few } \%.$$

Since $\psi(J)$ production is a rather prominent signal, it will be interesting to see if the decay $T \rightarrow \psi(J) + x$ is present.

OUTLOOK

In these lectures I have tried to describe the present theoretical picture of hadron physics in terms of QCD. It is clear that this picture is not yet a complete theory, as for example QED is a complete theory of all electromagnetic phenomena. Many problems have yet to be solved, in particular the conjectured color confinement property of QCD remains an unsolved issue. Nevertheless, I think the progress one has made during the last ten years in understanding the strong interactions is enormous. For the first time in the development of particle physics we confront a basic quantum field theory of hadrons directly with experiment. Thus far no place has been found where the theory has failed, although there are still many areas of strong interaction physics where nonperturbative aspects of QCD enter and where it is essentially impossible at the moment to make firm predictions.

Finally I would like to emphasize the importance of the jet phenomena for our understanding of the strong interactions. Quarks and gluons are confined entities in the chromodynamic theory, nevertheless quark jets and gluon jets are believed to be real and observable. If quark jets and gluon jets are really observed in the high energy experiments yet to be done, at PETRA or PEP, quarks and gluons would be a lot less mysterious to the layman as they appear today. Seeing a clean narrow quark jet in e^+e^- annihilation at say 20 GeV is almost as important as seeing a quark itself. Quarks and gluons, although permanently confined, become essentially as real as electrons and photons. I made up a little poem expressing the same idea:

Q C D

God made the quark,
God made the gluon,
But added the remark:
There shouldn't be the quark,
There shouldn't be the gluon
Be seen by any fool.
God made the quark,
God made the gluon,
But to see them, I bet,
You must belong to the jet set.

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