Filters for three-dimensional limited-angle tomography

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Abstract. One approach to the three-dimensional, tomographic reconstruction of data from a planar dual-detector stationary positron camera necessitates frequency-space filtering of the image obtained from back-projection. It is therefore useful to have a closed-form expression for the appropriate filter, but the derivation is complicated by the fact that the camera measures only a limited angular range of projections. Nevertheless, suitable expressions can be obtained and are here derived for detectors of rectangular and circular cross-section.

1. Introduction

The intensity profile of the annihilation coincident photon pairs emerging from an unknown three-dimensional distribution of a positron-emitting radionuclide reflects the internal distribution of the radionuclide. A positron camera consisting of dual large-area position-sensitive detectors operating in coincidence may be used to image this profile, from which the original distribution of the positron emitter can be reconstructed, in three-dimensions, using mathematical techniques. Each coincident event from positron annihilation defines a line through the unknown distribution, so that by back-projecting each event (see Colsher 1980) a three-dimensional function $\hat{f}(x, y, z)$ is obtained, which is related to the unknown distribution $f(x, y, z)$ by the equation

$$\hat{f}(x, y, z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x', y', z') h(x - x', y - y', z - z') dx' dy' dz'. \quad (1)$$

The function $h(x - x', y - y', z - z')$ is the spatially invariant system response defined by the back-projection process for a point source. Equation (1) is transformed into an algebraic equation by application of the Fourier convolution theorem in three dimensions:

$$\hat{F}(k_x, k_y, k_z) = F(k_x, k_y, k_z) H(k_x, k_y, k_z) \quad (2)$$

where

$$\hat{F}(k_x, k_y, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \hat{f}(x, y, z) \exp[-2\pi i(xk_x + yk_y + zk_z)] dx dy dz \quad (3a)$$

$$F(k_x, k_y, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y, z) \exp[-2\pi i(xk_x + yk_y + zk_z)] dx dy dz \quad (3b)$$

$$H(k_x, k_y, k_z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} h(x, y, z) \exp[-2\pi i(xk_x + yk_y + zk_z)] dx dy dz. \quad (3c)$$

Equation (2) may be solved formally for $F(k_x, k_y, k_z)$:

$$F(k_x, k_y, k_z) = \hat{F}(k_x, k_y, k_z) / H(k_x, k_y, k_z) \quad (4)$$

assuming that $H(k_x, k_y, k_z) \neq 0$. The unknown distribution $f(x, y, z)$ is then obtained from equation (4) by an inverse three-dimensional Fourier transform. However, for the limited-angle positron camera the condition $H(k_x, k_y, k_z) \neq 0$ cannot be satisfied at all frequencies k_x, k_y, k_z within the bandwidth of the system, and for those frequencies where H is zero the function $F(k_x, k_y, k_z)$ is not determined by equation (4). This arises because angular limitations in real space become frequency limitations in Fourier space and certain frequencies cannot be measured by the limited-angle camera. The values of $F(k_x, k_y, k_z)$ at these unmeasured frequencies may be to some extent estimated from the values of $F(k_x, k_y, k_z)$ at the measured frequencies (Papoulis 1975), but a full discussion of the procedure is beyond the scope of this paper.

The function $h(x, y, z)$ can often be expressed in a simple analytic form such that equation (3c) may be evaluated directly to provide a closed-form expression for $H(k_x, k_y, k_z)$. Such an expression would eliminate the need for numerical integration in equation (3c), and offer greater flexibility to the reconstruction method.

For practical positron cameras that have only limited angular acceptance, $h(x, y, z)$ is zero outside a three-dimensional region Ω , and equation (3c) becomes instead

$$H(k_x, k_y, k_z) = \iiint_{\Omega} h(x, y, z) \exp[-2\pi i(xk_x + yk_y + zk_z)] dx dy dz. \quad (5)$$

Expressions for $h(x, y, z)$ based on the inverse square law for isotropic emission from a point source have been described by Chu and Tam (1977). Two expressions are considered here:

$$h(x, y, z) = 1/(2\pi r^2), \quad r^2 = x^2 + y^2 + z^2, \quad (x, y, z) \in \Omega \quad (6)$$

and

$$h(x, y, z) = |\cos \theta|/(2\pi r^2), \quad \cos \theta = z/r, \quad (x, y, z) \in \Omega. \quad (7)$$

The second form, equation (7), is relevant to the limited-angle planar positron camera, because the response function is defined in terms of the number of coincidence lines per unit area intersecting planes parallel to the detectors of the camera.

