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INFRA-RED AND MASS REGULARIZATION IN AF FIELD THEORIES. I: ϕ_6^3

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ABSTRACT

In the framework of asymptotic free (AF) field theories we determine the correction terms in the renormalization group (RG) approach to deep inelastic (DI) scattering and to the Drell-Yan (DY) process. This leads us to an order g^2 analysis of the DI/DY parton cross-sections. Some of their contributions reveal ultra-violet (UV), infra-red (IR) and mass (M) divergences which, after regularization, are removed by renormalization, IR and (partially) M cancellation. The initial-state M divergences, persisting in both processes, are removed by M factorization. Consistent IR and M regularization is constrained by the double-cut-rule: its meaning and implications are explained. The calculations are performed in ϕ_6^3 theory which is AF, IR finite and technically simple. We use "on-shell", "off-shell" and "massless" (n dimensional) mass assignments and demonstrate explicitly the regularization independence of the correction term.

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1. INTRODUCTION

The understanding of strong interaction dynamics by an asymptotic free (AF) gauge field theory (QCD) is at the centre of present-day particle theory¹⁾. The scaling parton model is considered as the 0-th order approximation, and the perturbatively treated strong interaction corrections were so far limited to a leading-log approximation. The next-to-leading terms in deep-inelastic (DI) scattering following from the higher-order corrections to the anomalous dimensions γ_n and the β -function turned out to be relatively small²⁾. The order $\bar{g}^2$ correction of the Drell-Yan (DY) "coefficient function" leads to contributions³⁾ which for $Q^2 \lesssim 100 \text{ GeV}^2$ substantially surmount the "leading-log" approximation⁴⁾. This statement assumes scale-dependent momentum distributions $vW_2 \equiv u(x, q^2)$ without $\bar{g}^2$ corrections; thus logarithmic q^2 -dependence but no constant terms. The specification of the correction term requires evaluation of the DI and DY parton cross-sections. Whilst determining the contributions of all lowest-order Feynman graphs, one comes across ultraviolet (UV), mass (M), and infrared (IR) divergences⁵⁾. They are studied in their most simple form in ϕ^3 theories, where ϕ_g^3 is asymptotic-free and IR-finite⁶⁾.

The UV divergences are removed by dimensional regularization⁷⁾ and subsequent renormalization. All M divergences originate from angular integration. They are technically prevented by the introduction of (on/off-shell) parton masses or by dimensional regularization. In the latter technique all parton masses vanish and the M divergences reappear as poles in the complex n -plane. It is particularly attractive for higher-order calculations since all phase-space integrations can be carried out in closed form. The introduction of parton masses is not arbitrary. The main restriction follows from Kinoshita's *double-cut-rule*^{8a,9)}, which resulted from analysis of the analyticity properties of Feynman graphs with respect to their external and internal masses. All transition probabilities (product of two Feynman graphs) of any of the massive photon processes can be generated from a vacuum bubble with a closed quark-line and gluon interconnections. Cutting a set of lines generates the initial state. Cutting another set of lines in each of

these diagrams leads to the final state. To proceed correctly, the mass assignment must be specified in the vacuum bubble and is then consistently carried over to the double-cut-graphs. Unitarity ascertains that all final-state mass-singularities drop out after summing over all contributions. The KLN theorem⁸⁾ imposes that the initial M singularities persist. The IR divergences develop if the gluon momentum vanishes. From the KLN theorem we derive that they cancel among the set of gluon-corrected initial (or final) states. As a result the complete parton cross-sections, to a given order in g^2 , are IR-finite. The IR regularization either leaves all quarks on-shell, introducing an infinitesimal gluon mass λ , or chooses the quark masses unequal without any gluon mass; dimensional regularization as a third alternative is possible too. We conclude: the parton cross-sections have initial-state M divergences and depend on the regularization procedures. All this arbitrariness is removed by M factorization¹⁰⁾. It expresses the measured quantities as a function of the scale-dependent momentum distribution $W_2 = u(x, q^2)$ (chosen to consist of logs only) plus a correction term which is independent of any regularization.

With this paper we give an insight into the technical side of the calculations, using the most simple example ϕ_6^3 . By giving detailed results we emphasize the influence of the regularization procedure. All calculations are done for on-shell, off-shell, and for n -dimensional regularization, permitting us to demonstrate the regularization independence of the coefficient function after M factorization has been performed.

The paper is organized as follows: Section 2 is reserved for a discussion of the UV, M , and IR singularities. We expose our analysing methods, the procedure to regularize these singularities, and we explain how they are removed correctly. We are primarily concerned with the consistent mass assignments to prevent the M and IR infinities. In Section 3 we introduce the order g^2 Feynman graphs of DI scattering, and determine the corresponding parton structure functions using on-shell and off-shell mass assignment. The same scheme is followed in Section 4 where the DY parton cross-sections are worked out. The cross-sections

of both processes are recalculated in Section 5 using n -dimensional regularization with all parton masses vanishing. In Section 6 we comment^{9,11)} on an earlier proposed, incorrect off-shell mass assignment. Mass factorization is the main topic of Section 7; we explicitly demonstrate the regularization independence of the coefficient function for DI scattering and for the DY process. Section 8 summarizes our findings.

2. UV-, M-, AND IR-DIVERGENCES

This section discusses the origin of the UV, M, and IR singularities in the Feynman diagrams and cross-sections, and explains how they are removed.

For reasons of simplicity and clearness of presentation, our calculations are carried out in a simplified toy model ϕ^3 in six dimensions (ϕ_6^3) -- a renormalizable and asymptotic-free field theory. Its Lagrangian reads

$$\mathcal{L} = \frac{1}{2} Z_3 (\partial_\mu \phi_R)^2 + \frac{1}{2} Z_2 (\partial_\mu \psi_R^*)(\partial^\mu \psi_R) - Z_1 g_R \phi_R \psi_R^* \psi_R - i e_R Z_4 \psi_R^* \vec{\partial}_\mu \psi_R A_{\text{ext}}^\mu \quad (2.1)$$

where ϕ_R is the field of a neutral scalar particle (gluon) and ψ_R stands for a charged scalar particle (quark); A_{ext}^μ is an external vector source parametrizing the complete influence of the electromagnetic interactions (in six dimensions); the Z_i are renormalization constants which will be determined by n -dimensional regularization. For the strong coupling constant we set $\alpha_s = g_R^2/64\pi^2$.

Later we will specify the wave function renormalization constant by subtracting at the propagator mass. Z_2 is therefore determined by $1 - Z_2^{-1} = \partial\Sigma/\partial p^2|_{p^2=m^2}$. We do *not* use minimal subtraction. If all particles are on-shell the Ward identity imposes $Z_2 = Z_4$. In the "off-shell" case, where all charged particles have different masses, we impose $Z_4 = \frac{1}{2}[Z_2(\mu_1^2) + Z_2(\mu_2^2)]$; μ_i^2 generically stands for the off-shell masses of the initial- and final-state lines (in Figs. 1 and 2). It is well known that off-shell masses lead to a violation of the above Ward identity constraint. It has been stated in the literature^{11b)}, and it will again be exemplified in this paper, that the fully evaluated correction terms of DI scattering and of the DY process are *not* gauge-dependent even if the parton cross-sections

reveal a gauge dependence. The above insight can be obtained by making a comparison with regularization methods which do preserve gauge independence of the cross-sections. Despite this short-coming, the off-shell method may therefore still be considered as a valuable tool for determining the correction terms.

Feynman diagrams and cross-sections are known for their M and IR singularities. We give a simple method for their analysis, discuss their cancellations, and introduce the techniques for their M and IR regularization. First the analysis. Following the work of Kinoshita, Lee and Nauenberg^{8a,b)} we notice that an M and/or IR singularity always develops if one or several external lines become degenerate in mass and energy with a group of internal lines. The *M singularities* occur if all masses of the lines (internal and external) entering a vertex vanish. The divergences then appear if the momenta of the internal particles are collinear with the external ones. An *IR-singularity* develops if one of the internal particles attached to a vertex is massless and its momentum approaches the zero value. It is necessary that the remaining external and internal lines have equal mass, which may or may not vanish. The above conditions are necessary but not sufficient.

In order to gain more insight, we have analysed the lowest-order Feynman diagrams which are connected with the individual cross-section contributions. Time-ordered perturbation theory¹²⁾ was found to be a fruitful instrument. Consider a specific Feynman graph such as that shown in Fig. 3. All negative energy denominators may be dropped since they never vanish and therefore cannot generate a singularity in the integrand. All positive energy denominators with a dependence on the photon energy q_0 cannot vanish either, because of the large value of $q_0 \rightarrow \infty$. If all masses are ignored, the remaining factors are of the form $[k(1 - \cos \theta)]^{-1}$, thus indicating the IR and M singular terms. We have analysed all graphs by this method and summarize our results:

- i) Owing to the higher-dimensional phase space, ϕ_6^3 has no IR singularities and less M divergences than QCD.
- ii) In ϕ_6^3 the M divergences occur only if there are at least two equal energy denominators which contain the angular integration variable. For this reason the interference graphs can never be M-divergent.

- iii) In virtual Compton scattering, all IR and M singularities originate from the initial lines. They arise from the degeneracy in energy of the internal quark and gluon lines with the incoming quark if we consider a quark-initiated process (Fig. 3). The same, of course, is true for the gluon-initiated virtual Compton process, where the incoming gluon becomes degenerate with the internal quarks (Fig. 4).
- iv) DI and DY processes with a gluon in the initial state have no M and IR singularities originating from the final-state lines.
- v) A diagram is only IR-divergent if the internal gluon starts and ends on an internal line (provided there is only one gluon line; see Ref. 8c).
- vi) A diagram with a gluon in the initial state is never IR-divergent. Since there are enough quark momenta p_i in the numerator of the integrand, the integrals are regular for $p_i \rightarrow 0$. This phenomenon is due to helicity conservation of the quark at the strong vertex.

Some of the IR and M singularities drop out due to cancellations between different terms. The KLN theorem⁸⁾ states: M singularities are absent in the sum of radiatively corrected initial and final states containing both incoming *and* outgoing hard collinear gluons. The IR singularities instead drop out among the sub-set of virtual plus soft-radiative corrected in- (or out-) states. For the DI cross-section this implies that all M/IR singularities which originate from the final-state lines drop out in the complete sum. This cancellation follows from the unitarity relation between the virtual Compton amplitude and its cut-graphs in Fig. 3. The IR singularities which originate from the initial-state lines are also cancelled⁸⁾. The same insight applies for the DY cross-section, leading to the conclusion that the complete DI and DY cross-sections are IR-finite; the M singularities of the final state cancel, but those of the initial state persist.

In order to carry out the above cancellations formally, the divergences must first be regularized. This cannot be done at will but is constrained by consistency requirements. They emerge from the double-cut-rule which opens a more

general possibility for the analysis of the M and IR singular Feynman diagrams, including their cancellations. We present Kinoshita's arguments^{8a)} by using the DI and DY graphs under consideration as specific examples. Let us consider, in a given order of perturbation theory, the transition from a given initial state $|i\rangle$ to all possible final states $|f\rangle$. The order g^2 graphs of DI scattering (Fig. 1), as an example, lead to $|i\rangle = |\gamma^* q\rangle$ and $|f\rangle = |q\rangle$ or $|q, g\rangle$. The transition amplitudes associated with the Feynman diagrams are denoted as $T_{if}^{(k)}$. The total transition probability in a given order of g^2 is proportional to

$$P^d = \sum_f \int d\psi_f \left[\sum_k T_{if}^{(k)+} \right] \left[\sum_l T_{if}^{(l)} \right] \quad (2.2)$$

The superscripts (k) , (l) stand for the specific Feynman graphs with i and f as initial and final states; $T_{if}^{(k)+}$ corresponds to a diagram obtained from $T_{if}^{(k)}$ by interchanging past and future. Whilst dropping all summations in Eq. (2.2) we calculate a specific contribution; it is a product of two Feynman amplitudes with phase-space integration being carried out over all f -lines. This is indicated by a dashed vertical line (cut-line B) which crosses all f -lines. We thus have connected the f -state of diagram (k) with the corresponding f -state of diagram (l) . We call T^d the Feynman amplitude when cut-line B is removed. We now consider all diagrams contributing to Eq. (2.2) and select those which reduce to T^d if cut-line B is removed. Several sets can be formed, each being characterized by the index β . Examples are given in Fig. 3. The diagrams of virtual Compton scattering are denoted by T_α^d , and the cut-diagram A_α^d is composed of the corresponding set of DI graphs. One finds the following properties

- i) P^d is the sum over A_β^d and their elements;
 - ii) for a selected β , $\text{Im } T^d$ is the sum of all elements of A_β^d (unitarity);
 - iii) the set of diagrams A_β^d is free of M and IR singularities from the f -state.
- All such singularities come from the i -lines, and arise either from the self-energy and vertex parts attached to the external lines of A_β^d or from a gluon line which is emitted by the initial line and reabsorbed by the corresponding final external line of A_β^d .

Each A^d is characterized by a set of i -lines in its initial and its final state. In order to study the resulting M and IR singularities and their cancellations, we join the initial and final lines carrying equal momenta (dPS_i integration is, however, not carried out!). This step is indicated by a second dashed line (cut-line A) which crosses all i -lines. If in all graphs the cut-lines A and B are removed, all external lines are converted to internal lines; we arrive at a vacuum bubble which is described by a vacuum-to-vacuum transition amplitude T^V . Different types of vacuum bubbles can be formed, each being characterized by the index α . The set of diagrams with joint i -lines (and phase-space integrated f -lines), each reducing to the same T_α^V if both cuts are dropped, is called a *double-cut-diagram* Δ_α^V . Examples are given in Fig. 5. A particular vacuum bubble is denoted as T_α^V (e.g. Fig. 5a). Its corresponding double-cut-graph Δ_α^V is composed of the Feynman diagrams shown in Figs. 5a₁-a₄. Sub-sets of Δ_α^V can be considered as (single) cut-diagrams which are denoted by $A_{\alpha\beta}^d$. In the cited example, A_{11}^d is composed of Figs. 5a₁, a₃, and A_{12}^d of Fig. 5a₂, a₄. In shorthand notation this reads

$$T_i^V = \{f; g, 5a\} \rightarrow \Delta_i^V = \{A_{11}^d, A_{12}^d\} = \{(f; g, 5a_1, a_3), (f; g, 5a_2, a_4)\} \quad (2.3)$$

One realizes that the total transition probability Eq. (2.2) is the sum of the elements of several Δ_α . We conclude: the well-known relation between the cut-diagram A^d and its uncut-graph T^d (Cutkosky rule) can be formulated in analogous manner for the double-cut-diagram Δ^V and its corresponding vacuum-to-vacuum amplitude T^V (double-cut-rule).

