Ref.TH. 2952-CERN

WEAK NON-LEPTONIC DECAYS BEYOND LEADING LOGARITHMS IN QCD

G. Altarelli

Istituto di Fisica dell' Università di Roma
Istituto Nazionale di Fisica Nucleare - Roma

G. Curci

Scuola Normale Superiore - Pisa
Istituto Nazionale di Fisica Nucleare - Roma

G. Martinelli

CERN--Geneva

S. Petrarca

Istituto di Fisica dell' Università di Roma
Istituto Nazionale di Fisica Nucleare - Roma

ABSTRACT

We compute the two-loop anomalous dimensions of the four fermion operators relevant to weak non-leptonic decays and discuss the physical implications for strange and charm particle decays. In particular, we derive the complete order α_s corrections to the inclusive decay width of a heavy quark.

It is known that gluon effects, as obtained from the short-distance operator expansion¹⁾ and QCD in the leading logarithmic approximation (LLA) are important²⁾ for weak non-leptonic amplitudes. In particular, these effects work in favour of the $\Delta T = 1/2$ rule in strange particle decays. The predicted amplitude ratio $(A_{1/2}/A_{3/2})_{LLA} \approx 3 \div 4$ is, however, not sufficient to reproduce the observed value $(\sim 20)^{3)}$. The remaining enhancement is presumably due to low energy effects in the matrix elements⁴⁾ (including those of "penguin" operators⁵⁾).

For heavy quark decay, (especially for charm), a substantial increase in the non-leptonic width is obtained, which leads to a prediction⁶⁾ for the (quark) semi-leptonic branching ratio B_{SL} which is considerably smaller than the free field value. For charm, the prediction in the LLA is $B_{SL} \approx 13 \div 16\%$ as compared to the free field value $\sim 20\%$. Until recently, the results for a c quark were directly taken as relevant for charm particles, the light constituent quarks being taken as inert spectators. After the experimental finding⁷⁾ of a quite different lifetime for D^0 and D^+ by DELCO, also confirmed by MARK II and by observations in emulsions, one is now led to a picture of charm particle decays based on both c quark decay and W exchange between constituent quarks (in the s or t channel) with real gluon emission⁸⁾. However, the c quark decay prediction should remain essentially valid for D^+ (provided the spectator is really inert⁹⁾) because, in D^+ , the annihilation process can only occur at the Cabibbo suppressed level. Since a value of B_{SL} for D^+ close to 20% is being currently reported⁷⁾, it is important to verify whether or not the LLA is supported by a study of the next to leading corrections. We report here on the first complete calculation of the next to leading QCD corrections to the non-leptonic width of a heavy quark (for massless final state quarks and gluons). This includes, as a main ingredient, the evaluation in the massless theory of the two-loop anomalous dimensions for the four fermion operators of dimension six in the short-distance expansion for the product of two weak currents. These results are also relevant for strange particle decays. For heavy quark decay this paper supersedes an incomplete calculation of the non-leptonic width previously attempted in Ref. 10). When taken together with the known¹¹⁾ corrections of order α_s to the semi-leptonic width, our calculation leads to the expression of B_{SL} up to, and including terms of order α_s .

In the lowest order in the weak coupling the effective Hamiltonian for a non-leptonic weak process induced by charged currents is given by:

$$H_{FI} \approx g_w^2 \int d^4x D_w(x^2, m_w^2) \langle F | T(J^\mu(x) J_\mu(0)) | I \rangle \quad (1)$$

where M_W , g_W and D_W are the W boson mass, coupling and propagator respectively. For flavour changing amplitudes, the leading contributions in the limit $M_W \rightarrow \infty$ arise from the four fermion operators of dimension six in the short-distance expansion for the T-product of Eq. (1)^{2),3),12)}. For charm changing processes the relevant terms are of the form:

$$H_{FI}^{\Delta C=1} = \frac{G_F}{\sqrt{2}} \left\{ C_+(t, \alpha_s) \langle F | O_+(0) | I \rangle + C_-(t, \alpha_s) \langle F | O_-(0) | I \rangle + h.c. + \dots \right\} \quad (2)$$

where $t = 1N M_W^2/\mu^2$ and:

$$O_{\pm} = \frac{1}{2} \left[(\bar{s}'c)(\bar{u}d') \pm (\bar{u}c)(\bar{s}'d') \right] = \frac{N \pm 1}{2N} (\bar{s}'c)(\bar{u}d') \pm \sum_A (\bar{s}'t^A c)(\bar{u}t^A d') \quad (3)$$