For an imaging system with full angular acceptance ($\Omega = R^3$), equation (5) has been evaluated using generalised functions (Tanaka 1979) and $h(x, y, z)$ given by equation (6). Also, again for $h(x, y, z)$ given by equation (6), equation (5) has been evaluated for the rotating dual-detector planar camera (Colsher 1980), taking advantage of the rotational symmetry. In the next section, equation (5) will be evaluated for the stationary planar camera, with $h(x, y, z)$ given by both equations (6) and (7).

2. Evaluation of $H(k_x, k_y, k_z)$

Practical planar positron cameras consist either of a pair of multiwire proportional chambers (Jeavons 1979, Jeavons *et al* 1980) with square or rectangular acceptance, or a pair of Anger cameras (Atkins *et al* 1977, Colsher 1980) with circular acceptance. In the former case, the region Ω will be restricted to a pyramid of rectangular cross-section (see figure 1), while in the latter, Ω will be restricted to a cone of circular cross-section. Let the maximum acceptance angles in the x - z and y - z planes be ψ_1 and ψ_2 ,

Figure 1. The acceptance cone of the point response function for a positron camera with rectangular detectors; ψ_1 and ψ_2 are, respectively, the maximum acceptance angles in the x - z and y - z planes.

respectively, and set

$$\begin{aligned}\alpha &= \tan \psi_1, & 0 < \psi_1 < \pi/2 \\ \beta &= \tan \psi_2, & 0 < \psi_2 < \pi/2.\end{aligned}$$

For $n = 0, 1$ and $\gamma > 0$ we define the region $\Omega_{n,\gamma}$ in three-dimensional space by

$$\Omega_{n,\gamma} = \{(x, y, z) : 0 \leq |y| \leq \beta|z|, 0 \leq (x^2 + ny^2)^{1/2} \leq \alpha|z|, |z| \leq \gamma\}$$

with the convention that $\alpha = \beta$ if $n = 1$. Clearly $n = 0$ corresponds to a pyramid of rectangular cross-section, and $n = 1$ to a cone of circular cross-section. The special case of a square section is found by setting $\alpha = \beta$ if $n = 0$.

For $m = 0, 1$ we define the function:

$$h_{m,n}(x, y, z) = \begin{cases} \frac{1}{2\pi} \frac{|z|^m}{(x^2 + y^2 + z^2)^{1+m/2}} & \text{for } (x, y, z) \in \Omega_{n,\gamma} \\ 0 & \text{otherwise.} \end{cases} \quad (8)$$

It is easy to see that $h_{0,n}(x, y, z)$ is identical to the function of equation (6), and $h_{1,n}(x, y, z)$ is identical to the function of equation (7).

Substituting equation (8) into equation (5) it is easy to verify that $H(k_x, k_y, k_z)$ is real and can be written

$$\begin{aligned}H_{m,n}(k_x, k_y, k_z) &= \frac{1}{\pi} \lim_{\gamma \rightarrow \infty} \int_0^\gamma \int_{-\beta z}^{+\beta z} \int_{-\sqrt{(\alpha^2 z^2 - ny^2)}}^{+\sqrt{(\alpha^2 z^2 - ny^2)}} \frac{z^m}{(x^2 + y^2 + z^2)^{1+m/2}} \\ &\quad \times \cos[2\pi(xk_x + yk_y + zk_z)] dx dy dz. \end{aligned} \quad (9)$$

It can also be shown that $H_{m,n}(k_x, k_y, k_z)$ is an even function in the variables k_x, k_y, k_z and thus it is sufficient to evaluate the integral (9) for $k_x, k_y, k_z > 0$. For $k_x = k_y = k_z = 0$ the integral does not exist.

