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NEUTRINO MASSES IN SU(5)

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ABSTRACT

Neutrino masses of a few electron-volts can be naturally obtained in an SU(5) unified gauge model which contains superheavy fermions. Recent attempts to embed SU(5) in a larger theory suggest the existence of such superheavy particles.

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There are various reasons to suppose that the minimal $SU(5)$ gauge model^{*)} of Georgi and Glashow¹⁾ is incomplete. Let us list some of them^{**) :}

- i) The fermions belong to the reducible representation $3(\bar{5} + 10)_L$ of $SU(5)$. No explanation is given as to how this structure arises.
- ii) Mass relations which involve charge $-1/3$ quarks and charged leptons exist in $SU(5)$. However, nothing can be said about charge $2/3$ quarks.
- iii) Minimal $SU(5)$ predicts $(m_d/m_s) = (m_e/m_\mu)$ in the asymptotic limit²⁾. This is known to be badly violated.
- iv) The predicted baryon asymmetry in the Universe is several orders of magnitude smaller than the experimental value.
- v) Although $B + L$ is violated, $B - L$ survives as an accidental global symmetry of $SU(5)$. The masslessness of the neutrinos in minimal $SU(5)$ is therefore accidental, which is not very satisfactory.

It was recognized some time ago that at least some of these difficulties can be taken care of by embedding $SU(5)$ in a larger group such as $SO(10)$ ³⁾ or E_6 ⁴⁾. An attempt to construct a unified gauge theory which incorporates multiple flavours was presented by Georgi in Ref. 5). Recently, an alternative and much more intriguing suggestion that may ameliorate some (and perhaps all) of the issues raised above has been put forward by Ellis, Gaillard and Zumino⁶⁾. According to these authors, the fundamental particles that appear in $SU(5)$ are a subset of the $SU(8)$ ⁷⁾ bound states of preons that belong to $SO(8)$ extended supergravity. The subset is selected by imposing the requirement that there are no $SU(5)$ anomalies^{***)}. By considering only the basic composite $SU(8)$ supermultiplet, the authors of Ref. 6) are led to the following reducible representation of left-handed spin $1/2$ fields that is free of $SU(5)$ anomalies:

$$3(\bar{5} + 10) + (45 + \bar{45}) + 4(24) + 9(10 + \bar{10}) + 3(5 + \bar{5}) + 9(1). \quad (1)$$

*) The minimal model contains three $(\bar{5} + 10)_L$ spin $1/2$ fields as well as Higgs fields in the 24 and 5 dimensional representations.

**) We disregard here the very important, but at present unsolvable, gauge hierarchy problem.

***) This requirement is apparently based on an unpublished argument of Veltman. See Ref. 6).

The above spectrum mostly contains superheavy fermions, but, remarkably enough, also allows for three chiral families [see point i) above]. However, care is needed in arriving at this conclusion. Note that only a subset of the basic $SU(8)$ supermultiplet is retained. Thus, a mechanism must be found that gives superheavy masses ($\sim M_{\text{Planck}}$?) to the spin 1/2, spin 1, spin 3/2, spin 2 and spin 5/2 matter fields that are left out. This turns out to be not such an easy task. Indeed, it has been suggested⁸⁾ that an infinite number of supermultiplets may be required to give masses to all the particles that are left out from the final $SU(5)$ theory.

Be that as it may, the point we wish to note here is that the $SU(5)$ theory that may finally emerge from broken supergravity is expected to contain, in addition to the three (or possibly four) light generations, some (or mostly!) superheavy fermions. The presence of certain superheavy fermions, say in the $(5 + \bar{5})_L$ representation, necessarily breaks the accidental global $U(1)_{B-L}$ invariance of the minimal $SU(5)$ model. This has the important consequence that neutrinos may now acquire Majorana masses in the $SU(5)$ model^{*)} [see point v)]. The main purpose of this note is to show that, in the presence of suitable superheavy fermions, neutrino masses in the few electronvolt range can be naturally obtained (this applies in particular to the electron neutrino).

We shall assume that the effective $SU(5)$ gauge theory that emerges from broken supergravity^{**)} contains one 24-plet and one 5-plet of Higgs fields and also has left-handed spin 1/2 fields in the following reducible representation:

$$\mathcal{F} = 3(\bar{5} + 10) + l(5 + \bar{5}) + m(10 + \bar{10}) + n(1), \quad n \geq 3. \quad (2)$$

The fermions in $\mathcal{F}$ are assumed to be massless in the absence of $SU(5)$ symmetry breaking^{***)}. They acquire masses when the Higgs 24-plet and 5-plet acquire non-zero vacuum expectation values.

We shall show that, for the charge zero sector of the theory, one obtains the following mass spectrum:

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- *) Majorana masses for neutrinos in the $SU(5)$ model have previously been discussed in Ref. 9). The neutrino masses in these papers are estimated to be of order 10^{-5} eV and 10^{-3} eV respectively.
 - **) Clearly, superheavy fermions can be introduced in the $SU(5)$ model without referring to any underlying supergravity theory.
 - ***) We assume that $SU(8)$ breaks down to $SU(5)$ through anomalies rather than through an $SU(5)$ invariant Higgs expectation value.