The short-hand $(\bar{Q}q) = \bar{Q}\gamma_{\mu}(1-\gamma_5)q$ has been introduced here. s' and d' are the Cabibbo-like mixtures coupled to c and u respectively¹³⁾. t^A are the $SU(N)_{\text{COLOUR}}$ matrices in the quark representation ($t_r(t^A t^B) = 1/2 \delta^{AB}$). The second equality is obtained by Fierz' rearrangement of $(\bar{u}c)(\bar{s}'d')$. Note that $O_{\pm}$ are symmetric (antisymmetric) under the exchange $u \leftrightarrow c$ and/or $s \leftrightarrow d$. In the massless limit, $O_{\pm}$ are multiplicatively renormalizable because they transform as different irreducible representations of the flavour group which is a symmetry of the theory in this limit. In Eq. (2) the dots stand for non-leading terms including "penguin" diagrams^{*)} which, however, in charm decays only appear at the Cabibbo suppressed level (and are quite small even at that level). In the following we shall neglect these terms and, for simplicity, actually refer to the Cabibbo allowed transitions with all four quarks being different.

The coefficients $C_{\pm}$ in Eq. (2) are essentially determined by the massless theory. Note that, for these particular coefficients, the effects of the b and t quarks only amount to some distortions in the variation of the coupling α_s in its run from the W mass down to the charm mass. $C_{\pm}$ satisfy the renormalization group equation¹⁴⁾:

$$\left\{ -\frac{\partial}{\partial t} + \beta(\alpha_s) \frac{\partial}{\partial \alpha_s} + \gamma_{\pm}(\alpha_s) \right\} C_{\pm}(t, \alpha_s) = 0 \quad (4)$$

*) A study of higher order gluon effects in "penguin" diagrams was made in Ref. 18) and the general structure was clarified.

with the familiar solution:

$$C_{\pm}(t, \alpha_s) = C_{\pm}[\alpha_s(t)] \exp \left\{ \int_{\alpha_s}^{\alpha_s(t)} \frac{\gamma_{\pm}(\alpha)}{\beta(\alpha)} d\alpha \right\} \quad (5)$$

where $C_{\pm}[\alpha_s(t)] = C_{\pm}[0, \alpha_s(t)]$ and $\alpha_s = \alpha_s(0)$ is the renormalized coupling at the scale μ . By series expansion we define:

$$\begin{aligned} \beta(\alpha) &= -b \alpha^2 (1 + b' \alpha + \dots) \\ \gamma_{\pm}(\alpha) &= \gamma_{\pm}^{(1)} \alpha + \gamma_{\pm}^{(2)} \alpha^2 + \dots \\ C_{\pm}(\alpha) &= 1 + C_{\pm}^{(1)} \alpha + \dots \end{aligned} \quad (6)$$

with¹⁵⁾:

$$\begin{aligned} b &= \frac{11N - 2f}{12\pi} \\ b' &= \frac{1}{(4\pi)^2} b \left[\frac{34}{3} N^2 - \frac{13}{3} Nf + \frac{f}{N} \right] \end{aligned} \quad (7)$$