Introduce ξ, η, ζ as integration variables by setting

$$x = \alpha\zeta\xi, \quad y = \beta\zeta\eta, \quad z = \zeta.$$

Equation (9) then becomes

$$\begin{aligned}H_{m,n}(k_x, k_y, k_z) &= \frac{\alpha\beta}{\pi} \lim_{\gamma \rightarrow \infty} \int_0^\gamma \int_{-1}^{+1} \int_{-\sqrt{(1-\varepsilon_n\eta^2)}}^{+\sqrt{(1-\varepsilon_n\eta^2)}} \frac{\cos[2\pi(\alpha k_x \xi + \beta k_y \eta + k_z) \zeta]}{(1 + \alpha^2 \xi^2 + \beta^2 \eta^2)^{1+m/2}} d\xi d\eta d\zeta \end{aligned} \quad (10)$$

where

$$\varepsilon_n = (\beta^2/\alpha^2)n. \quad (11)$$

It is assumed that $k_x^2 + k_y^2 + k_z^2 > 0$. Changing the order of integration in equation (10) and integrating with respect to ζ we obtain

$$H_{m,n}(k_x, k_y, k_z) = \frac{\alpha\beta}{2\pi^2} \lim_{\gamma \rightarrow \infty} \int_{-1}^{+1} \int_{-\sqrt{(1-\varepsilon_n\eta^2)}}^{+\sqrt{(1-\varepsilon_n\eta^2)}} \frac{\sin[2\pi(\alpha k_x\xi + \beta k_y\eta + k_z)\gamma] d\xi d\eta}{(\alpha k_x\xi + \beta k_y\eta + k_z)(1 + \alpha^2\xi^2 + \beta^2\eta^2)^{1+m/2}}. \quad (12)$$

For $k_x > 0$, let

$$t = \alpha k_x\xi + \beta k_y\eta + k_z \quad \text{i.e.} \quad \xi = (1/\alpha k_x)(t - \beta k_y\eta - k_z)$$

then substituting into equation (12):

$$H_{m,n}(k_x, k_y, k_z) = \frac{1}{\pi} \lim_{\gamma \rightarrow \infty} \int_{-1}^{+1} \int_{a_n}^{b_n} p_m(t, \eta; k_x, k_y, k_z) (\sin 2\pi\gamma t/t) dt d\eta \quad (13)$$

where

$$p_m(t, \eta; k_x, k_y, k_z) = (\beta/2\pi) \frac{k_x^{1+m}}{[k_x^2(1 + \beta^2\eta^2) + (t - \beta k_y\eta - k_z)^2]^{1+m/2}} \quad (14)$$

$$a_n = \beta k_y\eta + k_z - \alpha k_x(1 - \varepsilon_n\eta^2)^{1/2} \quad (15)$$

$$b_n = \beta k_y\eta + k_z + \alpha k_x(1 - \varepsilon_n\eta^2)^{1/2}. \quad (16)$$

From Apostol (1975; theorem 11.8) as $\gamma \rightarrow \infty$, equation (13) becomes

$$H_{m,n}(k_x, k_y, k_z) = \int_{-1}^{+1} p_m(0, \eta; k_x, k_y, k_z) [(\text{sign}(b_n) - \text{sign}(a_n))/2] d\eta. \quad (17)$$

Consider the indefinite integral

$$P_m(\eta; k_x, k_y, k_z) = \int p_m(0, \eta; k_x, k_y, k_z) d\eta. \quad (18)$$

From equations (14) and (18), and using a table of standard integrals (Gradshteyn and Ryzhik 1965),

$$P_0(\eta; k_x, k_y, k_z) = (1/2\pi k_3) \tan^{-1}[(\beta k_2^2\eta + k_y k_z)/k_x k_3] \quad (19)$$

$$P_1(\eta; k_x, k_y, k_z) = (k_2/2\pi k_3^2) \{(\beta k_2^2\eta + k_y k_z)/[k_x^2 k_3^2 + (\beta k_2^2\eta + k_y k_z)^2]^{1/2}\} \quad (20)$$

where

$$k_2^2 = k_x^2 + k_y^2, \quad (21)$$

$$k_3^2 = k_x^2 + k_y^2 + k_z^2 \quad (22)$$

are the squares of the two-dimensional and three-dimensional frequency space radii, respectively.