Reconsidering Eq. (2.2) we find that P^d contains no M and IR singularities of the final state. They cancel among each other in a given set A^d . The IR singularities originating from the initial state cancel since appropriate sums of elements out of Δ_α could be shown to be IR-finite. We are left with M singularities of the initial state. There are two basic options for removing them:

- a) dPS_f and dPS_i integration is carried out;
- b) M factorization is applied.

The first possibility emerges from the KLN theorem^{8a,b)} which states that all M and IR singularities cancel if momentum integration over all particles belonging to the initial *and* final state is formed; summation over the elements in Δ_α is of course understood. Decay processes such as $\pi^- \rightarrow e^- \bar{\nu}_\gamma$ ^{8a)} and e^+e^- annihilation with massive particles in the initial state may be considered as special cases. Mass factorization makes use of the fact that the DY graphs, as for instance $q\bar{q} \rightarrow \gamma^* g$ (Fig. 7), can be related via the double-cut-rule to exactly the same vacuum bubble as the corresponding DI graphs such as $q\gamma^* \rightarrow qg$ (Fig. 5). We denote the set of double-cut-graphs for the DI (DY) process by Δ'_α (Δ_α). From both groups of diagrams Δ_α and Δ'_α , only the M singularities of the initial state persist in the respective cross-sections. In practical calculations the M singularity manifests itself in QCD and ϕ_6^3 by a logarithmic mass dependence, with the function multiplicative to this logarithm being identified as the M residue. We take advantage of the fact that the residue of the M singularity in Δ_α is exactly the same as in its counterpart Δ'_α . This universality of M singularities permits us, in the M factorization program, to cancel these initial-state M singularities of Δ'_α by those of Δ_α .

We comment on the initial gluon graphs of the DI and DY processes (Figs. 6,8). Considering the DI graphs as shown in Fig. 6, we notice the cut-diagram A^d and form the corresponding (uncut) virtual Compton graph T^d and the vacuum bubble. The latter is seen to be the same as for the final-gluon process. If the complete set of double-cut-diagrams is formed, some alien virtual gluon contributions have to be added; they are needed to cancel the initial-state M and IR singularities if dPS_i (besides of dPS_f) integration is carried out as indicated in point (a) above. For a motivation of such alien graphs, which do not belong to the initial-gluon process, see Ref. 8b. Completely analogous observations can be made for the DY process. Rejecting (a), the initial M singularities are removed via M factorization. Notice that all DI graphs in Fig. 6 are obtained by rotating the DY graphs in Fig. 7 on each side of the cut-line B by 180° in the plane of the paper. The same holds true between Figs. 5 and 8.

We now consider the methods of M and IR regularization. Cut- and uncut-diagrams are clearly distinct; the former ones furnish M and IR singularities

but the latter are non-singular. In order to determine the cancellation pattern explicitly, the divergent contributions need to be regularized in the intermediate stages of the calculation. Regulator masses may not be introduced at will but are subject to constraints, since each M and IR singularity must be regularized consistently. Take, for example, Figs. 3d₂, d₃, d₄ for A^d in DI scattering. We would proceed inconsistently to choose equal (quark) mass values in the intermediate states (indicated by the cut-line B) with all other masses vanishing. It can easily be checked that the result of such a calculation would not agree with the imaginary part of the corresponding virtual Compton amplitude; unitarity is violated. To proceed correctly, the regulator masses have to be introduced in the M-finite virtual Compton graph, with this mass assignment subsequently being carried over to all resulting cut-graphs. The analogy between A^d ↔ T^d (Cutkosky rule) and $\Delta_{\alpha}^V \leftrightarrow T_{\alpha}^V$ (double-cut-rule) guides us to proceed similarly for all double-cut-diagrams. The regulator masses are introduced in T_α^V, with the consistent mass assignment for all graphs of Δ_α and Δ_α' being derived by applying the double-cut-rule. This method guarantees that the M singularity which appears in the DI process as well as in the DY one is regularized in the same way. This step is conveniently carried out by giving numbers to all internal lines of the vacuum bubble (indicated by 1-14 in Figs. 5-8). These numbers represent the masses of those lines; they are carried over to all double-cut-graphs of the DI and the DY process, thus establishing a one-to-one correspondence.

Choosing masses in the vacuum bubble in order to regularize the M and IR singularities of the double-cut-graphs is in principle unconstrained, although three principle methods have, in the recent past, enjoyed particular attention: on-mass-shell, n-dimensional, and off-shell regularization. We expose their essential assumptions:

- a) On-shell: All quarks are given the same mass m which prevents all M singularities. The remaining IR singularities can be regularized in three ways:
 - i) The gluons are given a small regulator mass $\lambda \ll m$.
 - ii) The phase space is generalized to n dimensions with the IR singularity appearing as a pole in the

complex n -plane. From our earlier discussion, the DI and DY cross-sections are known to cancel all IR singularities. We therefore expect all λ - and n -poles to disappear in the two aforementioned regularization schemes. iii) We set $m = 0$, $\lambda \neq 0$. In this case λ plays the role of an M as well as an IR regulator; it will remain after all IR cancellations have been carried out in the cross-sections. This is distinct from (i) above, where λ is exclusively an IR regulator.

b) n -dimensional: All parton masses vanish and only the dynamical variables such as $\hat{s}$, q^2 , ... remain. The phase-space integrations are generalized to n -dimensions with the M and IR singularities reappearing as poles in the complex n -plane at $n = 4$ (6) for QCD (ϕ_6^3). Even double-poles occur, which drop out in the complete cross-sections owing to the IR cancellations.

c) Off-shell: The M singularities are prevented by giving the quarks masses. If at the strong vertices the masses are chosen to be slightly different, the IR divergences disappear. Instead of m^2 we therefore put the auxiliary mass $\sqrt{m^2 + \mu^2}$ ($\mu^2 < 0$ or $\mu^2 > 0$) in some appropriate lines of the vacuum bubble. The gluon mass vanishes. Considering the set of double-cut-graphs, one finds for a selected external line the off-shell mass $p^2 = m^2 + \mu^2$ in some of the diagrams, and $p^2 = m^2$ in others. If $|\mu^2| \ll m^2 \neq 0$, μ serves as the IR regulator and m as the M regulator; for $m^2 = 0$, μ assumes the role of both [compare with (iii) above].

The off-shell assignment always involves anomalous cuts since the masses of the quarks entering a gluon vertex always differ. Assuming $p_2^2 = m^2 + \mu^2$, $p_3^2 = m^2$, $p_5^2 = 0$, we refer to Figs. 7a as an example. If $\mu^2 > 0$ there is an anomalous cut in Fig. 7a₁ (since $p_2^2 > p_3^2 + p_5^2$). If, instead, $\mu^2 < 0$, it is shifted to Fig. 7a₂ (since $p_3^2 > p_2^2 + p_5^2$).

Since the existence of these anomalous cuts depends on the sign of μ^2 , we conclude that they disappear if μ^2 is cancelled. For $m^2 \neq 0$ they are indeed absent in the sum of contributions to the DI and the DY cross-section; here μ^2 regulates only the IR divergences. For $m^2 = 0$, however, both cross-sections are complex owing to the anomalous cuts. Mass factorization absorbs this complex number in the momentum distribution, leaving a real correction term.

In our subsequent calculations we use a specific off-shell mass assignment which we define now. For the diagrams with a gluon in the final state (Figs. 5,7) we use

$$\text{off-shell: } m_1^2 = m_3^2 = m^2 + \mu_1^2, \quad m_6^2 = q^2, \quad m_1^2 = m^2 \quad \text{and} \quad \mu_j^2 < 0.$$

$$m_2^2 = m_{12}^2 = m^2 + \mu_2^2.$$

Most of the calculations are given for $m^2 = 0$, and the case $m^2 \neq 0$ is treated only briefly. Other choices are equally possible⁹⁾. We need to distinguish the quark and antiquark lines in the vacuum bubble. Lines 1 and 4 in Fig. 5a are identified with a quark, and lines 2 and 3 with an antiquark. The above "symmetric" mass assignment has the advantage that the quark and antiquark structure functions are treated equally and lead to the same result. For the diagrams with a gluon in the initial state (Figs. 6,8) we use massless quarks and set $m_5^2 = \lambda^2$ for the gluon.

We briefly mention another issue. It concerns the gauge constraint. Gluon gauge invariance is no longer obeyed by our general mass assignment, which was introduced to pin-point clearly the origin of the M singularities and to recognize the difference between on-shell and off-shell calculation in a simple case. In all our calculations it is, however, understood that the on-shell limit is formed at the end of all manipulations in order to restore gluon gauge invariance in the DI/DY correction terms.

3. DI SCATTERING

In this section we aim at determination of the DI structure functions in the framework of the parton model. After introducing the Feynman diagrams and some initial definitions, the parton structure functions are specified. We follow the mass assignments (defined in Section 2) and take proper care of the appearing mass divergences. We follow the scheme: naive parton model, virtual gluon corrections, real gluon corrections leading to the quark structure function, and finally we specify the gluon structure function. We then discuss our results and point to some of their particularities.

3.1 Diagrams and definitions

The order g^2 Feynman graphs for DI scattering are shown in Fig. 1. We distinguish the scaling parton model (SPM), the virtual gluon corrections (V) composed of the vertex and self-energy diagrams (Figs. 1c-e; V_x, S_1, S_2), the production of real gluons (Figs. 1a,b; R), and the initial gluon graphs (Figs. 1f,g; g).

Deep-inelastic electron scattering is conventionally described by the structure functions $W_1, \nu W_2$ which in the parton model read

$$F_1 = M W_1 = \int_x^1 dy u_0(y) [\hat{F}_1^{(0)} + \hat{F}_1^{(2)} + \dots] \quad (z = \frac{x}{y}) \quad (3.1)$$

$$F_2 = \nu W_2 = \int_x^1 dy u_0(y) [\hat{F}_2^{(0)} + \hat{F}_2^{(2)} + \dots] \quad (z = \frac{x}{y}) \quad (3.2)$$

All structure functions (and kinematical variables) on the parton level are marked by the "hat" sign. We have absorbed the quark charges e_q^2 (in units of the electromagnetic charge) in the momentum distributions with $u_0 = e_q^2 u_q(y)$ (quark target) or $u_0 = \sum_q e_q^2 u_q(y)$ (gluon target). For purposes which will become clear later on (see Section 7), from now on we use the structure function notation

$$\mathcal{F}_i = (F_1, \frac{1}{x} F_2, F_3) \quad \text{with} \quad \mathcal{F}_i(x, q^2) = \sum_\ell \int_x^1 \frac{dz}{z} u_\ell(\frac{x}{z}) \hat{\mathcal{F}}_{i,\ell}(z, q^2) \quad (3.3)$$

The parton structure functions receive contributions from each order of perturbation theory $\hat{\mathcal{F}} = \hat{\mathcal{F}}^{(0)} + \hat{\mathcal{F}}^{(2)} + \dots$. The second-order contribution

$$\hat{\mathcal{F}}_2^{(2)} = \hat{\mathcal{F}}_{2,q} + \hat{\mathcal{F}}_{2,g} \quad (3.4)$$

is composed of a quark and gluon part. The former is given by

$$\hat{\mathcal{F}}_{2,q} = \hat{\mathcal{F}}_2^V + \hat{\mathcal{F}}_2^R \equiv \{ \hat{\mathcal{F}}_2^{V_x} + \hat{\mathcal{F}}_2^{S_1} + \hat{\mathcal{F}}_2^{S_2} \} + \{ \hat{\mathcal{F}}_2^{R\text{-hard}} + \hat{\mathcal{F}}_2^{R\text{-soft}} \} \quad (3.5)$$

(and similarly for the other structure functions). They are functions of the squared centre-of-mass energy $\hat{s}$, the photon momentum q^2 , and the parton masses. We proceed to the scaling limit by introducing the parton and nucleon scaling variables

$$z \equiv -\frac{q^2}{2pq} = \frac{x}{y} \quad x \equiv \frac{-q^2}{2p \cdot q} \quad (3.6)$$

P is the nucleon momentum of mass M, and p (or p_1 or k) stands for the momentum of the incoming parton. The external quark masses are denoted by m (or m_1, m_2), and $m_x(\lambda)$ was used to indicate the influence of the quark (gluon) propagator mass.

3.2 Parton structure functions

The structure functions in the 0-th order of perturbation theory (SPM in Fig. 1) are

$$\hat{F}_2^{(0)} = \delta(1-z) \quad \hat{F}_1^{(0)} = 0 \quad (3.7)$$

The virtual gluon contributions demand consideration of the vertex correction (Fig. 1c) and the initial-(final)-state self-energy corrections (Figs. 1d,e). These quantities are UV-divergent; renormalization is therefore carried out by dimensional regularization. The vertex correction is independent of the chosen mass assignment (see Appendix A) and reads

$$F_2^V(q^2) = \frac{g^2}{6(4\pi)^3} \left[\text{Ln} \frac{M_{\text{reg}}^2}{-q^2} - \zeta + \frac{11}{3} \right] \quad (3.8)$$

with ζ defined by

$$\zeta = \frac{2}{n-6} + \gamma - \text{Ln} 4\pi \quad (3.9)$$

M_{reg}^2 is the dimensional regulator mass, and n the dimension of the generalized space. Note the $n-6$ pole which appears via a Γ -function and reflects the UV divergence. The self-energy diagrams are dependent on the particular mass assignment. Formation of their derivative at the external mass point leaves

$$\Sigma_j^1(m^2) = -\frac{g^2}{6(4\pi)^3} \left[\text{Ln} \frac{M_{\text{reg}}^2}{m^2} - \zeta + \frac{5}{3} \right] \quad (\text{on-shell}), \quad (3.10)$$

$$\Sigma_j^1(-\mu_j^2) = -\frac{g^2}{6(4\pi)^3} \left[\text{Ln} \frac{M_{\text{reg}}^2}{-\mu_j^2} - \zeta + \frac{5}{3} - \pi i \right] \quad (\text{off-shell}), \quad (3.11)$$

where $j = 1, 2$ indicates the initial- and the final-state contributions, respectively. The off-shell result, obtained with $\mu_j^2 < 0$, is complex. The $-\pi i$ term

originates from an anomalous cut since $m_{\text{ext}}^2 > \sum m_{\text{int}}^2$ (see Appendix A for details). The contribution of the virtual gluon diagrams to the structure functions is

$$\hat{\mathcal{F}}_2^V = \delta(1-z) (2F^V + \Sigma'_1 + \Sigma'_2) \equiv \delta(1-z) 2F^V(q^2) \quad \hat{\mathcal{F}}_1^V = 0 \quad (3.12)$$

with the renormalized vertex correction

$$F^V(q^2) = \frac{g^2}{6(4\pi)^3} \left[\ln \frac{m^2}{-q^2} + 2 \right] \quad (\text{on-shell}) \quad (3.13)$$

$$F^V(q^2) = \frac{g^2}{6(4\pi)^3} \left[\frac{1}{2} \ln \frac{\mu_1^2}{q^2} + \frac{1}{2} \ln \frac{\mu_2^2}{q^2} + \frac{17}{6} + \pi i \right] \quad (\text{off-shell}) \quad (3.14)$$

and the log arguments changed appropriately. Any dependence on $n-6$ and M_{reg}^2 disappears since all UV divergence is cancelled. The non-log term of the renormalized vertex changes for different mass assignments since renormalization has been carried out at different mass points of the initial- and/or final-state momenta.