(f is the number of quark flavours). We also know that²⁾:

$$\gamma_{\pm}^{(1)} = \mp \frac{3}{4\pi} \frac{N \mp 1}{N} \quad (8)$$

We recall that, while $\gamma^{(2)}$ and $C^{(1)}$ are separately dependent on the regularization and the renormalization prescription for the local operator O , the combination $\gamma^{(2)} - bC^{(1)}$ is not¹⁴⁾. It depends, however, on the definition of α_s because, under the transformation $\alpha_s \rightarrow \alpha's$ where $\alpha_s = \alpha's (1 + K\alpha's)$, then $\gamma^{(2)} \rightarrow \gamma^{(2)} = \gamma^{(2)} + K\gamma^{(1)}$. It is therefore convenient to rearrange the expanded form of Eq. (2) as:

$$H_{FI} \simeq \frac{G_F}{\sqrt{2}} \sum_{i=\pm} \tilde{O}_i^{FI}(\alpha_s) [1 + C_i^{(1)} \alpha_s] L_i(t) \left\{ 1 + \frac{\alpha_s - \alpha_s(t)}{\pi} g_i \right\} \quad (9)$$

where $\tilde{O}_i^{FI} = \langle F | O_i(0) | I \rangle$ are the renormalized operator matrix elements and:

$$L_i(t) = \left[\frac{\alpha_s}{\alpha_s(t)} \right]^{\gamma_i^{(1)}/b} \quad g_i = \frac{\pi}{b} [\gamma_i^{(2)} - bC_i^{(1)} - b'\gamma_i^{(1)}] \quad (10)$$

p_i 's are prescription independent and, of course, independent of the external states. Thus, p_i can be computed by taking convenient, possibly not physical, external states. In our computation we took four massless off-shell quarks with equal virtual masses $-p^2$. On the other hand, the quantities:

$$O_i^{FI}(\alpha_s) = \tilde{O}_i^{FI}(\alpha_s) [1 + c_i^{(1)} \alpha_s] \quad (11)$$

are only well defined if H_{FI} is. In the previous example they would depend, not only on p^2 , but also on the gauge chosen (contrary to p_i) as H_{FI} is gauge dependent for off-shell external states. It is only after calculation of real physical quantities that one can get rid of the infra-red cut-off (like p^2) and gauge parameters. For heavy quark decays this is obtained after adding up the rates for a massive quark into $qq\bar{q}$ and into $qq\bar{q}G$ where q and G are real massless quarks and gluons. Note that we keep here, as the expansion parameter, $\alpha_s - \alpha_s(t)$ and not $\alpha_s(t)$ as is usually done for moments of structure functions in deep inelastic scattering¹⁴⁾. This is because we want here to expose the difference between operators defined at M_W and at μ (limit of the perturbative domain or energy scale of interest). As for the matrix element in front, it could be either non-perturbative (strange particle decays) or perturbative (heavy quark decay in the parton approximation).

The evaluation of $\gamma_{\pm}^{(2)}$ is a difficult task, not only because of the large number of two-loop diagrams involved (Fig. 1), but mainly because of practical problems in the application to this case of the method of dimensional regularization. These problems are due to virtual gluon exchanges that turn the initial $\gamma_{\mu}(1 - \gamma_5) \otimes \gamma^{\mu}(1 - \gamma_5)$ operator into operators with up to three (in one loop) or up to five (in two loops) γ matrices on each fermion line, such as $\gamma_{\alpha}\gamma_{\beta}\gamma_{\mu}(1 - \gamma_5) \otimes \gamma^{\alpha}\gamma^{\sigma}\gamma^{\mu}(1 - \gamma_5) \tilde{\gamma}_{\rho\sigma}^{\alpha\beta}$. In $n \neq 4$ dimensions these operators are independent of the starting one because for $n = 4$ the relation establishing their linear dependence involves the $\epsilon_{\mu\nu\rho\sigma}$ tensor. Thus, the calculation was performed in the dimensional reduction scheme of Ref. 16) which consists of continuing in $n < 4$ dimensions only coordinates and momenta while keeping in four dimensions all tensors and spinors (so that, in particular, γ matrices are 4×4 matrices). The gain in computational simplicity is paid for by the presence of a number of field theoretic subtleties in handling the resulting theory, which we discuss in detail elsewhere. Here we present only the final results.