Referring to equation (17), let $n = 0$, and using equations (11), (15), and (16) we obtain

$$a_0 = \beta k_y\eta + k_z - \alpha k_x \quad (23)$$

$$b_0 = \beta k_y\eta + k_z + \alpha k_x. \quad (24)$$

Analysis of the straight lines (23) and (24) for the range $-1 \leq \eta \leq +1$ yields for the integral in equation (17) for $k_x > 0$:

$$H_{m,0}(k_x, k_y, k_z) = 0 \quad \text{for } k_x > 0, 0 \leq \alpha k_x + \beta k_y < k_z \quad (25)$$

$$H_{m,0}(k_x, k_y, k_z) = P_m((\alpha k_x - k_z)/\beta k_y; k_x, k_y, k_z) - P_m(-1; k_x, k_y, k_z) \quad \text{for } k_x > 0, k_y > 0, |\beta k_y - \alpha k_x| \leq k_z \leq \alpha k_x + \beta k_y \quad (26)$$

$$H_{m,0}(k_x, k_y, k_z) = P_m((\alpha k_x - k_z)/\beta k_y; k_x, k_y, k_z) - P_m(-(\alpha k_x + k_z)/\beta k_y; k_x, k_y, k_z) \quad \text{for } \beta k_y > \alpha k_x > 0, 0 \leq k_z \leq \beta k_y - \alpha k_x \quad (27)$$

$$H_{m,0}(k_x, k_y, k_z) = P_m(1; k_x, k_y, k_z) - P_m(-1; k_x, k_y, k_z) \quad \text{for } \alpha k_x > \beta k_y \geq 0, 0 \leq k_z \leq \alpha k_x - \beta k_y. \quad (28)$$

For $n = 1$, with $\beta = \alpha$, equations (11), (15) and (16) yield the curves

$$a_1 = \alpha k_y \eta + k_z - \alpha k_x (1 - \eta^2)^{1/2} \quad (29)$$

$$b_1 = \alpha k_y \eta + k_z + \alpha k_x (1 - \eta^2)^{1/2}. \quad (30)$$

Analysis of these curves for the range $-1 \leq \eta \leq 1$ gives for the integral in equation (17), again for $k_x > 0$,

$$H_{m,1}(k_x, k_y, k_z) = 0 \quad \text{for } k_x > 0, \alpha k_2 < k_z \quad (31)$$

$$H_{m,1}(k_x, k_y, k_z) = P_m([-k_y k_z + k_x(\alpha^2 k_2^2 - k_z^2)^{1/2}]/\alpha k_2^2; k_x, k_y, k_z) - P_m(-[k_y k_z + k_x(\alpha^2 k_2^2 - k_z^2)^{1/2}]/\alpha k_2^2; k_x, k_y, k_z) \quad \text{for } k_x > 0, 0 \leq k_z \leq \alpha k_2. \quad (32)$$

From equations (19), (20), (25) to (28), (31) and (32) the following explicit expressions are obtained together with the corresponding cases for $k_x = 0$:

$$H_{m,0}(k_x, k_y, k_z) = 0 \quad \text{for } k_x \geq 0, 0 \leq \alpha k_x + \beta k_y < k_z, m = 0, 1 \quad (33)$$

$$H_{0,0}(k_x, k_y, k_z) = \frac{1}{2\pi k_3} \left(\tan^{-1} \frac{\alpha k_2^2 - k_x k_z}{k_y k_3} + \tan^{-1} \frac{\beta k_2^2 - k_y k_z}{k_x k_3} \right) \quad \text{for } k_x > 0, k_y > 0, |\beta k_y - \alpha k_x| \leq k_z \leq \alpha k_x + \beta k_y \quad (34)$$

$$H_{0,0}(k_x, k_y, k_z) = \frac{1}{2\pi k_3} \left(\tan^{-1} \frac{\alpha k_2^2 - k_x k_z}{k_y k_3} + \tan^{-1} \frac{\alpha k_2^2 + k_x k_z}{k_y k_3} \right) \quad \text{for } \beta k_y > \alpha k_x \geq 0, 0 \leq k_z \leq \beta k_y - \alpha k_x \quad (35)$$

$$H_{0,0}(k_x, k_y, k_z) = \frac{1}{2\pi k_3} \left(\tan^{-1} \frac{\beta k_2^2 + k_y k_z}{k_x k_3} + \tan^{-1} \frac{\beta k_2^2 - k_y k_z}{k_x k_3} \right) \quad \text{for } \alpha k_x > \beta k_y \geq 0, 0 \leq k_z \leq \alpha k_x - \beta k_y \quad (36)$$