Calculation of the structure functions resulting from the Feynman diagrams which describe real gluon production (Figs. 1a,b) is straightforward. For general mass assignment (see Appendix C) one finds

$$\hat{\mathcal{F}}_1^R = \frac{g^2}{6(4\pi)^3} (1-z)^2 \quad (3.15)$$

$$\hat{\mathcal{F}}_2^R = \frac{g^2}{(4\pi)^3} \left[z(1-z) \ln \frac{q^2}{z^2 M_1^2(z)} + \frac{11}{3} (1-z)^2 - 3(1-z) - \frac{1}{2} + \frac{1}{6(1-z+\dots)} \right] \quad (3.16)$$

where

$$M_1^2(z) = \frac{z(1-z)m_1^2 - z\lambda^2 - (1-z)m_2^2}{z(1-z)} \quad (3.17)$$

In expression (3.16) we have for pedagogical reasons proceeded to the large- q^2 limit, keeping z fixed in order to expose explicitly the $(1-z)$ mass-pole -- which, however, is spurious since m_1^2/q^2 corrections still exist. We therefore clearly distinguish $\hat{\mathcal{F}}_2^R$ from $\hat{\mathcal{F}}_2^{\text{R-hard}} \equiv \text{Bj-lim } \hat{\mathcal{F}}_2^R$, where all mass corrections are absent. We will come back to this definition below. For different mass assignments, only the log argument is changed whereas the non-log terms remain unmodified. So far

we have ignored the mass dependence coming from the seeming pole at $z = 1$, which now receives our attention.

Whilst performing $y(\equiv x/z)$ integration in Eq. (3.3), one notices that for $y \approx x$, $\hat{s} = m_1^2 + q^2[1 - (y/x)]$ is close to its threshold leading to $(1/\hat{s})$ divergence if all masses are ignored. This trouble is traced back to the $\hat{s}$ -pole diagram (Fig. 3d₂), where careful analysis reveals that the seeming divergence is prevented by masses. This fact was not taken into account in the above results. The y -integration and subsequent Bj-limit were improperly interchanged, which leads to the appearance of the $z = 1$ divergence.

In calculating the nucleon structure functions, the integration over the momentum fraction of the incoming partons is carried out as

$$\mathcal{F}_2^R = \text{Bj-lim} \int_{z_{\min}}^{z_{\max}} \frac{dz}{z} U_0\left(\frac{x}{z}\right) \hat{\mathcal{F}}_2^R(\hat{s}, q^2) \quad (3.18)$$

For presentation purposes, we have chosen z as the integration variable instead of y as in Eqs. (3.1) and (3.2); $\hat{\mathcal{F}}_2(\hat{s}, q^2)$ stands for the exact parton structure function with all masses finite; and $z_{\min} = x$, $z_{\max} \approx 1$ are the lower and upper integration limits which also can involve masses. In the region $z \sim 1$ we are close to the threshold value where the (seeming) divergence of the integrand is prevented by mass terms; z -integration and formation of the scaling limit cannot be interchanged. The introduction of a cutoff ϵ permits separation of the z -integration in a "hard"- and "soft"-gluon part

$$\mathcal{F}_2^R = \text{Bj-lim} \left\{ \int_x^{1-\epsilon} \frac{dz}{z} [\dots] + \int_{1-\epsilon}^{z_{\max}} \frac{dz}{z} [\dots] \right\} \equiv \mathcal{F}_2^{R\text{-hard}} + \mathcal{F}_2^{R\text{-soft}} \quad (3.19)$$

In the former, z -integration and the scaling limit may safely be interchanged since the $z = 1$ divergence is excluded by the cutoff ϵ ; thus

$$\hat{\mathcal{F}}_2^{R\text{-hard}}(z, q^2) \equiv \text{Bj-lim} \hat{\mathcal{F}}_2^R(\hat{s}, q^2) \quad (3.20)$$

is the parton structure function which we have calculated in Eq. (3.16). The z -integration of the soft part needs careful evaluation and is converted to $\hat{s}$ -integration by using

$$\begin{aligned} z \rightarrow \hat{s} &= m_1^2 + q^2(1 - \frac{1}{z}) & z_{\max} \rightarrow \hat{s} = \hat{s}_0 \text{ threshold} \\ \frac{dz}{z} \rightarrow \frac{d\hat{s}}{-q^2} & \quad (\hat{s} < -q^2) & 1 - \epsilon \rightarrow \hat{s} = -\epsilon q^2 \quad (\epsilon \ll 1) \end{aligned} \quad (3.21)$$

In the soft integral, one can safely factorize the parton distribution out of the integrand

$$\hat{F}_2^{R\text{-soft}} = u(x) B_j \text{-lim} \int_{\hat{s}_0}^{-\epsilon q^2} \frac{d\hat{s}}{-q^2} \hat{F}_2^R(\hat{s}, q^2) \quad (3.22)$$

We now have the possibility to define a new parton structure function valid for the soft region:

$$\hat{F}_2^{R\text{-soft}}(q^2, \epsilon) \equiv \delta(\hat{s} - m_2^2) \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s} \hat{F}_2^R(\hat{s}, q^2) = \delta(1-z) \int_{\hat{s}_0}^{-\epsilon q^2} \frac{d\hat{s}}{-q^2} \hat{F}_2^R(\hat{s}, q^2) \quad (3.23)$$

which permits us to draw conclusions already on the non-integrated level, and furthermore allows us to drop all quark mass dependence in the hard part. The soft contributions to the parton structure functions for on- and off-shell mass assignment are given by

$$\hat{F}_2^{R\text{-soft}} = \delta(1-z) \frac{q^2}{6(4\pi)^3} \left[\text{Ln} \frac{-\epsilon q^2}{m^2} - 1 \right] \quad (\text{on-shell}) \quad (3.24)$$

$$\hat{F}_2^{R\text{-soft}} = \delta(1-z) \frac{q^2}{6(4\pi)^3} \left[\text{Ln} \frac{\epsilon q^2}{\mu^2} - \frac{11}{6} - \pi i \right] \quad (\text{off-shell}) \quad (3.25)$$

The threshold value is $\hat{s}_0 = m^2 (= \mu_2^2)$ for on-shell (off-shell) calculations.

Notice that the complex term $-\pi i$ in $\hat{F}_2^{R\text{-soft}}$ is the reflection of an anomalous cut which is related to the one found in Σ'_2 of Eq. (3.11).

With the above formulas we have completely determined the quark part of the structure functions and can now specify its final form. For *on-shell* mass assignment we find

$$\begin{aligned}\hat{\mathcal{F}}_{2,q} &= \hat{\mathcal{F}}_2^{(0)} + \{ \hat{\mathcal{F}}_2^{R\text{-hard}} + \hat{\mathcal{F}}_2^{R\text{-soft}} + \hat{\mathcal{F}}_2^V \} \\ &= \delta(1-z) + \frac{g^2}{(4\pi)^3} \left\{ \left[z(1-z) \ln \frac{-q^2}{m^2 z(1-z)} + \frac{11}{3}(1-z)^2 - 3(1-z) - \frac{1}{2} \right. \right. \\ &\quad \left. \left. + \frac{1}{6(1-z)} \right] + \delta(1-z) \left[\frac{1}{6} \ln \frac{m^2}{-q^2} + \frac{1}{6} \ln \epsilon + \frac{1}{2} \right] \right\}\end{aligned}\quad (3.26)$$

The analogous sum of *off-shell* contributions reads

$$\begin{aligned}\hat{\mathcal{F}}_{2,q} &= \hat{\mathcal{F}}_2^{(0)} + \{ \hat{\mathcal{F}}_2^{R\text{-hard}} + \hat{\mathcal{F}}_2^{R\text{-soft}} + \hat{\mathcal{F}}_2^V \} \\ &= \delta(1-z) + \frac{g^2}{(4\pi)^3} \left\{ \left[z(1-z) \ln \frac{q^2}{\mu^2 z^2} + \frac{11}{3}(1-z)^2 - 3(1-z) - \frac{1}{2} + \frac{1}{\delta(1-z)} \right] \right. \\ &\quad \left. + \delta(1-z) \left[\frac{1}{6} \ln \frac{\mu^2}{q^2} + \frac{1}{6} \ln \epsilon + \frac{23}{36} + \frac{1}{6} \pi i \right] \right\}\end{aligned}\quad (3.27)$$

The structure functions resulting from the Feynman diagrams with a gluon in the initial state (Figs. 1f,g) are given for general mass assignment in Appendix C. For on- and off-shell mass regularization we obtain

$$\hat{\mathcal{F}}_{1,g} = \frac{g^2}{2(4\pi)^3} (1-z)^2 \quad (\text{on/off-shell}) \quad (3.28)$$

$$\hat{\mathcal{F}}_{2,g} = \frac{2g^2}{(4\pi)^3} \left[z(1-z) \ln \frac{-q^2(1-z)}{m^2 z} + \frac{1}{2}(1-z)(1-\eta z) \right] \quad (\text{on-shell}) \quad (3.29)$$

$$\hat{\mathcal{F}}_{2,g} = \frac{2g^2}{(4\pi)^3} \left[z(1-z) \ln \frac{q^2}{\lambda^2 z^2} + \frac{1}{2}(1-z)(1-\eta z) \right] \quad (\text{off-shell}) \quad (3.30)$$

We here have used $\lambda^2 < 0$. $\hat{\mathcal{F}}_{i,g}$ remains unchanged whilst going from on- to off-shell mass assignment. The same results emerge if the final-state quarks are changed to antiquarks. In the M factorization step we therefore use only half of the contribution given in Eqs. (3.28) to (3.30). $\hat{\mathcal{F}}_{1,g}$ is not influenced by the mass assignment, as is the case for $\hat{\mathcal{F}}_1^R$ [see Eq. (3.15)]. The coefficients of the log functions in Eqs. (3.26) and (3.27) and Eqs. (3.29) and (3.30) are equal, which follows from the absence of any spin in the ϕ_c^3 strong interaction model.

3.3 Analysis of the parton structure function

Since the structure functions are now known for different mass assignments, we can analyse the influence of the mass choices on the individual terms which were worked out in Subsection 3.2. This allows us to study specific combinations and to illustrate earlier insight.

We start with the M singularities originating from the intermediate lines of the cut-graphs in Fig. 3 (indicated by the dashed line B). Using the rules of Section 2 they are localized in Figs. 3d₂, d₃, d₄. The M regularization leads to finite expressions, although these still depend on the particularly chosen mass assignment. The self-energy contributions in Eqs. (3.10) and (3.11) serve as an illustrative example. In order to demonstrate M cancellation between the above-mentioned graphs, the z-divergent contributions to the structure functions are split into a soft- and a hard-gluon part. We keep all mass dependence in the soft parts but proceed to the scaling limit in the remaining terms. In ϕ_6^3 the only soft-gluon contribution comes from Fig. 3d₂.

Since the Compton graph T^d in Fig. 3d₁ is mass independent (in the scaling limit), the regulator masses in the cut-graphs 3d₂, d₃, d₄ have to cancel in a uniform way; the end result is independent of the chosen mass assignment. With the earlier on/off-shell results this is easily checked. Defining

$$\hat{\mathcal{F}}_2^{S_2} = \delta(1-z) \Sigma_2' \quad (3.31)$$

we observe that

$$\hat{\mathcal{F}}_2^{R\text{-soft}} + \hat{\mathcal{F}}_2^{S_2} = \delta(1-z) \left\{ \frac{g^2}{8(4\pi)^3} \left[\ln - \frac{\epsilon q^2}{M_{reg}^2} + \zeta - \frac{8}{3} \right] \right\} \quad (3.32)$$

is independent of m^2 and μ_2^2 . Since ζ contains the pole $1/(n-6)$, expression (3.32) still depends on UV regularization. Its influence can be removed by adding half of the vertex contribution $\hat{\mathcal{F}}_2^{Vx}$ which, as we have seen, does not introduce any new mass singularity. The expression

$$\frac{1}{2} \hat{F}_2^{Vx} + \hat{F}_2^{S_2} + \hat{F}_2^{R\text{-soft}} = \delta(1-z) \left\{ \frac{g^2}{6(4\pi)^3} [\ln \epsilon + 1] \right\} \quad (3.33)$$

is unique. Owing to the Ward-identity $Z_4 = Z_2$, it is UV regularization-independent.