- (a) Three Majorana neutrinos with masses of order M_W^2/M_{B-L} , where $M_{B-L} = (\alpha_g/\pi)M_X$, M_X and M_W are the masses of the superheavy gauge bosons and $W^\pm$ bosons respectively and α_g is the SU(5) gauge coupling.
- (b) n Majorana neutrinos with masses of order M_{B-L} .
- (c) 2ℓ Majorana neutrinos with masses of order M_X . These neutrinos can be grouped into ℓ pairs such that the mass splitting in each pair is of order M_W^2/M_X .

Taking $M_X \approx 4 \times 10^{14}$ GeV and $\alpha_g \approx 1/45$, we see that the three light neutrino masses can lie in the few electronvolt range. It is interesting to note that, in contrast to some SO(10) schemes¹⁰⁾, the neutrino masses in our case need not display any hierarchical structure. In particular, the electron neutrino can have a few electronvolt mass.

Let us begin by considering the coupling of 5_L and $(\bar{5})_L$ fields to ϕ_{24} . It takes the form^{*)}

$$\frac{1}{2} h_{i\alpha} \psi_{\bar{5}i} \phi_{24} \psi_{5\alpha} + \text{h. c.} \quad (3)$$

where $i = 1, 2, \dots, \ell + 3$ and $\alpha = 1, 2, \dots, \ell$. By suitable orthogonal transformations on $\psi_{\bar{5}i}$ and $\psi_{5\alpha}$, the matrix $h_{i\alpha}$ can be brought into the following form:

$$\begin{aligned} h_{i\alpha} &= 0 \quad \text{for } i \neq \alpha, \\ h_{\alpha\alpha} &= h_\alpha \quad (\text{no sum over } \alpha). \end{aligned} \quad (4)$$

Thus, there are three $\psi_{\bar{5}j}$ fields ($j = \ell + 1, \ell + 2, \ell + 3$) that decouple in Eq. (3). For $\langle \phi_{24} \rangle \approx M_X/g$, one obtains 5ℓ Dirac fermions with masses $m_a^{\alpha} = \langle (\phi_{24})_a \rangle h_\alpha$, $a = 1, 2, \dots, 5$. For $h_\alpha \sim g$, these masses are of order M_X ^{**)α}.

The simultaneous presence of 5_L and $(\bar{5})_L$ fields violates the global B - L symmetry of the minimal SU(5) model, and therefore permits the neutral lepton fields to acquire Majorana mass terms. Consider in particular the self-energy contributions to the SU(5) singlet fields ψ_{1p} ($p = 1, 2, \dots, n$). The leading contributions arise in one loop and are of order $M_{B-L} = (\alpha_g/\pi) M_X$. The relevant diagrams are shown in Figures (1a) and (1b). The following points should be noted:

*) For simplicity, we assume throughout that all the Yukawa couplings are real.

**) A similar analysis can be applied to the 10_L and $(\bar{10})_L$ fields. However, they do not play any role in the discussion of the neutral fermion mass matrix.

- 1) In Figure (1a) all possible $\langle \phi_{24} \rangle$ and $-\mu_{\phi_5}^2$ insertions are allowed on the ϕ_5 propagator. Diagrams with no $\langle \phi_{24} \rangle$ insertions are excluded. The internal fermion propagators are massless.
- 2) In Figure (1b) we have $2k + 1$ ($k \geq 1$) insertions of $\langle \phi_{24} \rangle$ on the internal fermion line. The internal fermion lines are massless. All possible insertions of $\langle \phi_{24} \rangle$ and $-\mu_{\phi_5}^2$ are allowed on the ϕ_5 propagator.

It is now easy to check that the mass matrix M_{pq} of the $SU(5)$ singlet fields ψ_{1p} ($p = 1, \dots, n$) is proportional to

$$\alpha_g \sum_{\alpha=1}^{\ell} \sum_{a=1}^5 m_{\alpha}^a \int d^4 k \frac{1}{k^2 - (m_{\alpha}^a)^2} \frac{1}{k^2 - \mu_a^2} \quad , \quad (5)$$

where μ_a is the mass of the a^{th} component of ϕ_5 (note that $\mu_4 = \mu_5 = 0$ for $\langle \phi_5 \rangle = 0$). Although the integral in (5) is superficially logarithmically divergent, the coefficient of the divergent term vanishes since $\sum_{a=1}^5 m_{\alpha}^a = 0$. Thus, one finds that^{*)}

$$M_{pq} \sim (\alpha_g / \pi) M_x = M_{B-L} \quad . \quad (6)$$

This mass matrix is real and symmetric and can be diagonalized by performing an orthogonal transformation on ψ_{1p} . One thereby obtains n Majorana fermions with masses of order M_{B-L} which are mass eigenstates for $\langle \phi_5 \rangle = 0$.