$$H_{1,0}(k_x, k_y, k_z) = \frac{k_2}{2\pi k_3^2} \left(\frac{\alpha k_2^2 - k_x k_z}{[k_y^2 k_3^2 + (\alpha k_2^2 - k_x k_z)^2]^{1/2}} + \frac{\beta k_2^2 - k_y k_z}{[k_x^2 k_3^2 + (\beta k_2^2 - k_y k_z)^2]^{1/2}} \right) \\ \text{for } k_x \geq 0, k_y > 0, |\beta k_y - \alpha k_x| \leq k_z \leq \alpha k_x + \beta k_y \quad (37)$$

$$H_{1,0}(k_x, k_y, k_z) = \frac{k_2}{2\pi k_3^2} \left(\frac{\alpha k_2^2 - k_x k_z}{[k_y^2 k_3^2 + (\alpha k_2^2 - k_x k_z)^2]^{1/2}} + \frac{\alpha k_2^2 + k_x k_z}{[k_y^2 k_3^2 + (\alpha k_2^2 + k_x k_z)^2]^{1/2}} \right) \\ \text{for } \beta k_y > \alpha k_x \geq 0, 0 \leq k_z \leq \beta k_y - \alpha k_x \quad (38)$$

$$H_{1,0}(k_x, k_y, k_z) = \frac{k_2}{2\pi k_3^2} \left(\frac{\beta k_2^2 + k_y k_z}{[k_x^2 k_3^2 + (\beta k_2^2 + k_y k_z)^2]^{1/2}} + \frac{\beta k_2^2 - k_y k_z}{[k_x^2 k_3^2 + (\beta k_2^2 - k_y k_z)^2]^{1/2}} \right) \\ \text{for } \alpha k_x > \beta k_y \geq 0, 0 \leq k_z \leq \alpha k_x - \beta k_y \quad (39)$$

$$H_{0,n}(k_x, k_y, k_z) = 0 \\ \text{for } k_x \geq 0, \alpha k_2 < k_z, n = 0, 1 \quad (40)$$

$$H_{0,1}(k_x, k_y, k_z) = \frac{1}{\pi k_3} \tan^{-1} \frac{(\alpha^2 k_2^2 - k_z^2)^{1/2}}{k_3} \\ \text{for } k_x \geq 0, 0 \leq k_z \leq \alpha k_2 \quad (41)$$

$$H_{1,1}(k_x, k_y, k_z) = \frac{(\alpha^2 k_2^2 - k_z^2)^{1/2}}{\pi(1 + \alpha^2)^{1/2} k_3^2} \\ \text{for } k_x \geq 0, 0 \leq k_z \leq \alpha k_2 \quad (42)$$

where k_2^2 and k_3^2 are given by equations (21) and (22).

This therefore completes the evaluation of a closed-form expression for the filter appropriate for use in equation (4) with stationary detectors of either rectangular or circular cross-section. Note that the results could easily be generalised to detectors of elliptic section.

It has been suggested (Chu and Tam 1977) that reconstructions can be improved by back-projecting with a weight proportional to some power of $\cos \theta$. This is incorporated into the above analysis by taking values of m other than 0 or 1, but it of course necessitates the evaluation of the corresponding indefinite integrals.

3. Behaviour of $H(k_x, k_y, k_z)$

Although the general expressions for $H_{m,n}(k_x, k_y, k_z)$ appear rather complicated, simplification is often possible in a particular case, and they are straightforward to implement on a computer. Thus, for the circular camera ($n = 1, \alpha = \beta$) and point response function given by equation (6) ($m = 0$), the appropriate filter is obtained from equations (40) and (41):

$$H(k_x, k_y, k_z) = 0 \quad \text{for } k_x \geq 0, \alpha k_2 < k_z \quad (43)$$

$$H(k_x, k_y, k_z) = (1/(\pi k_3)) \tan^{-1}[(\alpha^2 k_2^2 - k_z^2)^{1/2}/k_3] \quad \text{for } k_x \geq 0, 0 \leq k_z \leq \alpha k_2 \quad (44)$$

where k_2 and k_3 are given by equations (21) and (22) respectively, and $\alpha = \tan \psi$, where ψ is the acceptance angle of the circular system. From these equations it can be seen that $H(k_x, k_y, k_z)$ is defined at all frequencies within the bandwidth of the system, except for a circular cone of semi-angle $\pi/2 - \psi$ in which $H(k_x, k_y, k_z)$ is zero (equation 43). At these frequencies the function $F(k_x, k_y, k_z)$ is undetermined.