The difference between the on- and off-shell result is due to the way we have regularized the initial-state M divergences. They appear in the Compton graphs (Figs. 3a₁, e₁, f₁) and their corresponding cut-graphs (see Section 2). All mass dependence enters either via the log arguments in the hard part of graph 3a₂ or via the [$\delta(1-z)$ -proportional] self-energy contribution Σ'_1 (graphs 3e₂, f₂). Adding F^{Vx} and Σ'_1 exposes the latter insight with

$$F^{Vx}(q^2) + \Sigma'_1 = \frac{g^2}{6(4\pi)^3} \left[\ln \frac{m^2}{-q^2} + 2 \right] \quad (\text{on-shell}) , \quad (3.34)$$

$$F^{Vx}(q^2) + \Sigma'_1 = \frac{g^2}{6(4\pi)^3} \left[\ln \frac{M^2}{q^2} + \frac{17}{6} + \pi i \right] \quad (\text{off-shell}) . \quad (3.35)$$

We observe that the expressions (3.34) and (3.35) are the coefficients of the δ -term (apart from $\ln \epsilon + 1$) in Eqs. (3.26) and (3.27). We conclude: all mass dependence of the parton structure functions is fixed by the chosen mass assignment. Its influence is felt via the real-gluon graph 3a₂ and the virtual-gluon graphs 3e₂, f₂. The same holds true for the gluon structure function in Eqs. (3.29) and (3.30); its mass dependence can be traced back to the diagrams 4a₂, d₂. Finally, we remark that all log arguments of the virtual-gluon as well as the real-gluon graphs depend on the masses of the quarks and gluons jointly entering the strong interaction vertex (see Appendix B). The structure functions resulting from an antiquark in the initial state are the same.

4. THE DY PROCESS

In our analysis we proceed to the lowest-order corrections of DY massive lepton pair production. After introducing the Feynman diagrams, defining the DY cross-section and our notation, we specify the parton cross-sections according to the plan: naive parton model, virtual-gluon corrections, real-gluon corrections,

forming altogether the quark-antiquark contribution, and finally the cross-section with gluons in the initial state. In the third part of this section we discuss our results.

4.1 Diagrams and definitions

The order g^2 Feynman diagrams contributing to the DY process are shown in Fig. 2. We use the terminology of DI scattering: vertex (Vx), self-energy ($S_{1,2}$), real-gluon production (R), etc.

Massive lepton pair production in nucleon-initiated reactions is described in the parton model by

$$W(\tau, q^2) = \sum_{k, \ell} \int d\tau \int \frac{dx_1 dx_2}{x_1 x_2} \delta(\tau - \frac{\tau}{x_1 x_2}) u_k(x_1) u_\ell(x_2) \hat{W}_{k\ell}(\tau, q^2) \quad (4.1)$$

with the cross-section

$$s q^2 \frac{d\sigma}{dq^2} = \frac{4\pi\alpha^2}{3} W(\tau, q^2) \quad (4.2)$$

$\hat{W} = \hat{W}^{(0)} + \hat{W}^{(2)} + \dots$ is determined in a perturbative expansion, with the order g^2 contribution composed of the quark and gluon part as

$$\hat{W}^{(2)} = \hat{W}_{q\bar{q}} + \hat{W}_{qg} \quad (4.3)$$

The former is given by

$$\hat{W}_{q\bar{q}} = \{ \hat{W}^V + \hat{W}^R \} = \{ \hat{W}^{V_k} + \hat{W}^S + \hat{W}^{S_2} + \hat{W}^R \} \quad (4.4)$$

$\hat{W}$ can be directly compared (and later will be combined) with the structure functions of deep-inelastic scattering; u_ℓ stands for the quark (and/or gluon) momentum distribution multiplied by the relevant quark charges (in units of e). Our notation is explained in Figs. 1 and 2, with the kinematical quantities on the parton level indicated by the "hat" sign. We use

$$\hat{s} \approx x_1 x_2 s, \quad \tau = \frac{q^2}{\hat{s}} = \frac{\tau}{x_1 x_2}, \quad \tau = \frac{q^2}{s} \quad (4.5)$$

where s stands for the squared sum of the initial-state momenta, and $\hat{s}$ for that of the parton momenta; Q^2 represents the time-like photon mass.

4.2 Parton cross-sections

The 0-th order contribution (Fig. 2, SPM, scaling parton model) is given by

$$\hat{W}^{(0)} = \delta(1-z) \quad (4.6)$$

The vertex correction, independent of any mass choice, is obtained by analytic continuation of Eq. (3.8) to time-like Q^2 -values. For on- and off-shell mass assignment we simply take over the formulas for the self-energies as computed in DI scattering [Eqs. (3.10) and (3.11)]. The sum of virtual-gluon contributions reads

$$\hat{W}^V = \delta(1-z) \left\{ F_{(Q^2)}^{Vx} + F_{(Q^2)}^{Vx*} + \sum_1'(-\mu_1^2) + \sum_2'(-\mu_2^2) \right\} = \delta(1-z) 2 \operatorname{Re} F_{(Q^2)}^V \quad (4.7)$$

with the renormalized vertex correction given by

$$2 \operatorname{Re} F_{(Q^2)}^V = \frac{2g^2}{6(4\pi)^3} \left\{ \operatorname{Ln} \frac{m^2}{Q^2} + 2 \right\} \quad (\text{on-shell}) \quad (4.8)$$

$$2 \operatorname{Re} F_{(Q^2)}^V = \frac{g^2}{6(4\pi)^3} \sum_{j=1}^2 \left\{ \operatorname{Ln} \frac{M_j^2}{Q^2} + \frac{17}{6} + \pi i \right\} \quad (\text{off-shell}) \quad (4.9)$$

The imaginary part in Eq. (4.9) is due to the anomalous cuts of the self-energy diagrams [see Eq. (3.11)]. The notation $\operatorname{Re} F^V$ is symbolic. Had we determined the squared absolute value of a sum of amplitudes to find a cross-section, the result should clearly be real. Whilst using the off-shell assignment, we instead worked out the *amplitudes* corresponding to the cut-graphs in Figs. 7a₁, a₂, b₁, b₃, c₁, c₃. The unusual off-shell mass choices enforce anomalous cuts which manifest themselves in complex "cross-sections" such as those shown in Eq. (3.27).

The calculation of the real-gluon diagrams (Figs. 2a,b) proceeds as in DI scattering. The general result is

$$\hat{W}^R = \frac{g^2}{(4\pi)^3} \sum_{j=1}^2 \left[z(1-z) \operatorname{Ln} \frac{Q^2(1-z)}{z^2 M_j^2(z)} + (1-z)(2-3z) \right] \quad (4.10)$$

for details we refer to Appendix C.

The sum of contributions with a quark-antiquark pair in the initial state for on- and off-shell mass assignment is assembled in the following expression:

$$\hat{W}_{q\bar{q}} = \delta(1-z) + \frac{g^2}{(4\pi)^3} \left\{ \left[z(1-z) \ln \frac{Q^2}{m^2 z} + (1-z)(2-3z) \right] + \delta(1-z) \left[\frac{1}{6} \ln \frac{m^2}{Q^2} + \frac{1}{3} \right] \right\} \quad (\text{on-shell}) \quad (4.11)$$

$$\hat{W}_{q\bar{q}} = \delta(1-z) + \frac{g^2}{(4\pi)^3} \sum_{j=1}^2 \left\{ \left[z(1-z) \ln \frac{Q^2(1-z)}{\mu_j^2 z^2} + (1-z)(2-3z) \right] + \delta(1-z) \left[\frac{1}{6} \ln \frac{\mu_j^2}{Q^2} + \frac{17}{36} + \frac{1}{6} \pi i \right] \right\} \quad (\text{off-shell}) \quad (4.12)$$

Using, in the same way, on- and off-shell mass assignments, the Feynman diagrams with a gluon in the initial state (Figs. 2f,g) give

$$\hat{W}_{qg} = \frac{g^2}{(4\pi)^3} \left[z(1-z) \ln \frac{Q^2(1-z)^2}{m^2 z} - \frac{1}{6} (1-z)^4 + \frac{1}{2} (1-z)^3 + 3(1-z)^2 - 2(1-z) \right] \quad (\text{on-shell}) \quad (4.13)$$

$$\hat{W}_{qg} = \frac{g^2}{(4\pi)^3} \left[z(1-z) \ln \frac{Q^2(1-z)}{\lambda^2 z^2} - \frac{1}{6} (1-z)^4 + \frac{1}{2} (1-z)^3 + 3(1-z)^2 - 2(1-z) \right] \quad (\text{off-shell}) \quad (4.14)$$

We here have used $\lambda^2 < 0$.

4.3 Analysis of the parton cross-sections

Inspecting the expressions (4.10) to (4.13), we come to the same conclusions as in DI scattering. For different mass assignments there are different constant terms in the renormalized vertex correction $F^V(Q^2)$ [Eqs. (4.8) and (4.9)] due to the self-energy contributions of the initial state. They depend on the way we have chosen the off-shell masses. The log arguments in $\hat{W}^R$ [Eqs. (4.10)] and $\hat{W}_{qg}$ [Eqs. (4.13) and (4.14)] also depend on the chosen mass assignment. As in DI scattering, the coefficients in front of the logarithms in $\hat{W}^R$ and $\hat{W}_{qg}$ are equal; this is a reflection of the fact that in ϕ_6^3 the quarks and gluons have the same spin. There is one difference between the DI and DY results. In the DY cross-section there is no $z = 1$ pole; hard- and soft-gluon separation is not needed. The inner rainbow graph (Fig. 3d₂), contributing the $z = 1$ singularity in DI scattering, is absent in the set of analogous DY diagrams.

5. DIMENSIONAL REGULARIZATION OF THE M SINGULARITIES

In this section we demonstrate the use of n-dimensional regularization in order to prevent the M singularities. All partons are massless.

5.1 Generalities

In the preceding sections the M singularities were regularized by introducing masses on those lines which join the gluon vertex. All others are irrelevant. This procedure prevented M divergence of the integrated cross-sections and resulted in their logarithmic dependence on the mass ratios (q^2/m_i^2) , with m_i^2 representing one of the on/off-shell masses. Dimensional regularization offers another possibility. Its application is demonstrated in Ref. 7 where the decay process $W^- \rightarrow e^- + \bar{\nu} + \gamma$ is worked out. Here we use this method to evaluate the DI structure functions and the DY cross-section. Since all masses are equal (i.e. zero), dimensional-regularization can be considered as a special case of on-shell regularization. The transition amplitude and the complete cross-sections are gauge independent (contrary to off-shell regularization) enabling us to establish a one-to-one correspondence between on-shell and dimensional regularization.

The calculation follows the steps indicated in Sections 3 and 4 with the momenta and phase-space integrals generalized to n dimensions. Except for the calculation of the self-energy contribution we limit ourselves to the relevant results, exposing the technical details in Appendix C.

5.2 DI scattering

Our analysis starts with the virtual-gluon graphs (Figs. 1c-e) since the 0-th order diagram is unchanged [Eq. (3.7)]. The renormalized vertex correction is free of M singularities

$$F^V(q^2) = \frac{g^2}{6(4\pi)^3} \left[\ln \frac{M_{\text{reg}}^2}{-q^2} - \gamma + \frac{11}{3} \right] \quad (5.1)$$

where

$$\gamma = \frac{2}{n-6} + \gamma - \ln 4\pi \quad (5.2)$$

The $2/(n-6)$ pole indicates the UV divergence.

The calculation of the self-energy diagrams for all masses vanishing needs a more careful analysis since the self-energy integral, even if generalized to n dimensions, does not exist. This is a reflection of the fact that in the argument of the log-function there is no longer a mass which could scale the regulator mass

M_{reg}^2 . Although we employ n -dimensional techniques, renormalization is *not* enforced by minimal subtraction; instead, we subtract at the mass pole. However, Z_2 as given by Eq. (5.4) diverges for $m^2 \rightarrow 0$ and no longer exists for a completely massless theory. We therefore renormalize on the mass-shell $m^2 \neq 0$, and establish in the UV finite quantity Σ_f a connection between the $(n-6)$ mass pole and the logarithmic m^2 -dependence which subsequently allows us to proceed to the $m^2 = 0$ limit. The DI/DY cross-sections resulting from the Feynman diagrams with a gluon in the initial or final state (Figs. 1,2) manifest their M singularity by the logarithmic mass dependence $\ln m^2$ for on-shell calculation or by the pole $(n-6)^{-1}$ if dimensional regularization is used. We can therefore formulate a transformation rule: $\ln m^2 \leftrightarrow (1/n-6) + \text{const.}$ Such correspondence can also be established for the virtual contributions where dispersion techniques turned out to be of help. Using on-shell mass assignment, the renormalized self-energy correction Σ_f is given by

$$\Sigma_f(p_i^2, m^2) = \Sigma(p_i^2, m^2) - \Sigma(m^2) - (p_i^2 - m^2)(1 - Z_2^{-1}) \quad (5.3)$$

with

$$\delta m^2 = \Sigma(m^2) \quad \text{and} \quad 1 - Z_2^{-1} = \left. \frac{\partial \Sigma}{\partial p^2} \right|_{p^2 = m^2} \quad (5.4)$$

Writing for $\Sigma(s, m^2)$ an unsubtracted dispersion relation which, however, is UV divergent:

$$\Sigma(s, m^2) = \frac{1}{\pi} \int_{m^2}^{\infty} ds' \frac{\text{Im} \Sigma(s', m^2)}{s' - s} \quad (s \equiv p^2) \quad (5.5)$$

we are led to introduce subtractions and find

$$\Sigma_f(s, m^2) = \frac{(s - m^2)^2}{\pi} \int_{m^2}^{\infty} ds' \frac{\text{Im} \Sigma(s', m^2)}{(s' - s)(s' - m^2)^2} \quad (5.6)$$

The imaginary part is specified via unitarity

$$\text{Im} \Sigma(s, m^2) = \frac{g^2}{2} (2\pi i)^2 \int \frac{d^n p_2}{(2\pi)^n} \frac{d^n k}{(2\pi)^n} \delta^+(p_2^2 - m^2) \delta^+(k^2) (2\pi)^n \delta^n(p_1 - p_2 - k) \quad (5.7)$$

where $g_n = g_{\text{reg}}^{3-(n/2)}$ (see Appendix A) is the dimensional coupling constant. The integral in Eq. (5.7) has been worked out in Appendix B. Keeping m and n arbitrary, one finds

$$I_m \Sigma(s, m^2, n) = - \frac{g^2}{2} \frac{1}{(2\pi)^{n-2}} I_0(s, m^2, n) \quad (5.8)$$

Expression (5.8) is inserted in the twice subtracted dispersion relation Eq. (5.6) with the result,

$$\Sigma_f(s, m^2, 6) = \frac{g^2}{6(4\pi)^3} \left[\frac{(s-m^2)}{s^2} \ln \frac{m^2-s}{m^2} - \frac{(s-m^2)^2}{s} \right] \quad (5.9)$$

$$\Sigma_f(s, 0, n) = \frac{g^2}{6(4\pi)^3} s \left[\ln \frac{s}{M_{\text{reg}}^2} + \gamma - \frac{6}{3} \right] \quad (5.10)$$

Note that the pole of Eq. (5.10) is now explicitly appearing as a mass singularity.