In the $\overline{MS}$ prescription¹⁴⁾ for α_s we obtain*):

*) The algebraic programme SCHOONSHIP written by M. Veltman was an essential device for the completion of this work.

$$\left[\gamma_{\pm}^{(2)} - b c^{(1)} \right]_{\pm} = \frac{N+1}{(4\pi)^2 N} \left[\mp \frac{211}{12} N + \frac{21}{4} \mp \frac{57}{4N} \pm \frac{10}{3} f \right] + K \gamma_{\pm}^{(1)} \quad (12)$$

where $K = N/12\pi$ and $\gamma_{\pm}^{(1)}$ are given in Eq. (8). The last term in Eq. (12) can be eliminated by a change of $\Lambda^{(4)}$. In fact it converts the result from α_s defined by analogy with $\overline{MS}$ in dimensional reduction to the usual $\overline{MS}$ definition of α_s . It is numerically irrelevant, implying a change of Λ by 6%. Note that actually all the terms in the bracket, except for $21/4$ (which is quite small) could be reabsorbed in a change of Λ . We note the reflection property $+ \leftrightarrow -$ for $(N, f) \leftrightarrow (-N, -f)$. Note also that terms growing as N^2 (or Nf) are absent. This non-trivial cancellation of the leading terms in N , also true in the lowest order where $\gamma_{\pm}^{(1)} \approx 1$, implies the vanishing of leading and subleading logarithmic corrections in the limit $N \rightarrow \infty$ with $N\alpha_s$ fixed. A numerical evaluation for $N = 3$ and $f = 4$ leads to [see Eqs. (9), (10)]:

$$L_{\pm}(t) \left\{ 1 + \frac{\alpha_s - \alpha_s(t)}{\pi} g_{\pm} \right\} = \left(\frac{\alpha_s}{\alpha_s(t)} \right)^{\begin{matrix} -0.240 \\ +0.480 \end{matrix}} \left\{ 1 + \frac{\alpha_s - \alpha_s(t)}{\pi} \begin{pmatrix} -0.469 \\ +1.36 \end{pmatrix} \right\} \quad (13)$$

(the f -dependence is mild in the region of interest). The size of the subleading corrections is quite normal and the signs are such as to reproduce the enhancement-suppression pattern of the leading term. The present result considerably improves the basis of the short-distance argument on the $\Delta T = 1/2$ rule for strange particle decays^{*}) and allows us to increase the QCD enhancement factor $(A_{1/2}/A_{3/2})_{\text{QCD}}$ to $4 \div 5$ for current values of Λ . For heavy quark decays one must now evaluate matrix elements and rates associated with virtual (Fig. 2a) and real (Fig. 2b) gluon emission. While for the real diagrams the difference between $\tilde{O}_i(\alpha_s)$ and $O_i(\alpha_s)$ in Eqs. (9), (11) is of higher order, for virtual diagrams one needs to extract $O_i(\alpha_s)$. The latter can be obtained from the full correction of order α_s by subtracting the leading logarithms in H_{FI} . On the other hand, the real diagrams are to be evaluated with the full operators $O_{\pm}$ at the four fermion vertex.

^{*}) It is clear that mass effects become important when μ is given a value as close as possible to that relevant to strange particle decays, but the result in the massless theory is still significant in itself.

The present calculation can be made by using a regularization method which differs from the one used for the first stage leading to Eq. (12). In fact, we deal here with physical on-shell quarks (the initial heavy quarks being massive) and use dimensional reduction as a regularization of both ultra-violet and infra-red singularities. The final result for the heavy quark inclusive non-leptonic rate after the cancellation of double and single infra-red poles, can be cast in the following form:

$$\Gamma = \Gamma_{LLA} \left\{ 1 + \frac{2\alpha_s}{3\pi} \left(\frac{31}{4} - \pi^2 \right) + \frac{2\alpha_s}{3\pi} \frac{19}{4} \frac{L_-^2 - L_+^2}{2L_+^2 - L_-^2} + \right. \\ \left. + \frac{2(\alpha_s - \alpha_s(t))}{\pi} \frac{2L_+^2 g_+ + L_-^2 g_-}{2L_+^2 + L_-^2} \right\} = \Gamma_{LLA} \mathcal{M} \quad (14)$$