Next consider the self-energy contributions to ψ_{5i} ($i = 1, 2, \dots, \ell + 3$) fields. The leading diagram is shown in Fig. (2). If i and j both refer to the light multiplets ($i, j = \ell + 1, \ell + 2, \ell + 3$) the diagram vanishes. In all other cases the contribution is of order M_W^2 / M_x . Similarly, the self energy contributions to $\psi_{5\alpha}$ ($\alpha = 1, \dots, \ell$) are of order M_W^2 / M_x .

Let us now consider the mass matrix of the $2\ell + n + 3$ [see Eq. (2)] neutral left-handed fields $\psi_{L\sigma}$, $\sigma = 1, 2, \dots, 2\ell + n + 3$ ($\sigma = 1, \dots, \ell + 3$ correspond to the neutral components of ψ_5 's, $\sigma = \ell + 4, \dots, 2\ell + 3$ to the neutral components of ψ_5 's and $\sigma = 2\ell + 4, \dots, 2\ell + 3 + n$ to ψ_1 's). The mass term is given by:

^{*)} We assume here that ℓ is not too large. For instance, $\ell = 3$ in Eq. (1).

$$\frac{1}{2} \bar{\psi}_L^c M_{\sigma\tau} \psi_L + h.c. , \quad (7)$$

where the real symmetric mass matrix M takes the following form:

$$\begin{array}{c} 1, 2, \dots, \ell, \ell+1, \ell+2, \ell+3, \ell+4, \dots, 2\ell+3, 2\ell+4, \dots, 2\ell+n+3 \\ \begin{array}{c} 1 \\ 2 \\ \vdots \\ \ell \\ \ell+1 \\ \ell+2 \\ \ell+3 \\ \ell+4 \\ \vdots \\ 2\ell+3 \\ 2\ell+4 \\ \vdots \\ 2\ell+n+3 \end{array} \left[\begin{array}{c|c|c|c} & & \begin{array}{c} M_x \\ M_x \\ M_{B-L} \end{array} & \\ \hline \begin{array}{c} M_W^2/M_x \\ M_W^2/M_x \\ \vdots \\ M_W^2/M_x \end{array} & \begin{array}{c} M_W^2/M_x \\ \\ \\ \end{array} & \begin{array}{c} M_{B-L} \\ \\ M_x \end{array} & M_W \\ \hline \begin{array}{c} M_W^2/M_x \\ \\ \end{array} & 0 & M_{B-L} & M_W \\ \hline \begin{array}{c} M_x \\ M_x \\ M_{B-L} \\ \vdots \\ M_{B-L} \\ M_x \end{array} & M_{B-L} & \begin{array}{c} M_W^2/M_x \\ \\ \end{array} & M_W \\ \hline M_W & M_W & M_W & \begin{array}{c} M_{B-L} \\ M_{B-L} \\ 0 \\ \vdots \\ 0 \\ M_{B-L} \end{array} \end{array} \right] \quad (8)$$

In (8) only the order of magnitude of the matrix elements has been written. Note that the zero 3×3 submatrix of M receives contributions of order $(\alpha_g/\pi) \cdot M_W^2/M_x$ in one loop which have been neglected. By studying the order of magnitude of the coefficients of the characteristic polynomial of M , one can estimate the order of magnitude of its eigenvalues. This yields three Majorana neutrinos with masses of order $M_W^2/M_{B-L} \sim \text{few electronvolts}^*)$, n Majorana neutrinos with

*) If the Yukawa couplings of the $SU(5)$ singlet fields, the light ψ_{5L} and ϕ_5 display a strong generation dependence, then this will be reflected in the mass spectrum of the light neutrinos.

masses of order $M_{B-L} \sim 10^{12} - 10^{13}$ GeV and 24 Majorana neutrinos with masses of order $M_X \sim 10^{14}$ GeV.

We wish to conclude by commenting briefly on a few things. In the above analysis, we took the left-handed fermions to be in the $5, \bar{5}, 10, \bar{10}$ and the singlet representations of $SU(5)$. The introduction of a 24-plet of fermions, for instance, would give light neutrino masses of order $M_W^2/M_X \sim 10^{-2}$ eV. A possible cure for the unsatisfactory relation $m_d/m_s = m_e/m_\mu$ that may be relevant in the present framework has been suggested in Ref. 11). The mechanism suggested in this reference also contributes to the mass matrix (8). However, these contributions are of the same order as the two loop contributions and have been neglected. It is worth noting that the decay of the superheavy fermions may provide an important mechanism¹²⁾ for producing the observed baryon asymmetry. Finally, if sufficient superheavy ($\sim M_X$) fermions are present, the $SU(5)$ gauge coupling g starts to increase beyond M_X and approaches unity at the Planck mass^{*)}. Note however, that $\sin^2 \theta_W$ and M_X remain essentially unaffected.

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*) Also see second paper in Ref. 6).

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FIGURE CAPTIONS

- Fig. 1 : One loop diagrams that contribute to the self-energy of the SU(5) singlet fields ψ_{1p} ($p = 1, \dots, n$).
- Fig. 2 : Leading contribution to the self energy of the SU(5) $\bar{5}$ -plets $\psi_{\bar{5}i}$ ($i = 1, \dots, \ell + 3$). The double internal line is the chirality non-flipping part of the superheavy fermion propagator.

FIG. 1

FIG. 2