We assume the following limiting procedures to be interchangeable:

$$\lim_{m \rightarrow 0} \lim_{n \rightarrow 6} \Sigma_f(s, m^2, n) = \lim_{n \rightarrow 6} \lim_{m \rightarrow 0} \Sigma_f(s, m^2, n) \quad (5.11)$$

The replacement rule for the self-energy contribution reads

$$\ln m^2 \xrightarrow[n \rightarrow 6]{m \rightarrow 0} \ln M_{\text{reg}}^2 - \gamma + \frac{5}{3} \quad (5.12)$$

Z_2 and δm^2 are derived from Eq. (5.3) using Eq. (5.9) with the result

$$1 - Z_2^{-1} = - \frac{g^2}{6(4\pi)^3} \left[\ln \frac{M_{\text{reg}}^2}{m^2} - \gamma + \frac{5}{3} \right] \quad (5.13)$$

$$\delta m^2 = \frac{g^2}{3(4\pi)^3} m^2 \left[\ln \frac{M_{\text{reg}}^2}{m^2} - \gamma + \frac{5}{3} \right] \quad (5.14)$$

Substituting $\ln m^2$ in Eqs. (5.13) and (5.14) by the replacement rule of Eq. (5.12), we find $Z_2 = 1$ and $\delta m^2 = 0$. Therefore if we set $s \equiv p_1^2 \equiv m^2$ in Eq. (5.3), and furthermore use the substitution rule (5.12), we immediately derive $\Sigma(0,0) = 0$. This result implies that the self-energy graphs do not contribute to the DI/DY cross-sections since the $(n-6)^{-1}$ UV and M poles completely cancel.

We apply the substitution rule (5.12) on the renormalized vertex correction Eq. (3.13) and immediately find its dimensional regularized form

$$F^V(q^2) = \frac{g^2}{6(4\pi)^3} \left[\ln \frac{M_{reg}^2}{-q^2} - \gamma + \frac{11}{3} \right] \quad (5.15)$$

which exactly agrees with the unrenormalized vertex correction $F^{Vx}(q^2)$ given in Eq. (5.1). Notice, however, that the $(n-6)^{-1}$ -pole in F^V in Eq. (5.15) is due to the M singularity, whereas in F^{Vx} [in Eq. (5.1)] it can be traced back to the UV divergence. The result of Eq. (5.15) can also be derived by using a subtracted dispersion relation for the vertex function, with its absorptive part generalized to n dimensions. The sum of virtual contributions to the structure functions is equal to

$$\hat{F}_2^V = \delta(1-z) \cdot 2 F^V(q^2) \quad (5.16)$$

The parton structure function of the Feynman diagrams describing real-gluon production (Figs. 1a,b) is worked out in Appendix C with the result,

$$\hat{F}_2^R = \frac{g^2}{(4\pi)^3} \left[z(1-z) \left\{ \ln \frac{-q^2(1-z)}{M_{reg}^2 z} + \gamma \right\} + \frac{11}{3} (1-z)^2 z(1-z) - \frac{1}{2} + \frac{1}{6(1-z)} \right] \quad (5.17)$$

Whilst using dimensional regularization the scaling limit is applied right from the start since all parton masses are assumed to vanish.

We again split the contributions resulting from real-gluon production into soft- and hard-gluon pieces, and use $\hat{F}_2^{R\text{-soft}}$ as defined in Section 3. The result is [see Eq. (C.24)]

$$\hat{F}_2^{R\text{-soft}} = \delta(1-z) \frac{g^2}{6(4\pi)^3} \left[\ln \frac{-q^2}{M_{reg}^2} + \gamma - \frac{8}{3} \right] \quad (5.18)$$

We collect all terms which contribute to the parton structure functions and find

$$\hat{F}_{1,q} = \frac{g^2}{6(4\pi)^3} (1-z)^2 \quad (5.19)$$

$$\hat{F}_{2,q} = \delta(1-z) + \frac{g^2}{(4\pi)^3} \left\{ \left[z(1-z) \left\{ \ln \frac{-q^2}{M_{reg}^2} \frac{1-z}{z} + \gamma \right\} + \frac{11}{3} (1-z)^2 z(1-z) - \frac{1}{2} + \frac{1}{6(1-z)} \right] + \delta(1-z) \left[-\frac{1}{6} \left\{ \ln \frac{-q^2}{M_{reg}^2} + \gamma - \frac{8}{3} \right\} + \frac{1}{6} \ln \epsilon + \frac{1}{2} \right] \right\} \quad (5.20)$$

The calculation of the structure functions resulting from the Feynman diagrams with a gluon in the initial state (Figs. 1f,g) reveals the result

$$\hat{F}_{1,q} = \frac{g^2}{2(4\pi)^3} (1-z)^2 \quad (5.21)$$

$$\hat{F}_{2,q} = \frac{2g^2}{(4\pi)^3} \left[z(1-z) \left\{ \ln \frac{Q^2}{M_{reg}^2} \frac{(1-z)}{z} + \gamma \right\} + \frac{1}{2}(1-z)(1-\gamma z) \right] \quad (5.22)$$

5.3 The DY process

The parton cross-sections of the DY process are determined in precisely the same way as in DI scattering. For calculation details we refer to Appendix C and present here only the results.

The 0-th order term as given in Eq. (4.6) is unchanged. The renormalized vertex correction follows from Eq. (5.15) using analytic continuation:

$$\hat{W}^V = \delta(1-z) \{ F^V(Q^2) + F^{V*}(Q^2) \} \quad (5.23)$$

with

$$F^V(Q^2) = \frac{g^2}{6(4\pi)^3} \left[\ln \frac{M_{reg}^2}{Q^2} - \gamma + \frac{11}{3} - \pi i \right] \quad (5.24)$$

The cross-section describing real-gluon production (Figs. 2a,b) reads

$$\hat{W}^R = \frac{2g^2}{(4\pi)^3} \left[z(1-z) \left\{ \ln \frac{Q^2}{M_{reg}^2} \frac{(1-z)^2}{z} + \gamma \right\} + (1-z)(2-3z) \right] \quad (5.25)$$

We form the complete sum of quark-antiquark cross-sections and find

$$\begin{aligned} \hat{W}_{q\bar{q}} = & \delta(1-z) + \frac{2g^2}{(4\pi)^3} \left\{ \left[z(1-z) \left\{ \ln \frac{Q^2}{M_{reg}^2} \frac{(1-z)^2}{z} + \gamma \right\} + (1-z)(2-3z) \right] \right. \\ & \left. + \delta(1-z) \left[-\frac{1}{6} \left\{ \ln \frac{Q^2}{M_{reg}^2} + \gamma - \frac{5}{3} \right\} + \frac{1}{3} \right] \right\} \end{aligned} \quad (5.26)$$

The parton cross-section following from the gluon-initiated DY sub-processes shown in Figs. 2f,g reads

$$\hat{W}_{gg} = \frac{g^2}{(4\pi)^3} \left[z(1-z) \left\{ \ln \frac{Q^2}{M_{reg}^2} \frac{(1-z)^2}{z} + \gamma \right\} - \frac{1}{6}(1-z)^4 + \frac{1}{2}(1-z)^3 + 3(1-z)^2 - 2(1-z) \right] \quad (5.27)$$

6. INCORRECT OFF-SHELL MASS ASSIGNMENT

In this section we will have a brief insight into the effects of an earlier proposed, incorrect off-shell mass assignment¹¹).

In Section 2 of this paper we have emphasized that any method of M regularization has to obey the double-cut-rule [following from unitarity and analyticity^{8,9}]. In the recent past, an off-shell mass assignment was advocated and results were derived which do not conform with this rule. All initial-state M divergences were prevented by space-like off-shell masses of the incoming momenta. Considering a specific initial line, we notice that in *each* diagram it is assigned the *same* mass. This is *incorrect*. The double-cut-graphs in Figs. 5-8 show different numbers in different graphs⁹) for a specific initial line. The final (or equivalently intermediate)-state M singularities are prevented by auxiliary masses, which maintain unitarity between the virtual Compton graphs and the corresponding cut-diagrams.

We analyse where the changes occur if the correct and incorrect mass assignments are compared. In ϕ_6^3 theory the cut-graphs with an intermediate M singularity are shown in Figs. 3d₂, d₃, d₄. In Subsection 3.3 we have shown that the resulting combination $\hat{\mathcal{F}}_2^R + \hat{\mathcal{F}}_2^{S2}$ is M-finite. The initial M singularities can be traced back to Figs. 3a₂ and 3e₂, f₂. Whilst going from the correct to the incorrect mass assignment, the contribution of Fig. 3a₂ is unchanged apart from the change of notation, $\mu_1^2 \rightarrow p_1^2$. Only the initial self-energy contributions corresponding to Figs. 3e₂, f₂ are modified to

$$\Sigma'_1(p_1^2 < 0) = -\frac{g^2}{6(4\pi)^3} \left[\text{Ln} \frac{M_{\text{reg}}^2}{-p_1^2} - 5 + \frac{8}{3} \right] \quad (6.1)$$

The coefficient of the $\delta(1-z)$ function in Eq. (3.27) is consequently changed to

$$\left[-\frac{1}{6} \text{Ln} \frac{q^2}{p_1^2} + \frac{1}{6} \text{Ln} \epsilon + \frac{1}{3} \right] \quad (6.2)$$

The incorrect term (6.2) appears in all parton structure functions in unchanged form. The M factorization in DI scattering leads to the $\bar{g}^2$ correction term of the coefficient function, which is always a difference of two structure functions (in this order). We may therefore conclude that the incorrect term drops out in the correction term of DI scattering.

The same conclusion applies to the DY process. The self-energy contributions (Figs. 2d,e) are changed to Eq. (6.2) and the other terms remain unaltered. The coefficient of the $\delta(1-z)$ function in Eq. (4.12) is changed to

$$\sum_{j=1}^2 \left[-\frac{1}{6} \ln \frac{q^2}{p_j^2} + \frac{1}{2} \right] \quad (6.3)$$

One immediately recognizes that the DY correction term $\Delta_{q\bar{q}}^{DY} \equiv f_{q\bar{q}} - 2f_{2,q}$ (see Section 7) is unaffected since the only change occurs in the self-energy contributions which appear in the same form in the non-log terms f_j of the DI and DY cross-sections. The above formula for $\Delta_{q\bar{q}}^{DY}$ teaches us that they are cancelled. The actual form of Σ'_1 is unessential.

We briefly depart from ϕ_6^3 and comment on QCD. As we go from the correct to the incorrect off-shell assignment, $\Delta_{q\bar{q}}^{DY}$ will change. The origin of this discrepancy could be traced back to the overlapping M and IR divergences resulting from the diagrams $5a_1-a_4$ and $7a_1-a_4$ ^{9,13}). Remember that in ϕ_6^3 these diagrams are M/IR-finite. Because of the M/IR divergences (in QCD), every massive regularization scheme will introduce anomalous cuts. Since there are two singularities on top of each other in the above graphs, the discontinuities provide terms proportional to π^2 in addition to $i\pi$. The latter already appear in ϕ_6^3 . If the mass assignment does not conform with the double-cut-rule, the anomalous cuts in the DI/DY cross-sections are no longer consistent; the mutual correspondence is lost. Some spurious terms will therefore remain after M factorization has been carried out, leaving an incorrect correction term $\Delta_{q\bar{q}}^{DY}$.

7. THE CORRECTION TERMS

In the preceding sections we were mainly concerned with the technical evaluation of the parton cross-sections of DI scattering and of the DY process. We here carry out M factorization and determine the (non-log) correction terms of these two processes¹⁰).

The order g^2 parton structure functions $\hat{\mathcal{F}}_{i,\ell}(z, q^2)$ with $\ell = (q, \bar{q}, g)$ and $i = 1, 2, L$ ($\hat{\mathcal{F}}_L \equiv \hat{\mathcal{F}}_2 - 2\hat{\mathcal{F}}_1$) are parametrized as

$$\hat{\mathcal{F}}_{i,q} = \alpha_i \delta(1-z) + g^2 \left[\alpha_i P_q(z) \ln \frac{q^2}{p_i^2} + f_{i,q}(z) \right] \quad (7.1)$$

$$\hat{\mathcal{F}}_{i,g} = g^2 \left[\alpha_i P_g(z) \ln \frac{q^2}{p_i^2} + f_{i,g}(z) \right] \quad (7.2)$$

where $a_1 = 0$, $a_2 = 1$, $a_L = 1$; P_ℓ and $f_{i,\ell}$ can be derived from Eqs. (3.26) to (3.30) and (5.19) to (5.22); $f_{i,\ell}$ is M regularization dependent. Following Ref. 10, the M-factorized structure functions read

$$\mathcal{F}_i(x, q^2) = \sum_\ell \int_x^1 \frac{dz}{z} u_\ell\left(\frac{x}{z}, q^2\right) C_{i,\ell}(z, \bar{g}^2) \quad (7.3)$$

with the coefficient functions given by

$$C_{i,q} = a_i \delta(1-z) + \bar{g}^2 (f_{i,q} - a_i f_{2,q}) \quad (7.4)$$

$$C_{i,g} = \bar{g}^2 (f_{i,g} - a_i f_{2,g}) \quad (7.5)$$

$\bar{g}^2(q^2)$ is the running coupling constant. Here it is obvious that for any regularization scheme the same correction term $\Delta_{i,\ell}^{DI} \equiv f_{i,\ell} - a_i f_{2,\ell}$ emerges.