where $L_{\pm}$ and $\rho_{\pm}$ are defined in Eqs. (9), (10) and Γ_{LLA} is the width in the leading logarithmic approximation (including the LLA factor $2L_+^2 + L_-^2$). In Ref. 10) only the first correction was given. It arises from gluon corrections within the first and second current separately and therefore can be obtained directly from the μ -decay radiative corrections¹⁷⁾ $2\alpha_s/3\pi(25/4 - \pi^2)$ plus the first order correction to $R_{e^+e^-} \propto \alpha_s/\pi$. The second correction term is from the virtual gluon exchange between the two currents plus the corresponding real diagrams. It arises because the resummation of leading logarithms introduces the colour octet-octet terms which are absent in the free field Weak Hamiltonian. These correction terms are separately gauge invariant and prescription independent. The third term is from the two-loop anomalous dimensions. A numerical evaluation for $f = 4$, $\mu \approx m_c \sim 1.5$ GeV and $\Lambda_{\overline{MS}} = .25$ GeV leads^{*)} to $\Gamma/\Gamma_{LLA} = 1 - 0.19 + 0.25 + 0.16 = 1.22$. For $\Lambda_{\overline{MS}} = .50$ GeV one finds $\Gamma/\Gamma_{LLA} \approx 1.55$. In general, the first and third correction terms almost cancel (for all interesting Λ values) so that the dominant contribution is from the second term. Thus for realistic values of Λ we find a substantial correction in the direction of a further increase in the non-leptonic rate with respect to LLA. On the other hand, the semi-leptonic rate of a charm quark, in the same approximation where the final state quark masses are neglected is given by¹¹⁾:

*) Note, however, that Γ_{LLA} decreases somewhat when modifying the expression for the running coupling α_s by the inclusion of the b' non-leading term.

$$\Gamma_{SL} = \Gamma_{SL}^0 \left[1 + \frac{2\alpha_s}{3\pi} \left(\frac{25}{4} - \pi^2 \right) \right] \quad (15)$$

Finally, from Eqs. (14) and (15) we obtain, for the quark semi-leptonic branching ratio:

$$B_{SL} = \frac{\Gamma_{SL}}{2\Gamma_{SL} + (2L_+^2 + L_-^2)\gamma} \quad (16)$$

Thus, the non-leading corrections lead to a reduction in the semi-leptonic width and to an increase in the non-leptonic width. It is thus clear that the prediction for B_{SL} is considerably decreased with respect to LLA.

For charm, we conclude that, if B_{SL} for D^+ is confirmed to be larger than $12 \div 13\%$, it must necessarily be attributed to non-partonic effects. The best candidate of this sort is the interference between the two $\bar{d}$ anti-quarks in the final state of D^+ decay (the spectator and the c-quark decay product) as considered in Ref. 9). This effect tends to suppress the non-leptonic D^+ width by an amount which increases with $|\psi_D(0)|^2$ i.e., with the importance of the annihilation diagrams (in a qualitative way). Needless to say, the same formulae also hold for b quark decay and an experimental study of bottom particle decays would help considerably in separating perturbative effects from wave function effects.

ACKNOWLEDGEMENTS

G. Curci wishes to thank CERN for the warm hospitality afforded him when part of this work was being carried out.

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FIGURE CAPTIONS

Fig. 1 : The 28 independent two-loop diagrams for the anomalous dimension of the four fermion operators of dimension six. Replicas differing by up-down, left-right reflections of diagrams are not shown. "Penguin"-like diagrams are absent in the massless theory. They are irrelevant for transition involving four different flavours, as in $c \rightarrow s \bar{d} u$. The diagrams for the fermion wave function renormalization are not shown.

Fig. 2 : Diagrams for the inclusive non-leptonic rate of a heavy quark.

- a) Virtual diagrams: the W line (---) is explicitly shown because, as explained in the text, we are interested in the matrix elements of the effective Hamiltonian.
- b) Real diagrams: the full operators $O_{\pm}$ with the appropriate coefficients are at the four quark vertices.

fig. 1

fig. 2a

fig. 2b