As the acceptance angle of the system increases ($\psi \rightarrow \pi/2$), the semi-angle of the cone of unmeasured frequencies decreases, i.e. fewer frequencies remain undetermined. In the limit of full angular acceptance ($\psi = \pi/2$), all frequencies are measured and $H(k_x, k_y, k_z)$ is defined everywhere by equation (44), which may be reduced to

$$H(k_x, k_y, k_z) = 1/(2k_3), \quad \text{for } k_x \geq 0, k_z \geq 0. \quad (45)$$

This result is identical to that obtained (Tanaka 1979) using generalised functions for the three-dimensional Fourier transform without angular restrictions (i.e. $\Omega = R^3$) of the point response function given by equation (6). It is the equivalent in three dimensions of the familiar ramp filter used in x-ray computer tomography to reconstruct two-dimensional sections.

Similar effects are observed for detectors of square cross-section, with a pyramid of unmeasured frequencies given by equation (33). In this case the pyramid has a square cross-section and a semi-angle of $\pi/2 - \psi$, but with respect to the real-space acceptance pyramid (figure 1) the pyramid in frequency space is rotated about the k_z axis through an angle of $\pi/4$. As the acceptance angle increases, the pyramid of unmeasured frequencies decreases until, in the limit, equations (34), (35) and (36) reduce to equation (45), which defines $H(k_x, k_y, k_z)$ at all frequencies.

In summary, after transformation from Cartesian (k_x, k_y, k_z) to polar (k_3, Θ, Φ) coordinates, $H(k_3, \Theta, \Phi)$ is seen to be of the form

$$H(k_3, \Theta, \Phi) = (1/(2k_3))G(\Theta, \Phi; \psi) \quad (46)$$

where $G(\Theta, \Phi; \psi)$ is a function that depends only on the frequency-space angles and the angular acceptance of the system. The filter therefore behaves as a modified ramp filter in three dimensions, the angle-dependent modifying function arising from the limited angular acceptance of the camera. Clearly $G(\Theta, \Phi; \psi) \rightarrow 1$ as $\psi \rightarrow \pi/2$.

The behaviour of the filters for the point response function given by equation (7) is slightly more complicated, although overall features are similar. It should be noted however that this response function is strictly appropriate only for limited-angle systems, because the function vanishes for $\theta \rightarrow \pi/2$. For cameras of large angular acceptance ($\psi \approx \pi/2$), the point response function given by equation (6) should be used.

4. Conclusions

In this paper, the derivation of closed-form expressions for some filters suitable for limited-angle tomographic reconstruction has been described. Closed-form expressions have the advantage of flexibility, and eliminate tedious numerical integration, with its associated problems of precision and stability. The filters derived above are easily implemented on a computer and may be evaluated rapidly, and as required, during filtering, removing the need for pre-calculation and storage on some external device.

Reconstructed images from data obtained with a square-section proportional-chamber positron camera (Jeavons 1979), and using the filter described above, can be found elsewhere (Jeavons *et al* 1980, Townsend *et al* 1980).

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Résumé

Filtres utilisés en tomographie tri-dimensionnelle, à angle limité.

Une des manières de résoudre le problème de la reconstruction tri-dimensionnelle des données tomographiques, obtenues à partir d'une caméra à positrons formée de deux détecteurs plans stationnaires, nécessite un filtrage dans le domaine des fréquences spatiales de l'image obtenue par rétro-projection. Il est donc nécessaire d'avoir une forme analytique pour le filtre approprié mais l'élaboration est compliquée car la caméra ne mesure qu'un domaine angulaire limité de projections. Cependant, des formules valables peuvent être obtenues pour des détecteurs de section rectangulaire ou circulaire et sont développées dans cet article.

Zusammenfassung

Filter für dreidimensionale Tomographie bei begrenzten Winkeln.

Bei einer bestimmten Art der tomographischen Rekonstruktion von Daten aus einer feststehenden Positronkamera mit zwei ebenen Detektoren ist es erforderlich, das aus der Rückprojektion gewonnene Bild im Frequenzraum zu filtern. Es ist daher nützlich, einen geschlossenen Ausdruck für das geeignete Filter zu haben. Da die Kamera Projektionen nur in einem begrenzten Winkelbereich misst, ist die Herleitung des Filters aufwendig. Für rechteckige und kreisförmige Detektoren werden in dieser Arbeit entsprechende Filter hergeleitet.

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