We consider the DY process. Its order g^2 parton cross-sections are parametrized as

$$\hat{W}_{q\bar{q}} = \delta(1-z) + g^2 \left[P_q(z) L_n \frac{q^2}{p_1^2} + P_{\bar{q}}(z) L_n \frac{q^2}{p_2^2} + F_{q\bar{q}}(z) \right] \quad (7.6)$$

$$\hat{W}_{qg} = g^2 \left[P_g(z) L_n \frac{q^2}{p_2^2} + F_{qg}(z) \right] \quad (7.7)$$

where P_ℓ and $f_{k\ell}$ are given in Eqs. (4.11) to (4.14), (5.26) and (5.27). Mass factorization implies that the DY cross-section can be cast in the form

$$W(\tau, q^2) = \sum_{k,\ell} \int d\tau \int \frac{dx_1}{x_1} \int \frac{dx_2}{x_2} \delta\left(\tau - \frac{\tau}{x_1 x_2}\right) u_k(x_1, q^2) u_\ell(x_2, q^2) \Phi_{k\ell}(z, \bar{g}^2) \quad (7.8)$$

with

$$\Phi_{q\bar{q}} = \delta(1-z) + \bar{g}^2 \Delta_{q\bar{q}}^{DY} = \delta(1-z) + \bar{g}^2 [F_{q\bar{q}} - 2 F_{2,q}] \quad (7.9)$$

$$\Phi_{qg} = \bar{g}^2 \Delta_{qg}^{DY} = \bar{g}^2 [F_{qg} - f_{2,g}] \quad (7.10)$$

Substituting $f_{2,\ell}$ and $f_{k\ell}$ for all possible mass assignments in Eqs. (7.9) and (7.10), we find the M regularization independent correction terms

$$\Delta_{q\bar{q}}^{DY} = \frac{2}{(4\pi)^3} \left[z(1-z) \ln(1-z) - \frac{2}{3}(1-z)^2 + 2(1-z) + \frac{1}{2} - \frac{1}{8(1-z)} - 8(1-z) \left\{ \frac{1}{6} \ln \epsilon + \frac{1}{8} \right\} \right] \quad (7.11)$$

$$\Delta_{qg}^{DY} = \frac{1}{(4\pi)^3} \left[z(1-z) \ln(1-z) - \frac{1}{6}(1-z)^4 + \frac{1}{2}(1-z)^3 - \frac{1}{2}(1-z)^2 + (1-z) \right] \quad (7.12)$$

The above equations are one of the main results of this paper. Consider Eq. (7.8). We have absorbed the main Q^2 -dependence in the momentum distributions, and expand the corrections to the DY form in powers of the running coupling constant $\bar{g}^2(Q^2)$, with the lowest-order term given by $\Delta^{DY}(z \equiv \tau/x_1 x_2)$. Owing to the separation in hard and soft gluons, the upper integration limit in Eq. (7.8) is $1 - \epsilon$. All ϵ -dependence cancels after the integrations are performed. The $\Delta^{DI, DY}(z)$ are real functions of z only. All complex contributions of the DI/DY cross-sections drop out. The anomalous cuts in the DI structure function and in the DY cross-section therefore cancel.

8. SUMMARY AND CONCLUSION

In this paper we determined the order g^2 non-log corrections for DI scattering and for the DY process in the perturbative framework of ϕ_6^3 theory which is AF. We thus have the possibility to study the difficulties appearing in QCD in their most simple form.

We determined the cross-sections of the DI and the DY sub-processes for different mass assignments (on-shell, off-shell, massless) and exposed their regularization dependence. Our analysis was mainly concerned with the M and IR singularities and their consistent regularization, apart from an explicit demonstration of our calculations including n-dimensional regularization. The AF correction terms following from M factorization could be explicitly shown to be regularization independent. The M regularization, however, must be carried out in agreement with the double-cut-rule. Its meaning, origin, and implications were exposed. We commented on an incorrect off-shell mass assignment whose application in QCD leads to incorrect and ambiguous answers. That these ambiguities do not manifest themselves in ϕ_6^3 is an artifact of its simplicity since it is IR-finite.

In a theory with overlapping M and IR divergences such as QCD, the correction term will be incorrect. The demonstration is reserved for a forthcoming publication¹³⁾.

APPENDIX A

VIRTUAL GLUON CORRECTIONS IN ϕ_6^3 THEORY

We present the calculation of the virtual gluon corrections which appear in the processes $q\gamma^* \rightarrow q$ (Figs. 1c,d,e, ...) and $q\bar{q} \rightarrow \gamma^*$ (Figs. 2c,d,e, ...). The computations of the diagrams are carried out for the mass assignments which are discussed in Section 2. We first calculate the electromagnetic vertex correction for space- and time-like q^2 , and show that it is mass-regularization independent. We subsequently draw attention to the self-energy and expose its mass dependence. Using the off-shell mass assignment, we will notice anomalous cuts, as was pointed out in Section 2.

THE ELECTROMAGNETIC VERTEX CORRECTION

We remind the reader that the quark-gluon coupling constant is equal to

$$-ig_n \equiv -ig M_{ng}^{3-\frac{n}{2}} \quad (A.1)$$

The electromagnetic vertex in all orders of the coupling constant is defined by

$$-ie_q \Gamma_\mu = -ie_q (p_1 + p_2)_\mu F(q^2) \quad (q = p_1 - p_2) \quad (A.2)$$

In 0-th order we have $F(q^2) = 1$.

The second-order correction to Γ_μ is equal to (Fig. 2c).

$$-ie_q \Gamma_\mu^{(1)} = -ie_q (-ig_n)^2 \int \frac{d^n k}{(2\pi)^n} \frac{i^2 \cdot (p_1 + p_2 - 2k)_\mu}{(k^2 - \lambda^2) [(k - p_1)^2 - m_1^2] [(k - p_2)^2 - m_2^2]} \quad (A.3)$$

We proceed to the Feynman parameter form which immediately gives

$$\Gamma_\mu^{(1)} = g_n^2 \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} \Gamma(3 - \frac{n}{2}) \int_0^1 dx \int_0^1 dy y \frac{(1-2xy)p_{1\mu} + (1-2y(1-x))p_{2\mu}}{[-q^2 y^2 x(1-x) + M^2(x,y)]^{3-\frac{n}{2}}} \quad (A.4)$$

with

$$M^2(x, y) \equiv m_1^2 y x + m_2^2 y(1-x) + \lambda^2(1-y) - p_1^2 x y(1-y) - p_2^2(1-x) y(1-y) \quad (A.5)$$

For asymptotic q^2 values, explicit calculation reveals that the mass term $M^2(x, y)$ in the integrand may be neglected since it leads to corrections of the order $(m_1^2/q^2)[\ln(m_1^2/q^2)]$ which do not concern us here.

Expanding expression (A.4) around $3-n/2$ gives

$$F(q^2) = \frac{g^2}{6(4\pi)^3} \left[\ln\left(\frac{M_{\text{reg}}^2}{-q^2}\right) - \zeta + \frac{11}{3} \right] \quad (A.6)$$

$$\zeta = \frac{2}{n-6} + \gamma - \ln 4\pi \quad (A.7)$$

Note that this result is independent of all mass assignments. For time-like q^2 (Fig. 2c) we get

$$F(q^2) = \frac{g^2}{6(4\pi)^3} \left[\ln\left(\frac{M_{\text{reg}}^2}{q^2}\right) - \zeta + \frac{11}{3} - \pi i \right] \quad (A.8)$$

where $-\pi i$ is the normal cut ($Q^2 \geq 0$) contribution.

From (A.6) we obtain the vertex renormalization constant

$$Z_4 = 1 - \frac{g^2}{3(4\pi)^3} \frac{1}{6-n} \quad (A.9)$$

SELF-ENERGY CORRECTION

The self-energy (Figs. 1d and e) depends on the chosen mass assignment for the internal as well as external lines. It is given by the expression

$$\begin{aligned} -i\Sigma(p) &= (-ig_n)^2 \int \frac{d^n k}{(2\pi)^n} \frac{i^2}{[k^2 - \lambda^2][(k-p)^2 - m^2]} \\ &= i g_n^2 \frac{\pi^{\frac{n}{2}}}{(2\pi)^n} \Gamma(2 - \frac{n}{2}) \int_0^1 dx \frac{1}{[m^2 x + \lambda^2(1-x) - p^2 x(1-x)]} \end{aligned} \quad (A.10)$$

The mass and wave function renormalization constants are defined by

$$\begin{aligned}\Sigma(p) &= \delta m_q^2 + (1 - Z_2^{-1})(p^2 - m_q^2) + \Sigma_f(p^2, m_q^2, m^2) \\ &= (1 - Z_2^{-1})(p^2 - \bar{m}_q^2) + \Sigma_f(p^2, \bar{m}_q^2, m^2)\end{aligned}\quad (\text{A.11})$$

where $m_q^2(m^2)$ and $\bar{m}_q^2(\bar{m}^2)$ represent the un-renormalized and renormalized external (internal) masses. $\Sigma_f \sim (p^2 - \bar{m}_q^2)^2$ stands for the renormalized part of $\Sigma(p)$. The wave function renormalization constant, defined by

$$(1 - Z_2^{-1}) \equiv \frac{\partial \Sigma}{\partial p^2} \Big|_{p^2 = \bar{m}_q^2} \equiv \Sigma'(\bar{m}_q^2, \bar{m}^2) \quad (\text{A.12})$$

depends on the mass assignment. The masses are chosen as $p^2 = 0, \lambda^2 = 0, m^2 = \mu^2$ giving

$$1 - Z_2^{-1} \equiv \Sigma'(\mu^2) = -\frac{g^2}{6(4\pi)^3} \left[\text{Ln}\left(\frac{m_{\text{reg}}^2}{\mu^2}\right) - \zeta + \frac{5}{6} \right] \quad (\mu^2 > 0) \quad (\text{A.13})$$

$$1 - Z_2^{-1} \equiv \Sigma'(-\mu^2) = -\frac{g^2}{6(4\pi)^3} \left[\text{Ln}\left(\frac{m_{\text{reg}}^2}{-\mu^2}\right) - \zeta + \frac{5}{6} - \pi i \right] \quad (\mu^2 < 0) \quad (\text{A.14})$$

For the analytic continuation from $\mu^2 > 0$ to $\mu^2 < 0$ we have taken the convention $\ln \mu^2 \rightarrow \ln -\mu^2 + \pi i$. The anomalous cut contribution $-\pi i$ is due to the fact that the external mass in the self-energy is larger than the sum of the internal masses (i.e. $m_{\text{ext}}^2 > \Sigma m_{\text{int}}^2$). The on-shell case $p^2 = m^2, \lambda^2 = 0$ is simple with the result exposed in Eq. (3.10). Inspecting the above expressions (A.13) and (A.14) we observe that up to pole terms the Ward identity is satisfied:

$$Z_4 = Z_2 = 1 - \frac{g^2}{3(4\pi)^3} \frac{1}{6-n} \quad (\text{A.15})$$

The virtual gluon terms are corrections to the Born graph and contribute to the δ -pieces of the DI/DY parton cross-sections. The expressions are exposed in Eqs. (3.12) and (4.7), respectively.

APPENDIX B

PHASE-SPACE INTEGRALS I_i, J_i

In deep inelastic scattering we consider the process $p(m^2) + q(q^2) \rightarrow p_1(m_1^2) + p_2(m_2^2)$ (Fig. 1). In the brackets () we indicated the most general mass assignment; our results can be used for any regularization procedure. The two-particle phase-space is equal to

$$I_i(p_1) = \int d^n p_1 d^n p_2 \delta^+(p_1^2 - m_1^2) \delta^+(p_2^2 - m_2^2) \delta^n(p + q - p_1 - p_2) \frac{1}{[(p_1 - q)^2 - m_2^2]^i} \quad (B.1)$$

This integral is determined in the c.m. frame of the photon and the incoming parton. Subsequent integration over p_2 , p_{10} , and $|\vec{p}_1|$ gives

$$I_i(p_1) = \frac{\pi^{\frac{n}{2}-1}}{\Gamma(\frac{n}{2}-1)} \frac{|\vec{p}_1^*|^{n-3}}{2\sqrt{s}} \int_{-1}^1 dx \frac{(1-x^2)^{\frac{n}{2}-2}}{[A+Bx]^i} \quad (B.2)$$

with all angular integrations being carried out except for one (i.e. $x = \cos \theta$, see Ref. 6).

$$A = q^2 - 2 p_{10}^* q_0^* + m_1^2 - m_2^2 \underset{B_j}{\approx} -\frac{1}{2} (s - q^2) \quad (B.3)$$

$$B = 2 |\vec{p}_1^*| |\vec{q}^*| \underset{B_j}{\approx} \frac{1}{2} (s - q^2) \quad (B.4)$$

p_{10}^* , q_0^* and $|\vec{p}_1^*|$, $|\vec{q}^*|$ are the c.m. energies and momenta, respectively; $\sqrt{s}$ is c.m. energy, and with b_j we denote the scaling limit.

In the actual calculations we are only interested in the cases $i = -1, 0, 1, 2$, and $n = 4, 6$. For $n = 4$ we find

$$\begin{aligned}
 I_{-1} &= \pi \frac{|\vec{p}_1^*|}{\sqrt{\hat{s}}} \cdot A \approx_{B_j} -\frac{\pi}{4} (\hat{s} - q^2) \\
 I_0 &= \pi \frac{|\vec{p}_1^*|}{\sqrt{\hat{s}}} \approx_{B_j} \frac{\pi}{2} \\
 I_1 &= \frac{\pi}{2} \frac{|\vec{p}_1^*|}{\sqrt{\hat{s}}} \frac{1}{B} \ln \left(\frac{A+B}{A-B} \right) \approx_{B_j} -\frac{\pi}{2} \frac{1}{\hat{s} - q^2} \ln \left(\frac{(\hat{s} - q^2)^2}{M_0^4} \right) \\
 I_2 &= \frac{\pi}{2} \frac{|\vec{p}_1^*|}{\sqrt{\hat{s}}} \frac{2}{A^2 - B^2} \approx_{B_j} \frac{\pi}{2} \frac{1}{M_0^4}
 \end{aligned} \tag{B.5}$$

where

$$A^2 - B^2 \approx_{B_j} \frac{1}{\hat{s}} [\hat{s} q^2 m^2 - \hat{s}(\hat{s} - q^2) m_x^2 - q^2(\hat{s} - q^2) m_z^2] \equiv M_0^4 \tag{B.6}$$

Notice that the argument of the logarithm only depends on the masses of the particles which are entering the strong vertex. Similarly for $n = 6$ the I_i integrals read:

$$\begin{aligned}
 I_{-1} &= \frac{2\pi^2}{3} \frac{|\vec{p}_1^*|^3}{\sqrt{\hat{s}}} A \approx_{B_j} -\frac{\pi^2}{24} \hat{s}(\hat{s} - q^2) \\
 I_0 &= \frac{2\pi^2}{3} \frac{|\vec{p}_1^*|^3}{\sqrt{\hat{s}}} \approx_{B_j} \frac{\pi^2}{12} \hat{s} \\
 I_1 &= \frac{\pi^2}{2} \frac{|\vec{p}_1^*|^3}{\sqrt{\hat{s}}} \left[\frac{B^2 - A^2}{B^3} \ln \left(\frac{A+B}{A-B} \right) + \frac{2A}{B^2} \right] \approx_{B_j} -\frac{\pi^2}{4} \frac{\hat{s}}{\hat{s} - q^2} \\
 I_2 &= \frac{\pi^2}{2} \frac{|\vec{p}_1^*|^3}{\sqrt{\hat{s}}} \left[\frac{2A}{B^3} \ln \left(\frac{A+B}{A-B} \right) - \frac{4}{B^2} \right] \approx_{B_j} \frac{\pi^2}{2} \frac{\hat{s}}{\hat{s} - q^2} \left[\ln \left(\frac{(\hat{s} - q^2)^2}{M_0^4} \right) - 2 \right]
 \end{aligned} \tag{B.7}$$

For general n and all masses equal to zero we obtain

$$I_i(n) = (-1)^i \frac{\Gamma(\frac{n}{2} - i - 1)}{\Gamma(n - i - 2)} (\hat{s} - q^2)^{-i} \hat{s}^{\frac{n}{2} - 2} \tag{B.8}$$

For the calculation of the Drell-Yan phase-space integrals we use the momentum and mass assignment $p_1(m_1^2) + p_2(m_2^2) \rightarrow p(m^2) + q(q^2)$ (Fig. 2). The calculation is carried out in the c.m. system of the initial partons. The two-particle phase-space integrals are

$$J_i(p_i) = \int d^n q d^n p \delta^+(p^2 - m^2) \delta^+(q^2 - q^2) \delta^n(p_1 + p_2 - p - q) \frac{1}{[(p_1 - q)^2 - m_X^2]^i} \quad (B.9)$$

Subsequent integration over p , q_0 , $|\vec{q}|$ and all angles except for one gives

$$J_i = \frac{\pi^{\frac{n}{2}-1}}{\Gamma(\frac{n}{2}-1)} \frac{|\vec{q}^*|^{n-3}}{2\sqrt{s}} \int_{-1}^1 dx \frac{(1-x^2)^{\frac{n}{2}-2}}{[A+Bx]^i} \quad (B.10)$$

where

$$A = q^2 - 2p_{i0}^* q_0^* + m_1^2 - m_X^2 \approx \frac{1}{2}(Q^2 - \hat{s}) \quad (B.11)$$

$$B = 2|\vec{p}_1^*||\vec{q}^*| \approx -\frac{1}{2}(Q^2 - \hat{s}) \quad (B.12)$$

and p_{i0}^* , q_0^* and $|\vec{p}_1^*|$, $|\vec{q}^*|$ are the c.m. energies and momenta, respectively. We are again only interested in the special cases mentioned above. For $n = 4$, J_i becomes

$$\begin{aligned} J_{-1} &= \pi \frac{|\vec{q}^*|}{\sqrt{s}} A \approx_{B_j} -\frac{\pi}{4} \frac{(\hat{s} - Q^2)^2}{\hat{s}} \\ J_0 &= \pi \frac{|\vec{q}^*|}{\sqrt{s}} \approx_{B_j} \frac{\pi}{2} \frac{(\hat{s} - Q^2)}{\hat{s}} \\ J_1 &= \frac{\pi}{2} \frac{|\vec{q}^*|}{\sqrt{s}} \frac{1}{B} \ln \left(\frac{A+B}{A-B} \right) \approx_{B_j} -\frac{\pi}{2} \frac{1}{\hat{s}} \ln \left(\frac{(\hat{s} - Q^2)^2}{M_X^4} \right) \\ J_2 &= \frac{\pi}{2} \frac{|\vec{q}^*|}{\sqrt{s}} \frac{2}{A^2 - B^2} \approx_{B_j} \frac{\pi}{2} \frac{\hat{s} - Q^2}{\hat{s}} \frac{1}{M_X^4} \end{aligned} \quad (B.13)$$

M_0^2 is exactly the same as defined before in Eq. (B.6). Similarly for $n = 6$, the J_i integrals read:

$$\begin{aligned}
 J_{-1} &= \frac{2\pi^2}{3} \frac{|\vec{q}^*|^3}{V\hat{s}} A \approx_{B_j} -\frac{\pi^2}{24} \frac{(\hat{s}-\varphi^2)^4}{\hat{s}^2} \\
 J_0 &= \frac{2\pi^2}{3} \frac{|\vec{q}^*|^3}{V\hat{s}} \approx_{B_j} \frac{\pi^2}{12} \frac{(\hat{s}-\varphi^2)^3}{\hat{s}^2} \\
 J_1 &= \frac{\pi^2}{2} \frac{|\vec{q}^*|^3}{V\hat{s}} \left[\frac{B^2-A^2}{B^3} \ln\left(\frac{A+B}{A-B}\right) + \frac{2A}{B^2} \right] \approx_{B_j} -\frac{\pi^2}{4} \left(\frac{\hat{s}-\varphi^2}{\hat{s}} \right)^2 \\
 J_2 &= \frac{\pi^2}{2} \frac{|\vec{q}^*|^3}{V\hat{s}} \left[\frac{2A}{B^3} \ln\left(\frac{A+B}{A-B}\right) - \frac{4}{B^2} \right] \approx_{B_j} \frac{\pi^2}{2} \frac{(\hat{s}-\varphi^2)}{\hat{s}^2} \left[\ln\left(\frac{(\hat{s}-\varphi^2)^2}{M_0^4}\right) - 2 \right]
 \end{aligned} \tag{B.14}$$

Integration of Eq. (B.10) for all masses vanishing and general n, i gives

$$J_i(n) = (-1)^i \frac{\pi^{\frac{n}{2}-1}}{2} \frac{\Gamma(\frac{n}{2}-i-1)}{\Gamma(n-i-2)} \left(\frac{\hat{s}-\varphi^2}{\hat{s}} \right)^{n-3} (\hat{s}-\varphi^2)^{-i} \hat{s}^{\frac{n}{2}-2} \tag{B.15}$$

APPENDIX C

REAL GLUON CONTRIBUTIONS TO THE DI STRUCTURE FUNCTIONS/ DY CROSS-SECTION IN ϕ_6^3

We present the calculation details of the DI structure functions and the DY cross-sections following from real gluon production in ϕ_6^3 theory, which involves charged (neutral) scalar fields representing the quarks (gluons). The parton cross-sections depend (in the scaling limit) in a specific way on the chosen masses and therefore can be specified for any mass assignment. We will notice that the relevant mass dependence enters via the phase-space integrals which we have computed in Appendix B. Note that the numerator of the transition probabilities is independent (in the Bj limit) of all parton masses.

The hadronic structure tensor in deep inelastic scattering is defined by

$$\hat{W}_{\mu\nu}(p, q) = \frac{1}{4\pi m} \sum_{k=0}^{\infty} \int \prod_{i=1}^k \frac{1}{(2\pi)^{n-1}} \frac{d^n p_i}{2 p_{i0}} (2\pi)^n \delta^n(p+q-\sum p_i) M_{\mu}(k) M_{\nu}^*(k) \quad (C.1)$$

$M(k)$ is the amplitude describing the scattering process $\gamma^*(q) + p(p) \rightarrow p_1 + p_2 + \dots p_k$, where p is the incoming parton and p_i are the outgoing partons. However, from covariance and gauge invariance we infer that $\hat{W}_{\mu\nu}$ can be written like

$$\hat{W}_{\mu\nu}(p, q) = (-g_{\mu\nu} + \frac{q_{\mu} q_{\nu}}{q^2}) \hat{W}_1 + (p_{\mu} - \frac{p q_{\mu}}{q^2}) (p_{\nu} - \frac{p q_{\nu}}{q^2}) \frac{\hat{W}_2}{m^2} \quad (C.2)$$

$\hat{W}_{1,2}$ are determined by the projections $C \equiv \hat{W}_{\mu\mu}$, $D \equiv (p_{\mu} p_{\nu} / m^2) \hat{W}_{\mu\nu}$. We get the result

$$\left(\frac{n-2}{4}\right) m \hat{W}_1 = \frac{m}{\beta} [-\beta C + D] \underset{Bj}{\approx} \frac{m}{2} \left[-\frac{C}{2} + \frac{m^2}{v} D\right] \quad (C.3)$$

$$\left(\frac{n-2}{4}\right) v \hat{W}_2 = -\frac{v}{4\beta^2} [\beta C - (n-1) D] \underset{Bj}{\approx} m^2 \left[-\frac{C}{2} + (n-1) \frac{m^2}{v} D\right] \quad (C.4)$$

with $\beta \equiv (1 - v^2/q^2)$, $v = pq/m$, $z = -q^2/2pq$.

In the case that all masses are equal to zero and n is arbitrary we define $C = W_{\mu\mu}$, $D = p_\mu p_\nu W_{\mu\nu}$, and $v = pq$. One can check that (C_3) and (C_4) are unchanged except for the fact that we have to drop m . This holds also for the formulas which follow hereafter.

In the following we determine the contributions of the perturbative diagrams shown in Figs. 1 and 2 using general mass assignment, which provides easy and detailed insight into the results.

The 0-th order calculation (Fig. 1, SPM) leads to the results exposed in Eq. (3.7). In order g^2 where we have a gluon and a quark in the final state (Figs. 1a,b) the structure tensor is equal to

$$\hat{W}_{\mu\nu} = \frac{1}{2m_1} \frac{1}{(2\pi)^n} \int d^n p_2 d^n k \delta^+(p_2^2 - m_2^2) \delta^+(k^2 - \lambda^2) \delta^n(p_1 + q - p_2 - k) \quad (C.5)$$

The matrix element for this process is

$$M_\mu(2) = g_n \left[\frac{(2p_2 - q)_\mu}{(p_2 - q)^2 - m_x^2} + \frac{(2p_1 + q)_\mu}{(p_1 + q)^2 - m_x^2} \right] \quad (C.6)$$

leading to the projections

$$C = \frac{\pi}{m_1} \frac{g_n^2}{(2\pi)^n} \int d^n p_S \left[\frac{(2\hat{p} - q^2)}{(\hat{p} - m_x^2)^2} + \frac{(2\hat{s} - q^2)}{(\hat{s} - m_x^2)^2} + 2 \frac{(\hat{s} + \hat{p} - q^2)}{(\hat{p} - m_x^2)(\hat{s} - m_x^2)} \right] \quad (C.7)$$

$$D = \frac{\pi}{m_1^3} \frac{g_n^2}{(2\pi)^n} \int d^n p_S \left[\frac{(\frac{\hat{s} - \hat{p}}{2} + \hat{t})^2}{(\hat{p} - m_x^2)^2} + \frac{(\frac{\hat{s} - q^2}{2})^2}{(\hat{s} - m_x^2)^2} + 2 \frac{(\frac{\hat{s} - q^2}{2} + \hat{t})(\frac{\hat{s} - q^2}{2})}{(\hat{p} - m_x^2)(\hat{s} - m_x^2)} \right] \quad (C.8)$$

with $\hat{s} = (p_1 + q)^2$, $\hat{t} = (p_2 - q)^2$ and $d^n p_S$ representing the phase space. In the above expressions we have dropped all mass terms in the numerator since they disappear in the Bjorken limit. Using the phase-space integral defined in formula (B.1), C and D can be written as

$$C = \frac{\pi}{m_1} \frac{g_n^2}{(2\pi)^n} \left[\left\{ \frac{4\hat{s} - q^2}{(\hat{s} - m_x^2)^2} \right\} I_0(p_1) + \left\{ \frac{2(2\hat{s} - q^2)}{\hat{s} - m_x^2} \right\} I_1(p_1) + \{-q^2\} I_2(p_1) \right] \quad (C.9)$$

$$D = \frac{\pi}{m_1^3} \frac{g^2}{(2\pi)^n} \left[\left\{ \frac{3\hat{s}-q^2}{2(\hat{s}-m_x^2)} \right\}^2 I_0(p_1) + \left\{ \frac{(3\hat{s}-q^2)(\hat{s}-q^2)}{2(\hat{s}-m_x^2)} \right\} I_1(p_1) + \left\{ \frac{\hat{s}-q^2}{2} \right\}^2 I_2(p_1) \right] \quad (C.10)$$

The phase-space integrals $I_i(p_1)$ can be found for $n = 6$ in (B.7) and for arbitrary n in formula (B.8). One finds

$$C = \frac{1}{m_1} \frac{g^2}{(4\pi)^3} \left[\frac{1}{2} z(1-z) \cdot Lg_1 + \frac{z}{2} - \frac{z}{4} + \frac{1}{12} \cdot \frac{1}{1-z} \right] \quad (C.11)$$

$$D = \frac{1}{m_1^3} \frac{g^2}{(4\pi)^3} \left[\frac{1}{4} (1-z) \cdot Lg_1 + \frac{z}{3} - \frac{5}{12} + \frac{1}{24} \frac{1}{1-z} \right] \quad (C.12)$$

where

$$Lg_j = L_n \left(\frac{q^2}{z^2 M_j^2(z)} \right) - 2 \quad (n=6) \quad (C.13)$$

with

$$M_j^2(z) = \frac{M_0^4}{q^2} = \{z(1-z)\}^{-1} [z(1-z)m_j^2 - z\lambda^2 - (1-z)m_x^2] \quad (C.14)$$

$$Lg_1 = Lg_2 = L_n \left(\frac{-q^2}{M_{m_j}^2} \cdot \frac{1-z}{z} \right) - 2 + 5 \quad (n \text{ arbitrary}, m_1^2=0) \quad (C.15)$$

Remember that in the latter case we have expanded around $n = 6$.

For the definition of ζ , see formula (A.7). Substituting C and D in Eqs. (C.3) and (C.4) we obtain

$$\hat{F}_1^{\text{R-hard}} = \frac{g^2}{6(4\pi)^3} (1-z)^2 \quad (C.16)$$

$$\hat{F}_2^{\text{R-hard}} = \frac{g^2}{(4\pi)^3} \left[z(1-z) \cdot Lg_1 + \frac{5}{3} z^2 - \frac{7}{3} z + \frac{1}{6} \cdot \frac{1}{1-z} \right] \quad (C.17)$$

We now consider the soft contribution which originates from Fig. 3d₂.

$\hat{F}_2^{\text{R-soft}}$ is given by [see Eq. (3.19)].

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(\hat{s}-m_2^2) \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s} \hat{\mathcal{F}}_2^R(\hat{s}, q^2, m_i^2) \quad (\text{C.18})$$

Substituting for the integrand expression (C.4), we find

$$\frac{n-2}{4} \hat{\mathcal{F}}_2^{R\text{-soft}} = -\delta(\hat{s}-m_2^2) \int_{\hat{s}_0}^{-\epsilon q^2} d\hat{s} \left[\frac{m_i}{2} C(\hat{s}, q^2) + (n-1) \frac{2m_i^3}{q^2} D(\hat{s}, q^2) \right] \quad (\text{C.19})$$

with the most singular parts maintained in

$$C = \frac{\pi}{m_i} \frac{q_n^2}{(2\pi)^n} \frac{(2\hat{s}-q^2)}{(\hat{s}-m_2^2)^2} I_0(\hat{s}, q^2, m_i^2, n) \quad (\text{C.20})$$

$$D = \frac{\pi}{m_i^3} \frac{q_n^2}{(2\pi)^n} \frac{(\frac{\hat{s}-q^2}{2})^2}{(\hat{s}-m_2^2)^2} I_0(\hat{s}, q^2, m_i^2, n) \quad (\text{C.21})$$

The general formula of I_0 is given in Eq. (B.2). Since in the integrand $-q^2 \gg \hat{s}, m_i^2$ we keep in the numerator of C and D only terms leading in q^2 . We define $\hat{\mathcal{F}}_2^{R\text{-soft}}$ for the mass assignments introduced in Section 2.

a) Off-shell: $n = 6$, $m_6^2 = q^2$, $m_3^2 = \mu^2$; $m_i^2 = 0$ $i \neq 3$. One finds

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{q^2}{6(4\pi)^3} \left[\ln\left(-\frac{\epsilon q^2}{\mu^2}\right) - \frac{11}{8} \right] \quad (\mu^2 > 0) \quad (\text{C.22})$$

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{q^2}{6(4\pi)^3} \left[\ln\left(\frac{\epsilon q^2}{\mu^2}\right) - \frac{11}{6} - \pi i \right] \quad (\mu^2 < 0) \quad (\text{C.23})$$

b) On-shell: $p_i^2 = m^2$ for all i . The calculation proceeds in an analogous way. The result is found in Eq. (3.24).

c) Massless (n-dim. regul.): $m_i^2 = 0$ for all i and n arbitrary. The result is

$$\hat{\mathcal{F}}_2^{R\text{-soft}} = \delta(1-z) \frac{q^2}{6(4\pi)^3} \left[\ln\left(-\frac{\epsilon q^2}{M_{\text{reg}}^2}\right) + 5 - \frac{\sigma}{3} \right] \quad (\text{C.24})$$

We determine the structure functions of the DI process with a gluon in the initial state (Figs. 1f,g). We perform the calculation for general mass assignment and arbitrary n . The structure tensor is given by

$$\hat{W}_{\mu\nu} = \frac{1}{2\lambda} \frac{1}{(2\pi)^{n-1}} \int d^n p_1 d^n p_2 \delta^+(p_1^2 - m_1^2) \delta^+(p_2^2 - m_2^2) \delta^n(k+q-p_1-p_2) M_\mu(z) M_\nu^*(z) \quad (C.25)$$

and the amplitude reads

$$M_\mu(z) = g_n \left[\frac{(2p_1 - q)_\mu}{(p_1 - q)^2 - m_x^2} - \frac{(2p_2 - q)_\mu}{(p_2 - q)^2 - m_x^2} \right] \quad (C.26)$$

The invariants $\hat{s}, \hat{t}, \hat{u}$ are defined by $\hat{s} = (k + q)^2$, $\hat{t} = (p_2 - q)^2$, $\hat{u} = (p_1 - q)^2$.

The projections C and D are of the form

$$C = \frac{\pi}{\lambda} \frac{g_n^2}{(2\pi)^n} \int d^n p_S \left\{ \frac{2\hat{t} - q^2}{(\hat{t} - m_x^2)^2} + \frac{2\hat{u} - q^2}{(\hat{u} - m_x^2)^2} - \frac{2\hat{s}}{(\hat{t} - m_x^2)(\hat{u} - m_x^2)} \right\} \quad (C.27)$$

$$D = \frac{\pi}{\lambda^3} \frac{g_n^2}{(2\pi)^n} \int d^n p_S \left\{ \frac{\left(\frac{\hat{u} - \hat{t}}{2}\right)^2}{(\hat{t} - m_x^2)^2} + \frac{\left(\frac{\hat{u} - \hat{t}}{2}\right)^2}{(\hat{u} - m_x^2)^2} + \frac{2\left(\frac{\hat{u} - \hat{t}}{2}\right)^2}{(\hat{t} - m_x^2)(\hat{u} - m_x^2)} \right\} \quad (C.28)$$

Notice that all masses in the numerator can be neglected if we proceed to the scaling limit. Splitting the last term in a sum of $\hat{t}$ - and $\hat{u}$ -poles permits regrouping as

$$C = \frac{\pi}{\lambda} \frac{g_n^2}{(2\pi)^n} \sum_{j=1}^2 \left[2 \left(1 + \frac{\hat{s}}{\hat{s} - q^2} \right) I_1(p_j) - q^2 I_2(p_j) \right] \quad (C.29)$$

$$D = \frac{\pi}{\lambda^3} \frac{g_n^2}{(2\pi)^n} \sum_{j=1}^2 \left[\left(\frac{\hat{s} - q^2}{2} \right) I_1(p_j) + \left(\frac{\hat{s} - q^2}{2} \right)^2 I_2(p_j) \right] \quad (C.30)$$

which exhibits the additive nature of these contributions with respect to the two final-state quark momenta p_j . Using $I_i(p_j)$ as given by Eq. (B.2) and forming the scaling limit, we obtain

$$C = \frac{1}{\lambda} \frac{g^2}{(4\pi)^3} \sum_{j=1}^2 \left[\frac{1}{2} z(1-z) \overline{Lg}_j - \frac{1}{2} (1-z)(2-z) \right] \quad (C.31)$$

$$D = \frac{2}{\lambda^3} \frac{g^2}{(4\pi)^3} \sum_{j=1}^2 \left[\frac{1}{4} (1-z) \overline{Lg}_j - \frac{1}{4} (1-z) \right] \quad (C.32)$$

where

$$\overline{Lg}_j = \ln \left(\frac{q^2}{z^2 \overline{M}_j^2(z)} \right) - 2 \quad (n=6) \quad (C.33)$$

with

$$\overline{M}_j^2(z) = \frac{M_0^4}{q^2} = \{z(1-z)\}^{-1} [z(1-z)\lambda^2 - z m_j^2 - (1-z)m_x^2] \quad (C.34)$$

and

$$\overline{Lg}_1 = \overline{Lg}_2 = \ln \left(-\frac{q^2}{M_{uq}^2} \frac{(1-z)}{z} \right) - 2 + \gamma \quad (\text{for arbitrary } n, \text{ all } m_i^2=0) \quad (C.35)$$

From formulas (C.3) and (C.4) we obtain the DI structure functions

$$\hat{F}_{1,q} = \frac{g^2}{2(2\pi)^3} (1-z)^2 \quad (C.36)$$

$$\hat{F}_{2,q} = \frac{g^2}{(4\pi)^3} \sum_{j=1}^2 \left[z(1-z) \overline{Lg}_j + \frac{1}{2} (1-z)(1-3z) \right] \quad (C.37)$$

We now give details of our calculations concerning the cross-section for massive-lepton pair production, which on the parton level is defined by

$$s q^2 \frac{d\hat{\sigma}}{dq^2} = \frac{4\pi\alpha^2}{3} \hat{W}(s, q^2) \quad (C.38)$$

with

$$\hat{W} = - \sum_{k=0}^{\infty} \int d^n q \delta(q^2 - Q^2) \prod_{i=1}^k \frac{1}{(2\pi)^{n-1}} \frac{d^n k_i}{2k_{i0}} \delta^n(p_1 + p_2 - \sum k_i - q) M^\mu(k) M_\mu^*(k) \quad (C.39)$$

As in the DI scattering, we are only interested in the *real gluon processes* which we calculate for general mass assignment and arbitrary n . The 0-th order result is given in Eq. (4.6).

In next order we have processes with a gluon in the final state, i.e. $q\bar{q} \rightarrow \gamma^* g$ (Fig. 2a,b) and a gluon in the initial state, $gq \rightarrow \gamma^* q$ (Fig. 2f,g). The order g^2 cross-section is defined by

$$\hat{W}^R = - \frac{1}{(2\pi)^{n-1}} \int d^n q d^n k \delta^+(q^2 - Q^2) \delta^+(k^2 - \lambda^2) \delta^n(p_1 + p_2 - k - q) M_\mu(k) M_\mu^*(k) \quad (C.40)$$

The matrix element for the process with a gluon in the final state is given by

$$M_\mu(k) = g_n \left[\frac{(2p_1 - q)_\mu}{(p_1 - q)^2 - m_k^2} - \frac{(2p_2 - q)_\mu}{(p_2 - q)^2 - m_k^2} \right] \quad (C.41)$$

The invariants are $\hat{s} = (p_1 + p_2)^2$, $\hat{t} = (p_2 - q)^2$, $\hat{u} = (p_1 - q)^2$. Substituting Eq. (C.41) into (C.40) we obtain

$$\hat{W}^R = - \frac{g_n^2}{(2\pi)^{n-1}} \int d^n p_S \left[\frac{2\hat{t} - Q^2}{(\hat{t} - m_k^2)^2} + \frac{2\hat{u} - Q^2}{(\hat{u} - m_k^2)^2} - 2 \frac{\hat{s}}{(\hat{t} - m_k^2)(\hat{u} - m_k^2)} \right] \quad (C.42)$$

Splitting the last term in a sum of $\hat{t}$ - and $\hat{u}$ -poles we can write

$$\hat{W}^R = - \frac{g_n^2}{(2\pi)^{n-1}} \sum_{j=1}^2 \left[-Q^2 J_2(p_j) + 2 \frac{(2\hat{s} - Q^2)}{\hat{s} - Q^2} J_1(p_j) \right] \quad (C.43)$$

Substituting $J_i(p_j)$ of Eqs. (B.14) and (B.15) we obtain

$$\hat{W}^R = \frac{g^2}{(4\pi)^3} \sum_{j=1}^2 [z(1-z) Lg_j + z^2 - 3z + 2] \quad (C.44)$$

with Lg_j given in Eqs. (C.13) and (C.15).

The matrix element for the process with a gluon in the initial state reads

$$M_\mu(z) = g_n \left[\frac{(2p_2 - q)_\mu}{(p_2 - q)^2 - m^2} + \frac{(2p_1 + q)_\mu}{(p_1 + q)^2 - m^2} \right] \quad (C.45)$$

The invariants are $\hat{s} = (p_1 + q)^2 = (p_2 + k)^2$ and $\hat{t} = (p_2 - q)^2 = (p_1 - k)^2$. The corresponding cross-section (see C.39) is equal to

$$\hat{W}_{qg} = - \frac{g_n^2}{(2\pi)^{n-1}} \int d^n p_S \left[\frac{2\hat{t} - Q^2}{(\hat{t} - m^2)^2} + \frac{2\hat{s} - Q^2}{(\hat{s} - m^2)^2} + 2 \frac{\hat{s} + \hat{t} - Q^2}{(\hat{t} - m^2)(\hat{s} - m^2)} \right] \quad (C.46)$$

Using the standard formulas for the phase-space integration as given in (B.9) we obtain

$$\hat{W}_{qg} = - \frac{g_n^2}{(2\pi)^{n-1}} \left[-Q^2 J_2(p_2) + \frac{2(2\hat{s} - Q^2)}{\hat{s}} J_1(p_2) + \frac{4\hat{s} - Q^2}{\hat{s}^2} J_0(p_2) \right] \quad (C.47)$$

From Eqs. (B.15) and (B.14) we derive

$$\hat{W}_{qg} = \frac{g^2}{(4\pi)^3} [z(1-z) \overline{Lg}_1 - \frac{1}{6}(1-z)^4 + \frac{1}{2}(1-z)^3 + (1-z)^2] \quad (C.48)$$

with $\overline{Lg}_1$ given in Eqs. (C.33) and (C.35).

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Figure captions

- Fig. 1 : Order g^2 Feynman diagrams contributing to DI scattering.
- Fig. 2 : Order g^2 Feynman diagrams contributing to DY massive μ -pair production.
- Fig. 3 : Order g^2 Feynman diagrams of virtual Compton scattering $\gamma^*q \rightarrow \gamma^*q$ (left), and their cut-graphs contributing to DI scattering (right).
- Fig. 4 : Order g^2 Feynman diagrams of virtual Compton scattering on a gluon target $\gamma^*g \rightarrow \gamma^*g$ (left), and the corresponding cut-graphs contributing to DI scattering (right).
- Fig. 5 : Double-cut-graphs of DI scattering on a quark target (left) and on an antiquark target (right). The corresponding lines are indicated by numbers.
- Fig. 6 : Double-cut-graphs of DI scattering on a gluon target (left), and their alien graphs (right) which are not considered in the cross-section. The corresponding lines are indicated by numbers.
- Fig. 7 : Double-cut-graphs of the $q\bar{q}$ -initiated DY process. Corresponding lines are indicated by numbers.
- Fig. 8 : Double-cut-graphs of the qg - and $\bar{q}g$ -initiated DY process (left of principle dashed lines), and their alien graphs (right of principle dashed lines) which are not considered in the cross-section. The corresponding lines are indicated by numbers.

(SPM)

(a)

(b)

(c)

(d)

(e)

(f)

(g)

Fig. 1

Fig. 2

Fig. 3

VIRTUAL COMPTON AMPLITUDE

DEEP INELASTIC CROSS SECTION

Fig. 4

Fig. 5

Fig. 6

Fig. 7

Fig. 8